SOLVED EXAMPLES

Ex.1 If $f(x) = \begin{cases} x^2 + 2, & x \ge 1 \\ 2x + 1, & x < 1 \end{cases}$, then $\text{Lim}_{x \to 1}$ f(x) equals -

(A) 1 (B) 2

(C) 3 (D) Does not exist

Sol. $\lim_{x \to 1-0} f(x) \lim_{h \to 0} = [2(1-h)+1] = 3$

$\lim_{x \to 1+0} f(x) = \lim_{h \to 0} [(1+h)^2 + 2] = 3$

$\therefore$ LHL = RHL, so $\lim_{x \to 1} f(x) = 3$. **Ans.[C]**

Ex.2 $\lim_{x \to 0} \frac{1 + e^{-1/x}}{1 - e^{-1/x}}$ is equal to -

(A) 1 (B) –1

(C) 0 (D) Does not exist

Sol. LHL = $\lim_{h \to 0} \frac{1 + e^{1/h}}{1 - e^{1/h}}$

$= \lim_{h \to 0} = \frac{e^{-1/h} + 1}{e^{-1/h} - 1} - 1$

RHL = $\lim_{h \to 0} = \frac{1 + e^{-1/h}}{1 - e^{-1/h}} = \frac{1 + 0}{1 - 0} = 1$

LHL ≠ RHL, so given limit does not exist. **Ans.[D]**

Ex.3 $\lim_{x \to \infty} \frac{2x^2 + 3x}{3x^2 + 4}$ equals -

(A) 1/2 (B) 2/3

(C) 3/4 (D) 0

Sol. $= \lim_{x \to \infty} \frac{2 + (3/x)}{3 + (4/x^2)} = \frac{2}{3}$ **Ans.[B]**

Ex.4 $\lim_{x \to \infty} \left(\sqrt{x^2 + 1} - x\right)$ equals -

(A) –1 (B) 0

(C) 1 (D) None of these

Sol. Limit = $\lim_{x \to \infty} x \left[\left(1 + \frac{1}{x^2}\right)^{1/2} - 1\right]$

$= \lim_{x \to \infty} x \left[1 + \frac{1}{2x^2} - \frac{1}{8x^4} + ... - 1\right]$

$= \lim_{x \to \infty} \left[\frac{1}{2x} - \frac{1}{8x^3} + ...\right] = 0.$ **Ans.[B]**

Ex.5 $\lim_{x \to -1} \left(\frac{x^2 - 1}{x^2 + 3x + 2}\right)$ is equal to-

(A) –2 (B) 1/2

(C) 0 (D) 1

Sol. Limit = $\lim_{x \to -1} \frac{(x - 1)(x + 1)}{(x + 2)(x + 1)} = \frac{-1 - 1}{-1 + 2} = -2$

Ans.[A]

Ex.6 $\lim_{x \to a} \left[\frac{x^2 - (a+1)x + a}{x^3 - a^3}\right]$ is equal to -

(A) $\frac{a - 1}{3a^2}$ (B) a – 1

(C) a (D) 0

Sol. $\lim_{x \to a} \left[\frac{x^2 - (a+1)x + a}{x^3 - a^3}\right] \left(\frac{0}{0} \text{form}\right)$

$= \lim_{x \to a} = \frac{2x - a - 1}{3x^2} = \frac{a - 1}{3a^2}$

(D.L.Hospital rule) **Ans.[A]**

Ex.7 $\lim_{x \to 3} \frac{x - 3}{|x - 3|}$, is equal to -

(A) 1 (B) –1

(C) 0 (D) Does not exist

Sol. LHL = $\lim_{h \to 0} \frac{(3 - h) - 3}{|(3 - h) - 3|}$

$= \lim_{h \to 0} \frac{-h}{|-h|} = -1$

RHL = $\lim_{h \to 0} \frac{(3 + h) - 3}{|(3 + h) - 3|}$

$= \lim_{h \to 0} \frac{h}{|h|} = 1$

LHL ≠ RHL, so limit does not exist. **Ans.[D]**

Ex.8 If $f(x) = \frac{x + |x|}{x}$, then $\lim_{x \to 0} f(x)$ equals-

(A) 2 (B) 0

(C) 1 (D) Does not exist

Sol. LHL = $\lim_{h \to 0} \frac{-h + |h|}{-h} = \lim_{h \to 0} (0) = 0$

RHL = $\lim_{h \to 0} \frac{h + |h|}{h} = 2$

LHL ≠ RHL ⇒ does not exist. **Ans.[D]**

Ex.9 $\lim_{x \to 0} \frac{x}{\sqrt{1+x} - \sqrt{1-x}}$ is equal to -

(A) 1/2 (B) 2

(C) 1 (D) 0

Sol. Limit = $\lim_{x \to 0} \frac{x\left(\sqrt{1+x} + \sqrt{1-x}\right)}{(1+x) - (1-x)}$

$= \lim_{x \to 0} \frac{\sqrt{1+x} + \sqrt{1-x}}{2} = 1.$ **Ans.[C]**

Ex.10 $\lim_{x \to 0} \frac{xe^x - \log(1+x)}{x^2}$ equals -

(A) 1/2 (B) 1

(C) 3/2 (D) 2

Sol. $\lim_{x \to 0} \frac{x\left(1 + x + \frac{x^2}{2!} + ...\right) - \left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ...\right)}{x^2}$

$= \lim_{x \to 0} \left(\frac{3}{2} + \frac{1}{6}x + ...\right) = 3/2$ **Ans.[C]**

Ex.11 The value of $\lim_{x \to 0} \left[\frac{1}{x^2} - \frac{1}{\sin^2 x}\right]$ is -

(A) –1/2 (B) 1/2

(C) –1/3 (D) 1/3

Sol. Limit = $\lim_{x \to 0} \frac{\sin^2 x - x^2}{x^2 . \sin^2 x}$

$= \lim_{x \to 0} \frac{\left(x - \frac{x^3}{3!} + ...\right)^2 - x^2}{x^2\left(x - \frac{x^3}{3!} + ...\right)^2}$

$= \lim_{x \to 0} \frac{x^2 - \frac{1}{3}x^4 + ... - x^2}{x^4\left(1 - \frac{x^2}{3!} + ...\right)^2} = -1/3$ **Ans.[C]**

Ex.12 $\lim_{x \to 0} \frac{\tan 2x - x}{3x - \sin x}$ equals-

(A) 2/3 (B) 1/3

(C) 1/2 (D) 0

Sol. The given limit is in the form , therefore applying L 'Hospital's rule, we get

Limit = $\lim_{x \to 0} \frac{2\sec^2 2x - 1}{3 - \cos x} = \frac{2-1}{3-1} = \frac{1}{2}$ **Ans.[C]**

Ex.13 $\lim_{x \to 0} \frac{\sin x + \log(1-x)}{x^2}$ is equal to -

(A) 0 (B) 1/2

(C) –1/2 (D) Does not exist

Sol. It is in 0/0 form, so using Hospital rule, we have

Limit = $\lim_{x \to 0} \frac{\cos x - \frac{1}{1-x}}{2x}$ (0/0 form)

$= \lim_{x \to 0} \frac{-\sin x - \frac{1}{(1-x)^2}}{2} = -1/2$ **Ans.[C]**

Ex.14 $\lim_{x \to \infty} \frac{\sin x}{x}$ equals -

(A) 1 (B) 0

(C) ∞ (D) Does not exist

Sol. $\lim_{x \to \infty} \frac{\sin x}{x}$

$= \lim_{x \to \infty}$ (a finite number between –1 and 1)/∞

= 0 **Ans.[B]**

Ex.15 $\lim_{x \to 0} \left(\frac{\tan x}{x}\right)^{1/x^2}$ is equal to -

(A) e^3 (B) $e^{1/3}$ (C) 1 1(D) e

Sol. Limit = $\lim_{x \to 0} \left(\frac{x + x^3/3 + ...}{x}\right)^{1/x^2}$

$= \lim_{x \to 0} \left(1 + \frac{x^3}{3}\right)^{1/x^2}$

[∵ x → 0, so neglecting higher powers of x]

$= \lim_{x \to 0} \left[\left(1 + \frac{x^2}{3}\right)^{3/x^2}\right]^{1/3} = e^{1/3}$ **Ans.[B]**

Ex.16 If $f(x) = \sqrt{\dfrac{x - \sin x}{x + \cos^2 x}}$, then $\lim_{x\to\infty} f(x)$ equals -

(A) 0 (B) ∞

(C) 1 (D) None of these

Sol. $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \sqrt{\dfrac{\{1-(\sin x/x)\}}{\{1+(\cos^2 x/x)\}}}$

$= \sqrt{\dfrac{1-0}{1+0}} = 1.$ **Ans.[C]**

Ex.17 If $G(x) = -\sqrt{25 - x^2}$,

then $\lim_{x\to 1} \dfrac{G(x) - G(1)}{x - 1}$ equals -

(A) 1/24 (B) 1/5

(C) $-\sqrt{24}$ (D) None of these

Sol. Here $G(1) = -\sqrt{25 - x^2} = -\sqrt{24}$

$\therefore$ Given limit

$= \lim_{x\to 1} \dfrac{-\sqrt{25-x^2} + \sqrt{24}}{x-1}$ $\left(\dfrac{0}{0}\text{ form}\right)$

$= \lim_{x\to 1} \dfrac{x}{\sqrt{25-x^2}}$ (By L Hospital ruel)

$= \dfrac{1}{\sqrt{24}}$ **Ans.[D]**

Ex.18 If f(9) = 9 and f'(9) = 4, then $\lim_{x\to 9} \dfrac{\sqrt{f(x)} - 3}{\sqrt{x} - 3}$ is equal to -

(A) 1 (B) 3 (C) 4 (D) 9

Sol. Given limit is in 0/0 form, so using Hospital rule, we get

$$\text{Limit} = \lim_{x\to 9} \dfrac{\dfrac{1}{2\sqrt{f(x)}}.f(x)}{\dfrac{1}{2\sqrt{x}}}$$

$= \dfrac{f(9).\sqrt{9}}{\sqrt{f(9)}} = \dfrac{4.3}{3} = 4$ **Ans.[C]**

Ex.19 $\lim_{x\to\infty} \left(\dfrac{x+2}{x+1}\right)^{x+3}$ is equal to -

(A) 1 (B) e (C) e^2 (D) e^3

Sol. Limit $= \lim_{x\to\infty} \left(\dfrac{x+2}{x+1}\right)^x . \left(\dfrac{x+2}{x+1}\right)^3$.

$= \lim_{x\to\infty} \left(\dfrac{1+2/x}{1+1/x}\right)^x . \left(\dfrac{1+2/x}{1+1/x}\right)^3$

$= \dfrac{\lim_{x\to\infty} [(1+2/x)^{x/2}]^2}{\lim_{x\to\infty} (1+1/x)^x} . \lim_{x\to\infty} \left(\dfrac{1+2/x}{1+1/x}\right)^3$

$= \dfrac{e^2}{e}.1 = e$ **Ans.[B]**

Ex.20 The value of $\lim_{x\to\infty} \dfrac{x(\log x)^3}{1 + x + x^2}$ is -

(A) 0 (B) 1 (C) –1 (4) 1/2

Sol. $\lim_{x\to\infty} \dfrac{x(\log x)^3}{1+x+x^2}$ $\left(\dfrac{\infty}{\infty}\text{ form}\right)$

$= \lim_{x\to\infty} \dfrac{(\log x)^3 + 3(\log x)^2}{1 + 2x}$ $\left(\dfrac{\infty}{\infty}\text{ form}\right)$

$= \lim_{x\to\infty} \dfrac{3(\log x)^2.\dfrac{1}{x} + 6(\log x).\dfrac{1}{x}}{2}$

$=. \lim_{x\to\infty} \dfrac{3(\log x)^2 + 6\log x}{2x}$ $\left(\dfrac{\infty}{\infty}\text{ form}\right)$

$= \lim_{x\to\infty} \dfrac{6(\log x)\dfrac{1}{x} + \dfrac{6}{x}}{2}$

$= 3\lim_{x\to\infty} \dfrac{\log x + 1}{x}$ $\left(\dfrac{\infty}{\infty}\text{ form}\right)$

$= 3\lim_{x\to\infty} \dfrac{(1/x)}{1} = 0.$ **Ans.[A]**

Ex.21 $\lim_{x\to 0} \dfrac{\sin x^{o}}{x}$ is equal to -

(A) 1 (B) π (C) x (D) $\pi/180$

Sol. Limit $= \lim_{x\to 0} \dfrac{\sin(\pi/180)x}{x}$

$= \lim_{x\to 0} \dfrac{(\pi/180)\cos(\pi/180)x}{1}$

$= \dfrac{\pi}{180}$ **Ans.[D]**

Ex.22 If $f(x) = \begin{cases} x-1, & x<0 \\ 1/4, & x=0 \\ x^2, & x>0 \end{cases}$ then $\lim_{x\to 0} f(x)$ equals -

(A) 0 (B) 1

(C) –1 (D) Does not exist

Sol. Here $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x^2 = 0$

and $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (x-1) = -1$

$\because \lim_{x\to 0^+} f(x) \neq \lim_{x\to 0^-} f(x)$

$\therefore \lim_{x\to 0} f(x)$ does not exist. **Ans.[D]**

Ex.23 $\lim_{x\to 0} \frac{2^x - 1}{\sqrt{(1+x)} - 1}$ equals -

(A) log 2 (B) 2 log 2

(C) 1/2 log 2 (D) 2

Sol. Given Limit

$= \lim_{x\to 0} \frac{2^x - 1}{\sqrt{(1+x)} - 1} \times \frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}$

$= \lim_{x\to 0} \frac{2^x - 1}{x} \lim_{x\to 0} (\sqrt{1+x}+1)$

$= 2. \lim_{x\to 0} \frac{2^x \log 2}{1} = 2. \log 2$ **Ans.[B]**

Ex.24 If a,b,c,d are positive real numbers, then $\lim_{n\to\infty}\left(1+\frac{1}{a+bn}\right)^{c+dn}$ is equal to -

(A) $e^{d/b}$ (B) $e^{c/a}$

(C) $e^{(c+d)/(a+b)}$ (D) e

Sol. $\lim_{x\to\infty}\left(1+\frac{1}{a+bn}\right)^{c+dn}$ (1^∞ form)

$= e^{\lim_{x\to\infty}\left(1+\frac{1}{a+bn}-1\right) \times (c+dn)}$

$= e^{\lim_{x\to\infty} \frac{c+dn}{a+bn}}$

$= e^{\lim_{x\to\infty} \frac{\frac{c}{n}+d}{a/n+b}} = e^{d/b}$ **Ans.[A]**

Ex.25 $\lim_{x\to\infty}\left(\frac{\pi}{2} - \tan^{-1} x\right)^{1/x}$ equals -

(A) 0 (B) 1 (C) ∞ (D) –1

Sol. Let $y = \lim_{x\to\infty}\left(\frac{\pi}{2} - \tan^{-1} x\right)^{1/x}$

$= \lim_{x\to\infty} (\cot^{-1}x)^{1/x}$

$\therefore \log y = \lim_{x\to\infty} \frac{\log \cot^{-1} x}{x}$ $\left(\frac{\infty}{\infty}\text{ form}\right)$

$= \lim_{x\to\infty} -\frac{1}{(1+x^2)\cos^{-1} x}$ ($0 \times \infty$ form)

$= -\lim_{x\to\infty} \frac{(1+x^2)^{-1}}{\cot^{-1} x}$ $\left(\frac{0}{0}\text{ form}\right)$

$= -\lim_{x\to\infty} \frac{\frac{-2x}{(1+x^2)^2}}{\frac{-1}{1+x^2}}$ $= -2 \lim_{x\to\infty} \frac{x}{1+x^2}$

$= -2 \lim_{x\to\infty} \frac{1}{2x} = 0$ $\therefore y = e^0 = 1.$**Ans.[B]**

Ex.26 $\lim_{x\to 0} \frac{x(2^x - 1)}{1-\cos x}$ equals -

(1) 0 (2) log 2

(3) 2 log 2 (4) None of these

Sol. The given limit $= \lim_{x\to 0} \frac{2^x - 1}{x} . \frac{x^2}{1-\cos x}$

$= \lim_{x\to 0} \frac{2^x-1}{x} \lim_{x\to 0} \frac{x^2}{2\sin^2 \frac{x}{2}}$

$= \log x . 2 \lim_{x\to 0} \left(\frac{x/2}{\sin(x/2)}\right)^2$

= 2 log 2. **Ans.[C]**

Ex.27 The value of $\lim_{x\to 0}\left[\frac{a}{x} - \cot\frac{x}{a}\right]$ is -

(A) 0 (B) 1 (C) a (D) a/3

Sol. Given Limit $= \lim_{x\to 0}\left[\frac{a}{x} - \frac{\cos(x/a)}{\sin(x/a)}\right]$

$= \lim_{x\to 0}\left[\frac{a\sin(x/a) - x\cos(x/a)}{x\sin(x/a)}\right]$

$= a \lim_{x\to 0}\left[\frac{a\sin(x/a) - x\cos(x/a)}{x^2}\right] \times \frac{(x/a)}{\sin(x/a)}$

$= a \lim_{x\to 0}\left[\frac{a\sin(x/a) - x\cos(x/a)}{x^2}\right] \left(\frac{0}{0}\text{ form}\right)$

$= a \lim_{x\to 0}\left[\frac{\cos(x/a) - \cos(x/a) + (x/a)\sin(x/a)}{2x}\right]$

= 0 **Ans.[A]**

LEVEL-1

Question based on **Existence of limit**

Q.1 If $f(x) = \begin{cases} 4x, & x < 0 \\ 1, & x = 0 \\ 3x^2, & x > 0 \end{cases}$, then $\lim_{x \to 0} f(x)$ equals-

(A) 0 (B) 1
(C) 3 (D) Does not exist

Q.2 If $f(x) = \begin{cases} -1, & x < -1 \\ x^3, & -1 \le x \le 1 \\ 1 - x, & 1 < x < 2 \\ 3 - x^2, & x > 2 \end{cases}$ then-

(A) $f(x) = 1$ (B) $\lim_{x \to 1^+} f(x) = 1$
(C) $\lim_{x \to 2^+} f(x) = -1$ (D) $\lim_{x \to 2^-} f(x) = 0$

Q.3 $\lim_{x \to \infty} \sin x$ equals-

(A) 1 (B) 0
(C) ∞ (D) Does not exist

Q.4 $\lim_{x \to 0} \sin \frac{1}{x}$ equals-

(A) 0 (B) 1
(C) ∞ (D) Does not exist

Q.5 $\lim_{x \to 0} x \sin \frac{1}{x}$ equals-

(A) 1 (B) 0
(C) ∞ (D) None of these

Q.6 Let $f(x) = x\,(-1)^{[1/x]}$, $x \ne 0$ where [ ] represent greatest integer function then $\lim_{x \to 0} f(x)$ is -

(A) 2 (B) 0
(C) –1 (D) Does not exist

Q.7 Which of the following limits does not exist-

(A) $\lim_{x \to 0} \frac{|x|}{x}$ (B) $\lim_{x \to 0} \{x + |x|\}$
(C) $\lim_{x \to 0} |x|$ (D) $\lim_{x \to 0} \{x - |x|\}$

Q.8 If $f(x) = \begin{cases} x, & x < 0 \\ 1, & x = 0 \\ x^2, & x > 0 \end{cases}$ then, $\lim_{x \to 0} f(x)$ -

(A) 0 (B) 1
(C) 2 (D) does not exist

Q.9 $\lim_{x \to 3/2} x - [x]$ equals -

(A) 0 (B) 1 (C) 1/2 (D) 3/2

Q.10 Which of the following limits exists-

(A) $\lim_{x \to 0} x\,|x|$ (B) $\lim_{x \to 1/4} [x]$
(C) $\lim_{x \to 0} x \sin 1/x$ (D) All the above

Q.11 $\lim_{x \to a} \frac{1}{(x-a)^{2n-1}}$ $(n \in N)$ equals-

(A) ∞ (B) $-\infty$
(C) 0 (D) Does not exist

Q.12 If $f(x) = \begin{cases} \dfrac{e^{1/x} + e^{-1/x}}{e^{1/x} - e^{-1/x}}, & x \ne 0 \\ 0, & x = 0 \end{cases}$ then

$\lim_{x \to 0} f(x)$ equals-

(A) 1 (B) 2
(C) 3 (D) Does not exist

Q.13 If f is a odd function and $\lim_{x \to 0} f(x)$ exists then $\lim_{x \to 0} f(x)$ equals-

(A) 0 (B) 1
(C) –1 (D) None of these

Q.14 If [x] = greatest integer $\le$ x, then $\lim_{x \to 2} (-1)^{[x]}$ is equal to -

(A) 1 (B) –1
(C) ±1 (D) None of these

Question based on $x \to \infty$

Q.15 $\lim_{n \to \infty} \frac{n^2 + n + 1}{1 + 3 + 5 + \ldots\ldots + (2n-1)}$ equals-

(A) 1 (B) 4/3
(C) 3/4 (D) ∞

Q.16 The value of $\lim_{x\to\infty} \frac{2x^3 - 4x + 7}{3x^3 + 5x^2 - 4}$ is-

(A) 2/3 (B) –7/4

(C) –4/5 (D) ∞

Q.17 The value of $\lim_{n\to\infty} \frac{\sqrt{3n^2-1} - \sqrt{2n^2-1}}{4n+3}$ is-

(A) $\frac{1}{4}(\sqrt{3} - \sqrt{2})$ (B) $\frac{1}{4}(\sqrt{3} + \sqrt{2})$

(C) $(\sqrt{3} - \sqrt{2})$ (D) None of these

Q.18 $\lim_{x\to\infty} \frac{(2x-3)(3x-4)}{(4x-5)(5x-6)} =$

(A) 0 (B) 1/10

(C) 1/5 (D) 3/10

Q.19 $\lim_{x\to\infty} \frac{\sin 5x}{x}$ equals-

(A) 5 (B) 1/5 (C) 0 (D) 1

Q.20 The value of $\lim_{n\to\infty} \frac{\frac{1}{2} + 1 + \frac{3}{2} + \ldots \frac{n}{2}}{25n^2 + n + 3}$ is-

(A) 0 (B) 1/100

(C) ∞ (D) None of these

Q.21 $\lim_{n\to\infty} \frac{1 + 5 + 5^2 + \ldots.. + 5^{n-1}}{1 - 25^n}$ equals-

(A) 0 (B) –1 (C) 1 (D) ∞

Q.22 $\lim_{n\to\infty} (4^n + 5^n)^{1/n}$ equals-

(A) 4 (B) 5

(C) e (D) None of these

Q.23 $\lim_{x\to\infty} \frac{x - \sin x}{x + \cos^2 x}$ equals-

(A) 0 (B) 1

(C) ∞ (D) None of these

Q.24 $\lim_{n\to\infty} \left[\frac{1}{1-n^2} + \frac{2}{1-n^2} + \ldots.. + \frac{n}{1-n^2}\right]$ is equal to-

(A) 1 (B) 2 (C) –1/2 (D) 1/2

Q.25 $\lim_{x\to 0^+} \frac{xe^{1/x}}{1 + e^{1/x}}$ equals-

(A) 0 (B) 1

(C) ∞ (D) None of these

Q.26 $\lim_{n\to\infty} \frac{(n+2)! + (n+3)!}{(n+4)!}$ equals-

(A) 0 (B) ∞

(C) 1 (D) None of these

Q.27 $\lim_{n\to\infty} \left(\frac{1}{n^2} + \frac{2}{n^2} + \frac{3}{n^2} + \ldots.. + \frac{n}{n^2}\right)$ equals-

(A) 0 (B) 1/2

(C) 2n (D) 2^n

Q.28 The value of

$\lim_{n\to\infty} \left(\frac{1}{1-n^4} + \frac{8}{1-n^4} + \ldots.. + \frac{n^3}{1-n^4}\right)$ is -

(A) 1 (B) 0

(C) –1/4 (D) None of these

Q.29 $\lim_{n\to\infty} \left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ldots. + \frac{1}{3^n}\right)$ equals-

(A) 1/2 (B) 1/3 (C) 1 (D) 0

Question based on **Factorisation method**

Q.30 The value of $\lim_{x\to\pi/2} \frac{1 - \sin^3 x}{\cos^2 x}$ is-

(A) $-\frac{3}{2}$ (B) $\frac{3}{2}$

(C) 1 (D) 0

Q.31 The value of $\lim_{x\to 3} \left(\frac{x^4 - 81}{x - 3}\right)$ is -

(A) –27 (B) 108

(C) undefined (D) None of these

Q.32 $\lim_{x\to 1} \frac{x - 1}{2x^2 - 7x + 5}$ equals-

(A) 1/3 (B) –1/3

(C) 1/2 (D) – 1/2

Q.33 $\lim_{x\to 1} \frac{1 - x^{-1/3}}{1 - x^{-2/3}}$ equals-

(A) 1/3 (B) 1/2

(C) 2/3 (D) – 2/3

Question based on **Rationalisation method**

Q.34 $\lim_{x\to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x^2} - \sqrt{1-x^2}}$ equals-

(A) 1 (B) 1/2

(C) 0 (D) Does not exist

Q.35 $\lim_{x\to 3} \frac{x-3}{\sqrt{x-2} - \sqrt{4-x}}$ equals-

(A) 0 (B) 3/2

(C) 1/4 (D) None of these

Q.36 $\lim_{x\to \infty} \left[\sqrt{(a^2x^2 + ax + 1)} - \sqrt{(a^2x^2 + 1)}\right]$

equals-

(A) 1 (B) 2

(C) 0 (D) 1/2

Q.37 $\lim_{x\to 0} \frac{\sin 4x}{1 - \sqrt{(1-x)}}$ equals-

(A) 4 (B) 8

(C) 10 (D) None of these

Q.38 $\lim_{x\to 4} \frac{3 - \sqrt{5+x}}{1 - \sqrt{5-x}}$ equals-

(A) 0 (B) 1 (C) 1/3 (D) –1/3

Q.39 The value of $\lim_{x\to a} \frac{\sqrt{x-b} - \sqrt{a-b}}{x^2 - a^2}$ (a > b) is -

(A) $\frac{1}{4a}$ (B) $\frac{1}{a\sqrt{a-b}}$

(C) $\frac{1}{2a\sqrt{a-b}}$ (D) $\frac{1}{4a\sqrt{a-b}}$

Q.40 The value of $\lim_{x\to \infty} x^{3/2}\left(\sqrt{x^3+1} - \sqrt{x^3-1}\right)$ is-

(A) 1 (B) –1

(C) 0 (D) None of these

Q.41 $\lim_{x\to 0} \frac{\sqrt{2} - \sqrt{1+\cos x}}{\sin^2 x}$ equals-

(A) $\sqrt{2}$ (B) $\frac{\sqrt{2}}{8}$

(C) 0 (D) None of these

Q.42 $\lim_{x\to 2a} \frac{\sqrt{x-2a} + \sqrt{x} - \sqrt{2a}}{\sqrt{x^2 - 4a^2}}$ equals-

(A) $\frac{1}{\sqrt{a}}$ (B) $\frac{1}{2\sqrt{a}}$

(C) $\frac{1}{3\sqrt{a}}$ (D) $\frac{1}{4\sqrt{a}}$

Question based on **Expansion method**

Q.43 $\lim_{x\to 0} \frac{e^{\alpha x} - e^{\beta x}}{\sin \alpha x - \sin \beta x}$ equals-

(A) 0 (B) $\alpha - \beta$

(C) –1 (D) 1

Q.44 $\lim_{x\to 0} \frac{x\cos x - \sin x}{x^2 \cos x}$ equals-

(A) 1/3 (B) 0

(C) 3 (D) –3

Q.45 $\lim_{x\to 0} \frac{1 + \sin x - \cos x + \log(1-x)}{x^3}$ equals-

(A) 1/2 (B) – 1/2

(C) 0 (D) None of these

Q.46 $\lim_{x\to 0} \frac{\sin^{-1} x - \tan^{-1} x}{x^3}$ equals-

(A) 1 (B) –1

(C) 1/2 (D) –3/2

Q.47 $\lim_{x\to 0} \frac{e^x + e^{-x} - 2\cos x}{x \sin x}$ equals-

(A) 1 (B) 2 (C) –1 (D) –2

Q.48 $\lim_{x\to 0} \frac{x.2^x - x}{1 - \cos x}$ is equal to -

(A) log 2 (B) log 4

(C) 0 (D) None of these

Q.49 $\lim_{x\to 0} \frac{\sqrt{x}\tan x}{(e^x - 1)^{3/2}}$ equals-

(A) 0 (B) 1 (C) 1/2 (D) 2

Question based on **L' Hospital rule**

Q.50 $\lim_{x\to 0}$ x log x equals-

(A) e (B) 1/e (C) 1 (D) 0

Q.51 $\lim_{x\to a} \frac{x^m - a^m}{x^n - a^n}$ equals-

(A) m/n (B) 0

(C) $\frac{m}{n}a^{m-n}$ (D) $\frac{n}{m}a^{n-m}$

Q.52 $\lim_{x\to \pi/2}$ tan x log sin x equals-

(A) 0 (B) 1

(C) –1 (D) None of these

Q.53 $\lim_{n\to\infty} n[a^{1/n}-1]$ equals-

(A) a (B) $\log_e a$

(C) 1 (D) None of these

Q.54 Let $f(x) = \frac{1}{\sqrt{18 - x^2}}$, then the value of $\lim_{x\to 3} \frac{f(x) - f(3)}{x - 3}$ is-

(A) 0 (B) –1/9

(C) – 1/3 (D) None of these

Q.55 The value of $\lim_{x\to a} \frac{a^x - x^a}{x^x - a^a} = -1$, then a equals-

(A) 0 (B) 1

(C) e (D) –1

Q.56 The value of $\lim_{x\to 0} \frac{(16+5x)^{1/4} - 2}{(32+3x)^{1/5} - 2}$ is-

(A) 4/5 (B) 25/6

(C) 3/8 (D) None of these

Q.57 $\lim_{x\to 0} \frac{(1+\sin x)^{1/3} - (1-\sin x)^{1/3}}{x}$ equals-

(A) 0 (B) 1 (C) 2/3 (D) 1/3

Q.58 $\lim_{h\to 0} \left[\frac{(x+h)^{1/3} - x^{1/3}}{h}\right]$ equals-

(A) $\frac{1}{3}x^{2/3}$ (B) $\frac{1}{3}x^{-2/3}$ (C) $\frac{1}{3}x^{1/3}$ (D) $3x^{-2/3}$

Q.59 $\lim_{x\to 1} \frac{x + x^2 + ... + x^n - n}{x - 1}$ equals-

(A) n (B) 0

(C) $\frac{n^2}{2}$ (D) $\frac{n(n+1)}{2}$

Q.60 The value of $\lim_{x\to \pi/2}$ [x tan x– (π/2) sec x] is-

(A) –1 (B) 0

(C) 1 (D) None of these

Q.61 The value of $\lim_{h\to 0} \left[\frac{1}{h(8+h)^{1/3}} - \frac{1}{2h}\right]$ is-

(A) 1/12 (B) –4/3 (C) –16/3 (D) –1/48

Q.62 $\lim_{x\to \pi/2} \frac{1 - \sin x}{\left(x - \frac{\pi}{2}\right)^2}$ equals-

(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

Q.63 The value of $\lim_{x\to 1} \frac{\cos\left(\frac{\pi x}{2}\right)}{1 - \sqrt{x}}$ is-

(A) 0 (B) π/2 (C) 1 (D) π

Q.64 The value of $\lim_{x\to 1} \sec\frac{\pi}{2x} \log x$ is-

(A) π/2 (B) 2/ π (C) – π /2 (D) –2/ π

Q.65 The value of $\lim_{x\to \pi/2}$ cos x log (tan x) is-

(A) 1 (B) –1

(C) 0 (D) None of these

Q.66 $\lim_{x\to 1} \frac{1+\log x - x}{1-2x+x^2}$ equals-

(A) 1 (B) –1 (C) –1/2 (D) 1/2

Q.67 The value of $\lim_{h\to 0}$

$\frac{\sin(x+h)\log(x+h)-\sin x \log x}{h}$ is-

(A) $\frac{\cos x}{x}$ + log sin x (B) $\frac{\cos x}{x}$

(C) x cos x + log sin x (D) cos x log x + $\frac{\sin x}{x}$

Q.68 $\lim_{x\to \pi/4} \left(\frac{1-\tan x}{1-\sqrt{2}\sin x}\right)$ is equal to-

(A) 0 (B) 1 (C) –2 (D) 2

Q.69 If f(a) = 3, f ' (a) = –2, g(a) = –1, g'(a) = 4, then

$\lim_{x\to a} \frac{g(x)f(a)-g(a)f(x)}{x-a}$ equals-

(A) –5 (B) 10 (C) –10 (D) 5

Q.70 $\lim_{h\to 0} \frac{(a+h)^2\sin(a+h)-a^2\sin a}{h}$ is equal to -

(A) a^2 cos a + 2a sin a (B) a (cos a + 2 sin a)

(C) a^2 (cos a + 2 sin a) (D) None of these

Q.71 The value of $\lim_{x\to 0} \frac{\sqrt{(1+x)}-\sqrt{(1-x)}}{\sin^{-1}x}$ is-

(A) 0 (B) 1 (C) –1 (D) ∞

Q.72 $\lim_{x\to 1} \left[\frac{1}{1-x}-\frac{3}{1-x^3}\right]$ equals-

(A) 0 (B) – 1 (C) –2 (D) 1/3

Q.73 The value of $\lim_{x\to \infty} \frac{x^5}{5^x}$ is-

(A) 0 (B) 1 (C) e^5 (D) e^{-5}

Question based on **Some standard limit**

Q.74 The value of $\lim_{x\to 0} \frac{\log(1+kx^2)}{1-\cos x}$ is -

(A) 0 (B) 1 (C) k (D) 2k

Q.75 The value of $\lim_{x\to 0} \frac{\cot px}{\cot qx}$ is-

(A) 0 (B) 1 (C) q/p (D) p/q

Q.76 $\lim_{x\to -\infty} \frac{x^2\tan 1/x}{\sqrt{8x^2+7x+1}}$ is equal to -

(A) $-\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{2\sqrt{2}}$

(C) $\frac{1}{\sqrt{2}}$ (D) Does not exist

Q.77 $\lim_{x\to 0} \frac{\sin^2 x}{x\cos x}$ equals-

(A) 1 (B) 2 (C) 0 (D) 1/2

Q.78 $\lim_{x\to 0} \frac{\sqrt{1-\cos x^2}}{1-\cos x}$ equals-

(A) $\sqrt{2}$ (B) $1/\sqrt{2}$

(C) 1 (D) None of these

Q.79 The value of $\lim_{y\to 2}$ (y –2) cosec a (y –2) is-

(A) 0 (B) 1 (C) a (D) 1/a

Q.80 The value of $\lim_{n\to \infty}$ n[log (n+1) – log n] is-

(A) 1 (B) 0 (C) –1 (D) 2

Q.81 $\lim_{x\to 0} \frac{(1+x)^{1/x}-e}{x}$ equals-

(A) e (B) e/2 (C) –e (D) –e/2

Q.82 $\lim_{x\to 0} \left(\frac{\sin 2x}{x}\right)^{1+x}$ equals-

(A) 1 (B) 0

(C) 2 (D) None of these

Q.83 $\lim_{x\to 0} \frac{2\sin^2 3x}{x^2}$ equals-

(A) 9 (B) 18 (C) 6 (D) 1

Q.84 $\lim_{x\to \infty} \left(\frac{x+1}{2x+1}\right)^{x^2}$ equals-

(A) 0 (B) e (C) 1 (D) ∞

Q.85 $\lim_{x\to 0} \frac{1}{x}\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is equal to -

(A) 1 (B) 0 (C) 2 (D) 1/2

Q.86 If $\lim_{x \to 0} \frac{\tan kx}{\sin 5x} = 3$, then the value of k is-

(A) 1 (B) 3 (C) 5 (D) 15

Q.87 $\lim_{x \to \infty} x(e^{1/x} - 1)$ equals-

(A) 0 (B) 1 (C) –1 (D) ∞

Q.88 The value of $\lim_{x \to \infty} a^x \sin (b/a^x)$ is (a >1) -

(A) b log a (B) a log b
(C) b (D) None of these

Q.89 $\lim_{x \to 0} \frac{d}{dx} \int \frac{1 - \cos x}{x^2} dx$ is equal to-

(A) 1/2 (B) –1/2 (C) 0 (D) 1

Q.90 If $\lim_{x \to \infty} x \sin \left(\frac{\pi}{8x}\right) \cos \left(\frac{\pi}{8x}\right) = k$, then value of k is-

(A) π/4 (B) π /3 (C) π /2 (D) π /8

Q.91 $\lim_{x \to 0} \frac{1}{x^8} \left[1 - \cos \frac{x^2}{2} - \cos \frac{x^2}{4} + \cos \frac{x^2}{2} \cos \frac{x^2}{4}\right]$ equals-

(A) 1/16 (B) 1/24
(C) $\frac{1}{2^8}$ (D) $\frac{1}{2^9}$

Question based on **1^∞, ∞^0, 0^0 Forms**

Q.92 $\lim_{x \to 0} \left[\frac{\log (1 + x)}{x}\right]^{1/x}$ equals-

(A) e (B) e^{-1} (C) e^2 (D) $e^{-1/2}$

Q.93 $\lim_{x \to 0} [1 + \tan x]^{\cot x}$ equals -

(A) 1 (B) e
(C) e^{-1} (D) None of these

Q.94 $\lim_{x \to 0} (1+ x)^{1/x}$ equals-

(A) 1 (B) 0 (C) e (D) 1/e

Q.95 $\lim_{x \to 0} \left(\frac{1 + x}{1 - x}\right)^{1/x}$ equals-

(A) e (B) e^2
(C) 1/e (D) $1/e^2$

Q.96 $\lim_{x \to \pi/2} (\sec x)^{\cot x}$ equals-

(A) e (B) 1/e
(C) 1 (D) None of these

Q.97 The value of $\lim_{x \to 0} (\text{cosec } x)^{1/\log x}$ is -

(A) 1 (B) –1
(C) e (D) 1/e

Q.98 The value of $\lim_{x \to \pi/4} (\tan x)^{\tan 2x}$ is-

(A) e (B) e^{-1} (C) 0 (D) –1

Q.99 If $f(x) = \left(\frac{x}{2 + x}\right)^{2x}$, then-

(A) $\lim_{x \to \infty} f(x) = e^{-6}$ (B) $\lim_{x \to \infty} f(x) = 2$
(C) $\lim_{x \to \infty} f(x) = e^{-3}$ (D) $\lim_{x \to \infty} f(x) = e^{-4}$

Q.100 $\lim_{x \to \infty} \left(1 + \frac{a}{x}\right)^x$ equals-

(A) a^x (B) e (C) a (D) e^a

Q.101 $\lim_{x \to \infty} \left[1 + \frac{4}{x - 1}\right]^{x+3} =$

(A) e^2 (B) e (C) e^4 (D) e^3

Q.102 The value of $\lim_{x \to \infty} x^{1/x}$ is -

(A) 0 (B) 1
(C) ∞ (D) None of these

Q. 103 The value of $\lim_{x \to \infty} (x + e^x)^{2/x}$ is -

(A) 1 (B) 2
(C) e (D) e^2

LEVEL- 2

Q.1 If $f(x) = \begin{cases} \sin x, & x \neq n\pi, n \in Z \\ 2, & \text{otherwise} \end{cases}$ and

$g(x)=\begin{cases} x^2+1, & x \neq 0, 2 \\ 4, & x = 0 \\ 5, & x = 2 \end{cases}$ then $\lim_{x \to 0} g[f(x)] =$

(A) 0 (B) 1 (C) 2 (D) 5

Q.2 If [x] denotes the greatest integer ≤ x, then

$\lim_{n \to \infty} \frac{1}{n^3} \{[1^2 x] + [2^2 x] + [3^3 x] + ... + [n^2 x]$

equals -

(A) x/2 (B) x/3

(C) x/6 (D) 0

Q.3 $\lim_{n \to \infty} \left[\frac{1}{2.3} + \frac{1}{3.4} + ... + \frac{1}{n(n+1)}\right]$ equals-

(A) 1 (B) 0 (C) 1/2 (D) 2

Q.4 The value of $\lim_{x \to 1} \frac{1 - \sqrt{x}}{(\cos^{-1} x)^2}$ is-

(A) 1/2 (B) 1 (C) 1/4 (D) 4

Q.5 $\lim_{n \to \infty} \{\log_{n-1}(n) \log_n(n+1) ... \log_{n^k-1}(n^k)\}$,

k ∈ N is -

(A) 0 (B) k

(C) does not exist (D) None of these

Q.6 The value of $\lim_{x \to \frac{\pi}{2}} \frac{\log\left(x - \frac{\pi}{2}\right)}{\tan x}$ is-

(A) 0 (B) 1

(C) –1 (D) None of these

Q.7 $\lim_{h \to 0} 2\left[\frac{\sqrt{3}\sin\left(\frac{\pi}{6}+h\right) - \cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h(\sqrt{3}\cosh - \sin h)}\right]$ is equal to

(A) 2/3 (B) 4/3

(C) $-2\sqrt{3}$ (D) –4/3

Q.8 If $f(x) = \begin{cases} \frac{\sin(1+[x])}{[x]} & \text{for } [x] \neq 0 \\ 0 & \text{for } [x] = 0 \end{cases}$

where [x] denotes the greatest integer ≤ x, $\lim_{x \to 0^-} f(x)$ equals -

(A) 1 (B) 0

(C) – 1 (D) None of these

Q.9 $\lim_{x \to 0} \frac{\sqrt{1+x+x^2} - 1}{\sin 4x}$ equals-

(A) 1/8 (B) 1/4

(C) 1/2 (D) 1

Q.10 If g(x) is a polynomial satisfying

g(x) g(y) = g(x) + g(y) + g (xy) – 2 for all real x and y and g(2) = 5, then $\lim_{x \to 3} g(x)$ is -

(A) –8 (B) 10

(C) 8 (D) None of these

Q.11 $\lim_{x \to -\infty} \frac{x^5 \tan\left(\frac{1}{\pi x^2}\right) + 3|x|^2 + 7}{|x|^3 + 7|x| + 8}$ is equal to -

(A) $-\frac{1}{\pi}$ (B) 0

(C) ∞ (D) does not exist

Q.12 $\lim_{x \to 2} \frac{2^x + 2^{3-x} - 6}{(\sqrt{2})^{-x} - 2^{1-x}}$ equals-

(A) 0 (B) 1

(C) 8 (D) ∞

Q.13 $\lim_{n \to \infty} \frac{4n + (-1)^n}{5n + (-1)^n}$ equals-

(A) 0 (B) ∞

(C) 4/5 (D) Does not exist

Q.14 $\lim_{x \to \infty} \frac{\sqrt{x}}{\sqrt{x + \sqrt{x + \sqrt{x}}}}$ equals-

(A) 0 (B) 1

(C) ∞ (D) None of these

Q.15 The value of $\lim_{x \to 0} \frac{\sec 4x - \sec 2x}{\sec 3x - \sec x}$ is-

(A) 1 (B) 0

(C) 3/2 (D) ∞

Q.16 If x > 0 and g is a bounded function $\lim_{x \to \infty} \frac{f(x)e^{nx} + g(x)}{e^{nx} + 1}$ is -

(A) 0 (B) f(x)

(C) g(x) (D) None of these

Q.17 $\lim_{x \to 0} \frac{x(1 - \sqrt{1 - x^2})}{\sqrt{1 + x^2}(\sin^{-1} x)^3}$ equals-

(A) 0 (B) 1

(C) 1/2 (D) 1/4

Q.18 $\lim_{n \to \infty} \left[\frac{1}{1 - n^2} + \frac{2}{1 - n^2} + ... + \frac{n}{1 - n^2}\right]$ is equal to-

(A) 0 (B) – 1/2

(C) 1/2 (D) None of these

Q.19 The value of $\lim_{x \to 0} \frac{\cos(\sin x) - \cos x}{x^4}$ equals-

(A) $\frac{1}{5}$ (B) $\frac{1}{6}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$

Q.20 $\lim_{x \to 0} \frac{x}{|x| + x^2}$ equals-

(A) 1 (B) –1

(C) 0 (D) Does not exist

Q.21 $\lim_{x \to \infty} \left\{\frac{(x+1)^{10} + (x+2)^{10} + ... + (x+100)^{10}}{x^{10} + 10^{10}}\right\}$ is equal to-

(A) 10^2 (B) 10^3 (C) ∞ (D) 10^4

Q.22 $\lim_{x \to -\infty} \frac{x^4 \sin(1/x) + x^2}{1 + |x|^3}$ equals-

(A) 0 (B) 1

(C) –1 (D) ∞

Q.23 $\lim_{x \to 0} \frac{x \sin x + \log(1 - x)^x}{x^3}$ equals-

(A) 1/2 (B) –1/2

(C) 1/4 (D) –1/4

Q.24 The value of $\lim_{x \to \tan^{-1} 3} \frac{\tan^2 x - 2\tan x - 3}{\tan^2 x - 4\tan x + 3}$ is -

(A) 0 (B) 2

(C) ∞ (D) None of these

Q.25 The value of $\lim_{x \to \infty} \left(\frac{x^2 + 5x + 3}{x^2 + x + 2}\right)^x$ is-

(A) e^2 (B) 2^4

(C) e^3 (D) e^4

Q.26 $\lim_{x \to 0} \frac{\sin x^n}{(\sin x)^m}$ (m < n) is equal to-

(A) 0 (B) 1

(C) n/m (D) m/n

Q.27 $\lim_{x \to \pi} \frac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2}$ equals-

(A) 1/2 (B) 1/3

(C) 1/4 (D) 1/8

Q.28 $\lim_{x \to 1} (\log_5 5x)^{\log_x 5}$ equals -

(A) 1 (B) e

(C) –1 (D) None of these

Q.29 $\lim_{x \to 0} \frac{a^{\sqrt{x}} - a^{1/\sqrt{x}}}{a^{\sqrt{x}} + a^{1/\sqrt{x}}}$ (a > 1, x > 0) is equal to -

(A) 1 (B) –1

(C) 0 (D) None of these

Q.30 $\lim_{x \to \pi/2} \frac{\cot x - \cos x}{(\pi - 2x)^3}$ is equal to-

(A) 1 (B) $\frac{1}{16}$

(C) 16 (D) None of these

Q.31 If $\lim_{x \to 0} \frac{x^n - \sin x^n}{x - \sin^n x}$ is non-zero definite, then n must be -

(A) 1 (B) 2

(C) 3 (D) None of these

Q.32 $\lim_{x \to 1/\sqrt{2}} \frac{x - \cos(\sin^{-1} x)}{1 - \tan(\sin^{-1} x)}$ equals-

(A) $\frac{1}{\sqrt{2}}$ (B) $-\frac{1}{\sqrt{2}}$ (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

Q.33 If f " (0) = 4, then the value of

$\lim_{x \to 0} \frac{2f(x) - 3f(2x) + f(4x)}{x^2}$ is-

(A) 11 (B) 12 (C) 2 (D) 0

Q.34 $\lim_{x \to 2^-} (x + (x - [x])^2)$ equals-

where [x] represent greatest integer function.

(A) 0 (B) 1 (C) 2 (D) 3

Q.35 $\lim_{x \to \infty} x \left(\tan^{-1}\left(\frac{x+1}{x+2}\right) - \tan^{-1}\left(\frac{x}{x+2}\right) \right)$ equals-

(A) 1 (B) –1

(C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

Q.36 $\lim_{x \to 0} \frac{e^x - e^{\sin x}}{x - \sin x}$ equals-

(A) 0 (B) 1

(C) ∞ (D) None of these

LEVEL- 3

Q.1 If [x] denotes the greatest integer less than or equal to x, then

$\lim_{n \to \infty} \frac{[x] + [2x] + [3x] + ... + [nx]}{n^2}$ equals -

(A) x/2 (B) x/3 (C) x (D) 0

Q.2 The value of $\lim_{x \to 1} (2 - x)^{\tan \frac{\pi x}{2}}$ is equal to -

(A) $e^{-2/\pi}$ (B) $e^{1/\pi}$ (C) $e^{2/\pi}$ (D) $e^{-1/\pi}$

Q.3 If { } → represent fractional part of x then

$\lim_{x \to [a]} \frac{e^{\{x\}} - \{x\} - 1}{\{x\}^2}$ is equal to where [·] represent G.I.F.

(A) 0 (B) 1/2

(C) e^{-2} (D) None of these

Q.4 Let f(x), $\lim_{n \to \infty} \frac{x^{2n} - 1}{x^{2n} + 1}$ then -

(A) f(x) = 1, for |x| > 1

(B) f(x) = –1 for |x| < 1

(C) f(x) is not defined for any value of x

(D) f(x) = 1 for |x| = 1

Q.5 If $f(x) = \frac{2}{x - 3}$, $g(x) = \frac{x - 3}{x + 4}$ and

$h(x) = -\frac{2(2x + 1)}{x^2 + x - 12}$ then

$\lim_{x \to 3}$ [f(x) + g(x) +h(x)] is-

(A) –2 (B) –1

(C) $-\frac{2}{7}$ (D) 0

Q.6 If $A_i = \frac{x - a_i}{|x - a_i|}$, i = 1, 2,..., n and if

$a_1 < a_2 < a_3 < ... a_n$. Then $\lim_{x \to a_m} (A_1 A_2 .....A_n)$,

$1 \le m \le n$

(A) is equal to $(-1)^m$

(B) is equal to $(-1)^{m+1}$

(C) is equal to $(-1)^{m-1}$

(D) does not exist

Q.7 Let a = $\min^m \{x^2 + 2x + 3\}$ x ∈ R and

b = $\lim_{\theta \to 0} \frac{1 - \cos\theta}{\theta^2}$ the value of $\sum_{r=1}^{n} a^r b^{n-r}$ is -

(A) $\frac{2^{n+1} - 1}{3.2^n}$ (B) $\frac{2^{n+1} + 1}{3.2^n}$

(C) $\frac{4^{n+1} - 1}{3.2^n}$ (D) None of these

Q.8 The value of $\lim_{x \to 0} \left(\frac{\sin x}{x}\right)^{\frac{\sin x}{x - \sin x}}$ -

(A) e^{-1} (B) e

(C) 1 (D) None of these

Q.9 $\lim_{x \to 0} \left(\frac{1^x + 2^x + 3^x + ... + n^x}{n}\right)^{1/x}$ is equal to -

(A) $(n!)^n$ (B) $(n!)^{1/n}$

(C) n ! (D) $\ell n(n!)$

Q.10 $\lim_{x \to \infty} \frac{2\sqrt{x} + 3x^{1/3} + 4x^{1/4} + ... + nx^{1/n}}{(2x - 3)^{1/2} + (2x - 3)^{1/3} + ... + (2x - 3)^{1/n}}$

(A) 1 (B) ∞

(C) $\sqrt{2}$ (D) None of these

Q.11 $\lim_{x \to 0} [(\min^m (y^2 - 4y + 11)) \frac{\sin x}{x}]$ where [ ] represent greatest integer function is -

(A) 5 (B) 6

(C) 7 (D) None of these

Q.12 If f(x) is the integral of $\frac{2\sin x - \sin 2x}{x^3}$, x ≠0 then find $\lim_{x \to 0} f'(x)$ -

(A) 1 (B) 1/2

(C) 3/2 (D) None of these

Q.13 If f (x) is a continuous function from f : R → R and attains only irrational value's then $\sum_{r=1}^{100} f(r)$ is equal to -

(A) 100 (B) $\sum_{r=101}^{200} f(r)$

(C) $\sum_{r=1}^{10} f(r)$ (D) None of these

Q.14 The value of $\lim_{x\to1}\left(\frac{x^3+2x^2+x+1}{x^2+2x+3}\right)^{\frac{1-\cos(x-1)}{(x-1)^2}}$ is -

(A) e (B) $e^{1/2}$
(C) 1 (D) None of these

Q.15 Given a real valued function f such that

$$f(x)=\begin{cases}\dfrac{\tan^2\{x\}}{(x^2-[x]^2)}, & x>0\\ 1, & x=0\\ \sqrt{\{x\}\cot\{x\}}, & x<0\end{cases}$$

where [ ] represent G.I.F. and { } represent fractional part of x

(A) $\lim_{x\to0^+} f(x)=1$

(B) $\lim_{x\to0^-} f(x)=\cot 1$

(C) $\tan^{-1}\left(\lim_{x\to0^+} f(x)\right)=\pi/4$

(D) All of the above

Q.16 $\lim_{x\to0}\frac{\sin[\cos x]}{1+\cos[\cos x]}$ is -

(A) 1 (B) 0
(C) does not exist (D) None of these

Q.17 The value of $\lim_{x\to0}\left(\left[100\frac{x}{\sin x}\right]+\left[99\frac{\sin x}{x}\right]\right)$

where [ ] represent greatest integer function -

(A) 199 (B) 198
(C) 0 (D) None of these

Q.18 If $\lim_{x\to\infty}\left(\frac{x^3+1}{x^2+1}-(ax+b)\right)=2$ then

(A) a = 1, b = 1 (B) a = 1, b = 2
(C) a = 1, b = – 2 (D) None of these

➢ Statement type Questions

All questions are Assertion & Reason type questions. Each of these questions contains two statements: Statement-1 (Assertion) and Statement-2 (Reason). Answer these questions from the following four option.

(A) Statement-I and Statement-II are true Statement-II is the correct explanation of Statement-I

(B) Statement-I Statement-II are true but Statement-II is not the correct explanation of Statement-I.

(C) Statement-I is true but Statement-II is false

(D) Statement-I is false but Statement-II is true.

Q.19 **Statement – I :** $\lim_{x\to0}[x]\left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$ (where [ ] represent greatest integer function) does not exist.

Statement-II : $\lim_{x\to0}\left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$ does not exist.

Q.20 **Statement-I :** The graph of the function y = f(x) has a unique tangent at the point (a, 0) through which the graph passes then

$$\lim_{x\to a}\frac{\log_e(1+6(f(x))}{3f(x)}=2$$

Statement-II : Since the graph passes through (a, 0). Therefore f(a) = 0, when f(a) = 0 given limit is zero by zero form. So that it can be evaluate by using L'Hospital's rule.

Q. 21 **Statement-I :** when | x | < 1,

$$\lim_{n\to\infty}\frac{\log(x+2)-x^{2n}\cos x}{x^{2n}+1}=\log(x+2)$$

Statement-II : For –1 < x < 1,

as $n\to\infty$, $x^{2n}\to0$.

Q.22 **Statement -I :** $\lim_{x\to0^+} x\sin\left(\frac{1}{x}\right)=1$

Statement -II : $\lim_{y\to\infty} y\sin\left(\frac{1}{y}\right)=1$

Q.23 **Statement -I :** $\lim_{x\to0}\frac{\sqrt{\frac{1-\cos 2x}{2}}}{x}$ exist's.

Statement -II : $\lim_{x\to a} f(x)$ exists if the left hand limit is equal to right hand limit.

Q.24 **Statement -I :** Value of $\lim_{x\to\pi/2}(\sin x)^{\tan x}$ is 1.

Statement -II: $\lim_{x\to a}(1+f(x))^{g(x)}$ is $e^{\lim_{x\to a} f(x)g(x)}$,

If $\lim_{x\to a} f(x)=0$ and $\lim_{x\to a} g(x)=\infty$

➢ Passage Based Questions

Passage :-

Let m, n are non zero integers and

$\lim_{x \to 0} \frac{\tan mx - n \sin x}{x^3}$ = an integer.

On the basis of above information, answer the following questions-

Q.25 Which of the following statement is true –

(A) m is should be an even but n is odd

(B) both m & n should be odd

(C) m is odd and n is even

(D) both m & n are even integers

Q.26 The value of limit in terms of m & n is –

(A) $\frac{2m + n^2}{6}$ (B) $\frac{2m^3 + n}{6}$

(C) ∞ (D) None of these

Q.27 Is m & n are related as –

(A) $m^2 = n$ (B) $m = n^2$

(C) m = n (D) None of these

Q.28 The value of limit for m = 2 is –

(A) 3 (B) 2

(C) $\frac{16 + n}{12}$ (D) None of these

Q.29 If $\lim_{x \to 0} \frac{\tan(mx) - n \sin x}{x^3}$ = not an integer then for m = n = 1, the value of limit is–

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$

(C) 2 (D) None of these

➢ Column Matching Questions

Match the entry in Column 1 with the entry in Column 2.

Q.30 $\lim_{x \to 0}$ f(x) is less than equal to, where

Column-I	Column-II
(A) $f(x) = \frac{e^x - e^{2x}}{x}$	(P) e
(B) $f(x) = \frac{e^x - e^{-x}}{\sin x}$	(Q) – 2
(C) $f(x) = \frac{e^{2x} - e^{4x}}{x}$	(R) – 1
(D) $(1 + \sin x)^{\text{cosec } x}$	(S) 2

Q.31 $\lim_{x \to 0}$ f(x), where f(x) is as in column-I is-

Column-I	Column-II
(A) $f(x) = \frac{\tan[e^2]x^2 - \tan[-e^2]x^2}{\sin^2 x}$	(P) $\sqrt{2}/8$
(B) $f(x) = \frac{[5/2 + \tan x + \tan^2 x] - [5/2]}{\tan x}$ where [x] is the greatest integer function	(Q) 15
(C) $f(x) = \frac{x \cos x - \log(1 + x)}{x^2}$	(R) 0
(D) $f(x) = \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin^2 x}$	(S) 1/2

LEVEL- 4

(Question asked in previous AIEEE and IIT-JEE)

SECTION –A

Q.1 If f (1) = 1, f ′ (1) = 2, then $\lim_{x \to 1} \frac{\sqrt{f(x)} - 1}{\sqrt{x} - 1}$ = **[AIEEE 2002]**

(A) 2 (B) 1 (C) 3 (D) 4

Q.2 The value of $\lim_{x \to 0} \frac{(1 - \cos 2x)\sin 5x}{x^2 \sin 3x}$ is- **[AIEEE 2002]**

(A) 10/3 (B) 3/10
(C) 6/5 (D) 5/6

Q.3 $\lim_{x \to \infty} \left(\frac{x^2 + 5x + 3}{x^2 + x + 3}\right)^x$ = **[AIEEE 2002]**

(A) e^4 (B) e^2 (C) e^3 (D) e

Q.4 $\lim_{x \to \infty} \frac{\log x^n - [x]}{[x]}$, n ∈ N, (where [x] denotes greatest integer less than or equal to x) **[AIEEE-2002]**

(A) has value – 1 (B) has value 0
(C) has value 1 (D) does not exist

Q.5 If $\lim_{x \to 0} \frac{\log(3 + x) - \log(3 - x)}{x}$ = k, the value of k is - **[AIEEE 2003]**

(A) $-\frac{2}{3}$ (B) 0 (C) $-\frac{1}{3}$ (D) $\frac{2}{3}$

Q.6 Let f(a) = g(a) = k and their n^{th} derivatives $f^n(a)$, $g^n(a)$ exist and are not equal for some n. Further if $\lim_{x \to a} \frac{f(a)g(x) - f(a) - g(a)f(x) + g(a)}{g(x) - f(x)}$ = 4 then the value of k is- **[AIEEE 2003]**

(A) 0 (B) 4 (C) 2 (D) 1

Q.7 $\lim_{x \to \pi/2} \frac{\left[1 - \tan\left(\frac{x}{2}\right)\right][1 - \sin x]}{\left[1 + \tan\left(\frac{x}{2}\right)\right][\pi - 2x]^3}$ is- **[AIEEE 2003]**

(A) ∞ (B) $\frac{1}{8}$

(C) 0 (D) $\frac{1}{32}$

Q.8 If $\lim_{x \to \infty} \left(1 + \frac{a}{x} + \frac{b}{x^2}\right)^{2x} = e^2$, then the values of a and b, are- **[AIEEE 2004]**

(A) a ∈ R, b ∈ R (B) a = 1, b ∈ R
(C) a ∈ R, b = 2 (D) a = 1 and b = 2

Q.9 Let α and β be the distinct roots of $ax^2 + bx + c = 0$, then

$\lim_{x \to \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - \alpha)^2}$ is equal to - **[AIEEE-2008]**

(A) $\frac{a^2}{2}(\alpha - \beta)^2$ (B) 0

(C) $\frac{-a^2}{2}(\alpha - \beta)^2$ (D) $\frac{1}{2}(\alpha - \beta)^2$

Q.10 $\lim_{x \to 2}\left(\frac{\sqrt{1 - \cos\{2(x - 2)\}}}{x - 2}\right)$ **[AIEEE-2011]**

(A) does not exist (B) equals $\sqrt{2}$

(C) equals $-\sqrt{2}$ (D) equals $\frac{1}{\sqrt{2}}$

SECTION-B

Q.1 $\lim_{x \to \pi/4} \frac{\sqrt{2}\cos x - 1}{\cot x - 1}$ = **[IIT-1990]**

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$

(C) $\frac{1}{2\sqrt{2}}$ (D) 1

Q.2 $\lim_{x \to \infty} \frac{(2x + 1)^{40}(4x - 1)^5}{(2x + 3)^{45}}$ = **[IIT-1990]**

(A) 16 (B) 24
(C) 32 (D) 8

Q.3 $\lim\limits_{x \to 0} \frac{\sqrt{\frac{1}{2}(1 - \cos 2x)}}{x} =$ **[IIT -1991]**

(A) 1 (B) –1

(C) 0 (D) None

Q.4 $\lim\limits_{x \to \infty} \frac{x^n}{e^x} = 0$ for **[IIT-1992]**

(A) no value of n

(B) n is any whole number

(C) n = 0 only

(D) n = 2 only

Q.5 $\lim\limits_{x \to 0} \left(\frac{x}{\tan^{-1} 2x} \right) =$ **[IIT-1992]**

(A) 0 (B) 1/2

(C) 2 (D) ∞

Q.6 $\lim\limits_{x \to 0} \left\{ \tan \left(\frac{\pi}{4} + x \right) \right\}^{1/x} =$ **[IIT- 1993]**

(A) 1 (B) –1

(C) e^2 (D) e

Q.7 $\lim\limits_{x \to 0} \left(\frac{1 + 5x^2}{1 + 3x^2} \right)^{1/x^2} =$ **[IIT- 1996]**

(A) e^2 (B) e

(C) e^{-2} (D) e^{-1}

Q.8 The value of $\lim\limits_{h \to 0} \frac{\log(1 + 2h) - 2\log(1 + h)}{h^2}$ is-

[IIT-1997]

(A) 1 (B) – 1

(C) 0 (D) None of these

Q.9 $\lim\limits_{x \to 1} \frac{\sqrt{1 - \cos 2(x - 1)}}{x - 1} =$

[IIT-1998 similar to IIT- 1991]

(A) does not exist because LHL ≠ RHL

(B) exists and it equals $-\sqrt{2}$

(C) does not exist because $x - 1 \to 0$

(D) exists and it equals $\sqrt{2}$

Q.10 $\lim\limits_{x \to 0} \frac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2}$ is- **[IIT-1999]**

(A) $\frac{1}{2}$ (B) –2

(C) 2 (D) $-\frac{1}{2}$

Q.11 For $x \in R$, $\lim\limits_{x \to \infty} \left(\frac{x - 3}{x + 2} \right)^x =$ **[IIT Scr. 2000]**

(A) e (B) e^{-1}

(C) e^{-5} (D) e^5

Q.12 $\lim\limits_{x \to 0} \frac{\sin(\pi \cos^2 x)}{x^2}$ equals - **[IIT Scr. 2001]**

(A) –π (B) π

(C) π/2 (D) 1

Q.13 The value of Integer n; for which $\lim\limits_{x \to 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n}$ is a finite non zero number- **[IIT Scr. 2002]**

(A) 1 (B) 2

(C) 3 (D) 4

Q.14 Let f : R → R such that f(1) = 3 and f '(1) = 6. then $\lim\limits_{x \to 0} \left(\frac{f(1 + x)}{f(1)} \right)^{1/x}$ equals -

[IIT Scr. 2002]

(A) 1 (B) $e^{1/2}$

(C) e^2 (D) e^3

Q.15 If $\lim\limits_{x \to 0} \frac{(\sin nx)[(a - n)nx - \tan x]}{x^2} = 0$ then the value of a is- **[IIT Scr.2003]**

(A) $\frac{1}{n + 1}$ (B) $\frac{n}{n + 1}$

(C) $n + \frac{1}{n}$ (D) n

Q.16 If f(x) is a differentiable function and $f'(2) = 6$, $f'(1) = 4$, $f'(c)$ represents the differentiation of f(x) at x = c, then $\lim_{h \to 0} \frac{f(2+2h+h^2) - f(2)}{f(1+h^2+h) - f(1)}$

[IIT Scr.2003]

(A) may exist (B) will not exist
(C) is equal to 3 (D) is equal to –3

Q.17 Let f(x) be strictly increasing and differentiable, then $\lim_{x \to 0} \frac{f(x^2) - f(x)}{f(x) - f(0)}$ is-

[IIT Scr.2004]

(A) 1 (B) –1
(C) 0 (D) 2

Q.18 $\lim_{x \to 0} \left((\sin x)^{1/x} + \left(\frac{1}{x}\right)^{\sin x} \right)$, for x > 0-

[IIT-2006]

(A) 0 (B) –1
(C) 2 (D) 1

Q.19 Let $L = \lim_{x \to 0} \frac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}$, a > 0. If L is finite, then **[IIT- 2009]**

(A) a = 2 (B) a = 1
(C) $L = \frac{1}{64}$ (D) $L = \frac{1}{32}$

Q.20 If $\lim_{x \to 0} [1 + x \ln(1 + b^2)]^{\frac{1}{x}} = 2b\sin^2\theta$, b > 0 and $\theta \in (-\pi, \pi]$, then the value of θ is - **[IIT- 2011]**

(A) $\pm\frac{\pi}{4}$ (B) $\pm\frac{\pi}{3}$
(C) $\pm\frac{\pi}{6}$ (D) $\pm\frac{\pi}{2}$

ANSWER KEY

LEVEL-1

Q.No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	A	C	D	D	B	B	A	A	C	D	D	D	A	D	A	A	A	D	C	B
Q.No.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
Ans.	A	B	B	C	A	A	B	C	A	B	B	B	B	D	D	D	B	D	D	A
Q.No.	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
Ans.	B	B	D	B	B	C	B	B	B	D	C	A	B	D	B	B	C	B	D	A
Q.No.	**61**	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
Ans.	D	C	D	B	C	C	D	D	B	A	B	B	A	D	C	A	C	A	D	A
Q.No.	**81**	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**	**91**	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**
Ans.	D	C	B	A	C	D	B	C	A	D	C	D	B	C	B	C	D	B	D	D
Q.No.	**101**	**102**	**103**																	
Ans.	C	B	D																	

LEVEL-2

Q.No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	B	B	C	C	B	A	B	B	A	B	A	C	C	B	C	B	C	B	B	D
Q.No.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**				
Ans.	A	C	B	B	D	A	C	B	B	B	A	B	B	D	C	B				

LEVEL- 3

Q.No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	A	C	D	A,B	C	D	C	A	B	C	B	A	B	D	D	B	B	C	B	A
Q.No.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**											
Ans.	A	D	D	A	D	B	C	A	A											

30. (A) $\rightarrow$ P,R,S ; (B) $\rightarrow$ P,S ; (C) $\rightarrow$ P,Q,R,S ; (D) $\rightarrow$ P **31.** (A) $\rightarrow$ Q ; (B) $\rightarrow$ R ; (C) $\rightarrow$ S ; (D) $\rightarrow$ P

LEVEL- 4

SECTION-A

Q.No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	A	A	A	A	D	B	D	B	A	A

SECTION-B

Q.No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Ans.	B	C	D	B	B	C	A	B	A	A	C	B
Q.No.	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**				
Ans.	C	C	C	C	B	D	A,C	D				