SOLVED EXAMPLES

Ex.1 Function $f(x) = \begin{cases} -1, \text{ when } & x < -1 \\ -x, \text{ when } & -1 \le x \le 1 \\ 1, \text{ when } & x > 1 \end{cases}$ is continuous -

(A) Only at x = 1
(B) Only at x = – 1
(C) At both x = 1 and x = – 1
(D) Neither at x = 1 nor at x = – 1

Sol. $f(-1-0) = -1,\ f(-1) = -(-1) = 1$

$\Rightarrow f(-1-0) \ne f(-1)$

$\Rightarrow$ f(x) is not continuous at x = –1

Further, $f(1) = -1$

$f(1+0) = 1 \qquad \Rightarrow f(1) \ne f(1+0)$

$\Rightarrow$ f(x) is not continuous at x = 1. **Ans.[D]**

Ex.2 If $f(x) = \begin{cases} x^k \cos(1/x), & x \ne 0 \\ 0, & x = 0 \end{cases}$ is continuous at x = 0, then -

(A) k < 0 (B) k > 0 (C) k = 0 (D) k ≥ 0

Sol. Since f(x) is continuous at x = 0

$\therefore \lim_{x \to 0} f(x) = f(0)$

but f(0)= 0 (given)

$\therefore \lim_{x \to 0} f(x) = \lim_{x \to 0} x^k \cos(1/x)$

= 0, if k > 0. **Ans.[B]**

Ex.3 If $f(x) = \begin{cases} \frac{1}{2} - x, & 0 < x < \frac{1}{2} \\ 0, & x = 0 \\ \frac{1}{2}, & x = \frac{1}{2} \\ \frac{3}{2} - x, & \frac{1}{2} < x < 1 \\ 1, & x = 1 \end{cases}$

then wrong statement is -

(A) f(x) is discontinuous at x = 0
(B) f(x) is continuous at x = 1/2
(C) f(x) is discontinuous at x = 1
(D) f(x) is continuous at x = 1/4

Sol. Obviously function f(x) is discontinuous at x = 0 and x= 1 because the function is not defined, when x< 0 and x> 1, therefore f(0–0) and f(1+0) do not exist. Again

$f\left(\frac{1}{2}+0\right) = \lim_{x \to 1/2}\left(\frac{3}{2} - x\right) = 1$

$f\left(\frac{1}{2}-0\right) = \lim_{x \to 1/2}\left(\frac{1}{2} - x\right) = 0$

$\because f\left(\frac{1}{2}+0\right) \ne f\left(\frac{1}{2}-0\right)$

function f(x) is discontinuous at $x = \frac{1}{2}$ **Ans.[B]**

Ex.4 If $f(x) = \begin{cases} \dfrac{x^3 + x^2 - 16x + 20}{(x-2)^2}, & x \ne 2 \\ k, & x = 2 \end{cases}$ is - continuous for all values of x, then the value of k is -

(A) 5 (B) 6 (C) 7 (D) 8

Sol. $\because$ f(x) is continuous at x =2

$\therefore f(2-0) = f(2+0) = f(2) = k$

But f(2+0)

$= \lim_{h \to 0} \frac{(2+h)^3 + (2+h)^2 - 16(2+h) + 20}{(2+h-2)^2}$

$= \lim_{h \to 0} \frac{h^3 + 7h^2}{h^2} = 7$ **Ans. [C]**

Ex.5 If the function $f(x) = \begin{cases} 1, & x \le 2 \\ ax + b, & 2 < x < 4 \\ 7, & x \ge 4 \end{cases}$ is continuous at x = 2 and 4, then the values of a and b are-

(A) 3, 5 (B) 3, –5
(C) 0, 3 (D) 0, 5

Sol. Since f(x) is continuous at x = 2

$\therefore f(2) = \lim_{x \to 2^+} f(x)$

$\Rightarrow 1 = \lim_{x \to 2^+} (ax + b)$

$\therefore 1 = 2a + b$ ...(1)

Again f(x) is continuous at x = 4,

$\therefore f(4) = \lim_{x \to 4^-} f(x)$

$\Rightarrow 7 = \lim_{x \to 4} (ax + b)$

$\therefore 7 = 4a + b$ ...(2)

Solving (1) and (2), we get a= 3, b = –5.**Ans.[B]**

Ex.6 If $f(x) = \begin{cases} x, \text{ when } & x \in Q \\ -x, \text{ when } & x \notin Q \end{cases}$, then f(x) is continuous at -

(A) All rational numbers

(B) Zero only

(C) Zero and 1 only

(D) No where

Sol. Let us first examine continuity at x = 0.

$f(0) = 0 \qquad (\because 0 \in Q)$

$= f(0-0) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} f(-h)$

$= \lim_{h \to 0} \{-h \text{ or } h \text{ according as } -h \in Q \text{ or } -h \notin Q)$

$= 0$

$f(0+0) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} f(h)$

$= \lim_{h \to 0} \{h \text{ or } -h\} = 0$

$f(0) = f(0-0) = f(0+0)$

$\Rightarrow$ f(x) is continuous at x = 0.

Now let $a \in R$, $a \neq 0$, then

$f(a-0) = \lim_{h \to 0} f(a-h)$

$= \lim_{h \to 0} \{(a-h) \text{ or } -(a-h)\}$

= a or –a, which is not unique.

$\Rightarrow$ f(a–0) does not exist

$\Rightarrow$ f(x) is not continuous at $a \in R_0$.

Hence f(x) is continuous only at x = 0. **Ans.[B]**

Ex.7 f(x) = x –[x] is continuous at -

(A) x = 0 (B) x = –1

(C) x = 1 (D) x = 1/2

Sol. We know that [x] is discontinuous at every integer. Therefore it is continuous only at x = 1/2, while the function x is continuous at all points x= 0, –1, 1, 1/2. Thus the given function is continuous only at x = 1/2. **Ans.[D]**

Ex.8 If $f(x) = \begin{cases} \dfrac{1-\sin^3 x}{3\cos^2 x}, & x < \pi/2 \\ a, & x = \pi/2 \\ \dfrac{b(1-\sin x)}{(\pi - 2x)^2}, & x > \pi/2 \end{cases}$ is continuous at $x = \pi/2$, then value of a and b are-

(A) 1/2, 1/4 (B) 2,4

(C) 1/2,4 (D) 1/4,2

Sol. $f\left(\frac{\pi}{2} - 0\right) = \lim_{h \to 0} \dfrac{1 - \sin^3\left(\frac{\pi}{2} - h\right)}{3\cos^2\left(\frac{\pi}{2} - h\right)}$

$= \lim_{h \to 0} \dfrac{1 - \cos^3 h}{3\sin^2 h}$

$= \lim_{h \to 0} \dfrac{(1-\cos h)(1 + \cos h + \cos^2 h)}{3(1-\cos h)(1+\cos h)}$

$= 1/2$

$f\left(\frac{\pi}{2} + 0\right) = \lim_{h \to 0} \dfrac{b\left[1 - \sin\left(\frac{\pi}{2} + h\right)\right]}{\left[\pi - 2\left(\frac{\pi}{2} + h\right)\right]}$

$= \lim_{h \to 0} \dfrac{b(1 - \cos h)}{4h^2}$

$= \lim_{h \to 0} \dfrac{2b\sin^2 h/2}{4h^2} = \dfrac{b}{8}$

Now f(x) is continuous at $x = \frac{\pi}{2}$

$\Rightarrow f\left(\frac{\pi}{2} - 0\right) = f\left(\frac{\pi}{2} + 0\right) = f\left(\frac{\pi}{2}\right) \Rightarrow \frac{1}{2} = \frac{b}{8} = a$

$\therefore$ a = 1/2, b = 4 **Ans.[C]**

Ex.9 If the function

$f(x) = \begin{cases} 1 + \sin\frac{\pi}{2} \times \text{ for } -\infty < x \le 1 \\ ax + b \text{ for } 1 < x < 3 \\ 6\tan\frac{x\pi}{12} \text{ for } 3 \le x < 6 \end{cases}$

is continuous in the interval (– ∞, 6), then the value of a and b are respectively -

(A) 0, 2 (B) 1, 1 (C) 2, 0 (D) 2, 1

Sol. Obviously the function f(x) is continuous at x = 1 and 3. Therefore $\lim_{x \to 1^+} f(x) = f(1)$

$\Rightarrow a + b = 2$ ...(1)

and $\lim_{x \to 3^-} f(x) = f(3)$

$\Rightarrow 3a + b = 6$ ...(2)

Solving (1) and (2) , we get a = 2, b = 0.**Ans.[C]**

Ex.10 If $f(x) = \begin{cases} \dfrac{1-\cos 4x}{x^2}, & x < 0 \\ a, & x = 0 \\ \dfrac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}, & x > 0 \end{cases}$

then at x = 0 -

(A) f(x) is continuous, when a = 0
(B) f(x) is continuous, when a = 8
(C) f(x) is discontinuous for every value of a
(D) None of these

Sol. $f(0-0) = \lim_{x\to 0} \dfrac{1-\cos 4x}{x^2} = \dfrac{2\sin^2 2x}{x^2} = 8$

$f(0+0) = \lim_{x\to 0} \dfrac{\sqrt{x}}{(\sqrt{16+\sqrt{x}}-4)} \times \dfrac{\sqrt{16+\sqrt{x}}+4}{\sqrt{16+\sqrt{x}}+4}$

$= \lim_{x\to 0} \dfrac{\sqrt{x}(\sqrt{16+\sqrt{x}}+4)}{16+\sqrt{x}-16} = 8$

$\because$ f (0 + 0) = f(0 – 0)

$\therefore$ f(x) can be continuous at x = 0, if

f (0) = a = 8. **Ans.[B]**

Ex.11 If $f(x) = \lim_{x\to 0} \begin{cases} \dfrac{\sin[x]}{[x]+1}, & x > 0 \\ \dfrac{\cos\dfrac{x}{2[x]}}{[x]}, & x < 0 \\ k, & x = 0 \end{cases}$

(Where [x] = greatest integer ≤ x) is continuous at x = 0, then k is equal to -

(A) 0 (B) 1
(C) –1 (D) Indeterminate

Sol. As given f (0 – 0) = f(0 + 0) = k

Now $f(0-0) = \lim_{h\to 0} \dfrac{\cos\dfrac{(-h)}{2[-h]}}{[-h]}$

$= \lim_{h\to 0} \dfrac{\cos\left(\dfrac{-h}{2(-1)}\right)}{-1} = -1$

$f(0+0) = \lim_{h\to 0} \dfrac{\sin[h]}{[h]+1} = \lim_{h\to 0} \dfrac{\sin 0}{0+1} = 0$

$\because$ f(0 – 0) ≠ f(0 + 0), so k is indeterminate.

Ans.[D]

Ex.12 If $f(x) = \begin{cases} (1+|\sin x|)^{a/|\sin x|}, & -\pi/6 < x < 0 \\ b, & x = 0 \\ e^{\tan 2x/\tan 3x}, & 0 < x < \pi/6 \end{cases}$

is continuous at x = 0, then value of a, b are -

(A) 2/3, $e^{2/3}$ (B) 1/3, $e^{1/3}$
(C) 2/3, 1/3 (D) None of these

Sol. $f(0-0) = \lim_{h\to 0} (1+|\sin(-h)|)^{a/|\sin(-h)|}$

$= (1+\sin h)^{a/\sin h} = e^a$

$f(0+0) = \lim_{h\to 0} e^{\frac{\tan 2h}{\tan 3h}} = e^{\lim_{h\to 0}\left(\frac{\tan 2h}{\tan 3h}\right)}$

$= e^{\lim_{h\to 0}\frac{2\sec^2 2h}{3\sec^2 3h}} = e^{2/3}$

Now f(x) is continuous at x = 0

⇒ f(0-0) = f(0+0) = f(0)

⇒ $e^a = e^{2/3} = b$

$\therefore$ a = 2/3, b= $e^{2/3}$ **Ans.[A]**

Ex.13 f(x) = |x| is not differentiable at -

(A) x = –1 (B) x = 0
(C) x = 1 (D) None of these

Sol. at x = 0:

$f'(0-0) = \lim_{h\to 0} \dfrac{|0-h|-0}{-h} = -1$

$f'(0+0) = \lim_{h\to 0} \dfrac{|0+h|-0}{h} = 1$

Now, since f ' (0–0) ≠ f ' (0+0)

⇒ f(x) is not differentiable at x = 0. **Ans.[B]**

Ex.14 Function $f(x) = \lim_{h\to 0} \begin{cases} -x, & \text{if } x < 0 \\ x^2, & \text{if } 0 \le x \le 1 \\ x^3 - x + 1, & \text{if } x > 1 \end{cases}$, is differentiable at -

(A) x = 0 but not at x = 1
(B) x = 1 but not at x = 0
(C) x = 0 and x = 1 both
(D) neither x = 0 nor x = 1

Sol. Differentiability at x = 0

$R[f'(0)] = \lim_{h\to 0} \dfrac{f(0+h)-f(0)}{h}$

$= \lim_{h\to 0} \dfrac{(0+h)^2-0}{h} = \lim_{h\to 0} h = 0$

$L[f'(0)] = \lim_{h\to 0} \dfrac{f(0-h)-f(0)}{-h}$

$= \lim_{h\to 0} \dfrac{-(0-h)-0}{-h} = -1$

$\because$ R [f ' (0)] ≠ L [f '(0)]

$\therefore$ f(x) is not differentiable at x = 0

Differentiability at x = 1

$R[f'(1)] = \lim_{h \to 0} \frac{f(1+h)^3 - f(1)}{h}$

$= \lim_{h \to 0} \frac{(1+h)^3 - (1+h) + 1 - 1}{h}$

$= \lim_{h \to 0} \frac{2h + 3h^2 + h^3}{h} = 2$

$L[f'(1)] = \lim_{h \to 0} \frac{f(1-h) - f(1)}{-h}$

$= \lim_{h \to 0} \frac{(1-h) - 1}{-h}$

$= \lim_{h \to 0} \frac{-2h + h^2}{-h} = 2$

Thus R [f ' (1)] = L f '(1)]

$\therefore$ function f(x) is differentiable at x = 1 **Ans.[B]**

Ex.15 If $f(x) = \begin{cases} 3^x, -1 \le x \le 1 \\ 4 - x, 1 < x < 4 \end{cases}$

then at x = 1, f(x) is -

(A) Continuous but not differentiable

(B) Neither continuous nor differentiable

(C) Continuous and differentiable

(D) Differentiable but not continuous

Sol. Since $f(1–0) = \lim_{x \to 1} 3^x = 3$

$f(1+0) = \lim_{x \to 1} (4–x) = 3$

and $f(1) = 3^1 = 3$

f(1–0) = f(1+0) = f(1)

$\therefore$ f(x) is continuous at x = 1

$\Rightarrow$ Again $f'(1+0) = \lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1}$

$= \lim_{x \to 1} \frac{3^x - 3}{x - 1}$

$= \lim_{h \to 0} \frac{3^{1+h} - 3}{h}$

$= 3 \lim_{h \to 0} \frac{3^h - 1}{h}$

= 3 log 3

and $f'(1+0) \quad \lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1}$

$= \lim_{x \to 1} \frac{4 - x - 3}{x - 1} = -1$

$\therefore$ f ' (1+0) ≠ f '(1–0)

≠ f(x) is not differentiable at x = 1.**Ans.[A]**

Ex.16 Function $f(x) = \frac{x}{1 + |x|}$ is differentiable in the set-

(A) (– ∞, ∞) (B) (–∞, 0)

(C) (–∞, 0) ∪ (0, ∞) (D) (0, ∞)

Sol. When x < 0, $f(x) = \frac{x}{1 - x}$

$f'(x) = \frac{1}{(1-x)^2}$ ...(1)

which exists finitely for all x < 0

Also when x > 0, $f(x) = \frac{1}{1 + x}$

$\Rightarrow f'(x) = \frac{1}{(1+x)^2}$ ...(2)

which exists finitely for all x > 0. Also from (1) and (2) we have

$\begin{cases} f'(0-0) = 1 \\ f'(0+0) = 1 \end{cases} \Rightarrow f'(0) = 1$

Hence f(x) is differentiable ∀ x ∈ R **Ans.[A]**

Ex.17 If $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, x \ne 0 \\ 0, \quad x = 0 \end{cases}$, then

(A) f and f ' are continuous at x = 0

(B) f is derivable at x = 0

(C) f and f ' are derivable at x = 0

(D) f is derivable at x = 0 and f ' is continuous at x = 0

Sol. When x ≠ 0

$f'(x) = 2x \sin \frac{1}{x} + x^2 \cos \frac{1}{x} . \left(-\frac{1}{x^2}\right)$

$= 2x \sin \frac{1}{x} - \cos \left(\frac{1}{x}\right)$

which exists finitely for all x ≠ 0

and $f'(0) = \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0} \frac{x^2 \sin 1/x}{x} = 0$

$\therefore$ f is also derivable at x = 0. Thus

$f'(x) = \begin{cases} 2x \sin \frac{1}{x} - \cos \frac{1}{x}, x \ne 0 \\ 0, \quad x = 0 \end{cases}$

Also $\lim_{x \to 0} f'(x) = \lim_{x \to 0} \left(2x \sin \frac{1}{x} - \cos \frac{1}{x}\right)$

$= 2 - \lim_{x \to 0} \cos \frac{1}{x}$

But $\lim_{x \to 0} \cos \frac{1}{x}$ does not exist, so $\lim_{x \to 0} f'(x)$ does not exist. Hence f ' is not continuous (so not derivable) at x = 0. **Ans.[B]**

LEVEL-1

Question based on **Continuity of a function at a point**

Q.1 Function $f(x) = \begin{cases} 1+x, & \text{when } x < 2 \\ 5-x, & \text{when } x > 2 \end{cases}$; $x = 2$ is continuous at $x = 2$, if f(2) equals -

(A) 0 (B) 1 (C) 2 (D) 3

Q.2 If $f(x) = \begin{cases} x\cos\frac{1}{x}, & x \neq 0 \\ k & x = 0 \end{cases}$ is continuous at x = 0, then

(A) k > 0 (B) k < 0

(C) k = 0 (D) $k \geq 0$

Q.3 If function $f(x) = \begin{cases} x^2+2, & x > 1 \\ 2x+1, & x = 1 \end{cases}$ is continuous at x = 1, then value of f(x) for x < 1 is-

(A) 3 (B) 1– 2x

(C) 1– 4x (D) None of these

Q.4 Which of the following function is continuous at x = 0-

(A) $f(x) = \begin{cases} \sin\frac{2x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

(B) $f(x) = \begin{cases} (1+x)^{1/x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

(C) $f(x) = \begin{cases} e^{-1/x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

(D) None of these

Q.5 If $f(x) = \begin{cases} 6 \times 5^x, & x \leq 0 \\ 2a+x, & x > 0 \end{cases}$ is continuous at x = 0, then the value of a is -

(A) 1 (B) 2

(C) 3 (D) None of these

Q.6 If $f(x) = \begin{cases} \frac{x^2-(a+2)x+a}{x-2} & x \neq 2 \\ 2, & x = 2 \end{cases}$ is continuous at x = 2, then a is equal to-

(A) 0 (B) 1 (C) –1 (D) 2

Q.7 If $f(x) = \begin{cases} \frac{\sin^{-1}ax}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at x = 0, then k is equal to-

(A) 0 (B) 1

(C) a (D) None of these

Q.8 What is the value of $(\cos x)^{1/x}$ at x = 0 so that it becomes continuous at x = 0-

(A) 0 (B) 1 (C) –1 (D) e

Q.9 If $f(x) = \begin{cases} \frac{k\cos x}{\pi-2x}, & x \neq \pi/2 \\ 1, & x = \pi/2 \end{cases}$ is a continuous function at $x = \pi/2$, then the value of k is-

(A) –1 (B) 1 (C) –2 (D) 2

Q.10 If function $f(x) = \frac{x^3-a^3}{x-a}$, is continuous at x = a, then the value of f(a) is -

(A) 2a (B) $2a^2$ (C) 3a (D) $3a^2$

Q.11 If $f(x) = \begin{cases} \sin\frac{1}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at x = 0, then k is equal to -

(A) 8 (B) 1

(C) –1 (D) None of these

Q.12 Function $f(x) = \left(1+\frac{x}{a}\right)^{1/x}$ is continuous at x = 0 if f(0) equals-

(A) e^a (B) e^{-a}

(C) 0 (D) $e^{1/a}$

Q.13 If $f(x) = \frac{1 - \cos 7(x-\pi)}{x-\pi}$, $(x \neq \pi)$ is continuous at $x = \pi$, then $f(\pi)$ equals-

(A) 0 (B) 1 (C) –1 (D) 7

Q.14 If $f(x) = \begin{cases} \frac{\tan x}{\sin x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$, then f(x) is -

(A) continuous everywhere
(B) continuous nowhere
(C) continuous at x = 0
(D) continuous only at x = 0

Q.15 If $f(x) = \frac{2x + \tan x}{x}$ is continuous at x = 0, then f(0) equals-

(A) 0 (B) 1 (C) 2 (D) 3

Q.16 If $f(x) = \frac{\sqrt{1+x} - \sqrt[3]{1+x}}{x}$, $(x \neq 0)$ is continuous at x = 0, then the value of f(0) is-

(A) 1/6 (B) 1/4 (C) 2 (D) 1/3

Q.17 If $f(x) = \begin{cases} ax^2 - b & \text{when } 0 \le x < 1 \\ 2 & \text{when } x = 1 \\ x + 1 & \text{when } 1 < x \le 2 \end{cases}$ is continuous at x = 1, then the most suitable values of a, b are-

(A) a = 2, b = 0 (B) a = 1, b = –1
(C) a = 4, b = 2 (D) All the above

Q.18 If $f(x) = \begin{cases} |x|, & \text{when } x < 0 \\ x, & \text{when } 0 \le x < 1 \\ 1, & \text{when } x > 1 \end{cases}$ then f is -

(A) continuous for every real number x
(B) discontinuous at x = 0
(C) discontinuous at x = 1
(D) discontinuous at x = 0 and x =1

Q.19 If $f(x) = \begin{cases} \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$, then it is discontinuous at -

(A) x = 0 (B) All points
(C) No point (D) None of these

Q.20 Function f(x) = x – |x| is-

(A) discontinuous at x = 0
(B) discontinuous at x = 1
(C) continuous at all points
(D) discontinuous at all points

Q.21 Function f(x) = tan x, is discontinuous at-

(A) x = 0 (B) $x = \pi/2$
(C) $x = \pi$ (D) $x = -\pi$

Q.22 Function f(x) = [x] is discontinuous at-

(A) every real number
(B) every natural number
(C) every integer
(D) No where

Q.23 Function $f(x) = 3x^2 - x$ is-

(A) discontinuous at x = 1
(B) discontinuous at x = 0
(C) continuous only at x = 0
(D) continuous at x = 0

Q.24 If $f(x) = \begin{cases} x^2, & \text{when } x \le 0 \\ 1, & \text{when } 0 < x < 1 \\ 1/x, & \text{when } x \ge 1 \end{cases}$, then f(x) is-

(A) continuous at x = 0 but not at x = 1
(B) continuous at x = 1 but not at x = 0
(C) continuous at x = 0 and x = 1
(D) discontinuous at x = 0 and x = 1

Q.25 Function $f(x) = \begin{cases} -1, & x \in Q \\ 1, & x \notin Q \end{cases}$ is-

(A) continuous at x = 0
(B) continuous at x = 1
(C) every where continuous
(D) every where discontinuous

Q.26 If $f(x) = \begin{cases} -x^2, & x \le 0 \\ 5x - 4, & 0 < x \le 1 \\ 4x^2 - 3x, & 1 < x < 2 \\ 3x + 4, & x \ge 2 \end{cases}$, then f(x) is-

(A) continuous at x = 0 but not at x = 1
(B) continuous at x = 2 but not at x = 0
(C) continuous at x = 0,1, 2
(D) discontinuous at x = 0,1, 2

Q.27 Function $f(x) = \begin{cases} \dfrac{x^2 - 4x + 3}{x^2 - 1}, & x \neq 1 \\ 2, & x = 1 \end{cases}$ is-

(A) continuous at x = 1
(B) continuous at x = –1
(C) continuous at x = 1 and x = –1
(D) discontinuous at x = 1 and x = –1

Q.28 Let f(x) = 3– | sin x|, then f(x) is-

(A) Everywhere continuous
(B) Everywhere discontinuous
(C) Continuous only at x = 0
(D) Discontinuous only at x = 0

Q.29 The function $f(x) = \begin{cases} x - 1, & x < 2 \\ 2x - 3, & x \geq 2 \end{cases}$ is a continuous function for-

(A) all real values of x
(B) only x = 2
(C) all real values of x ≠ 2
(D) only all integral values of x

Q.30 If $f(x) = \begin{cases} x \sin x, & 0 < x \leq \pi/2 \\ \dfrac{\pi}{2}\sin(\pi + x), & \dfrac{\pi}{2} < x < \pi \end{cases}$, then -

(A) f(x) is discontinuous at x = π/2
(B) f(x) is continuous at x = π/2
(C) f(x) is continuous at x = 0
(D) None of these

Q.31 The value of k so that

$f(x) = \begin{cases} k(2x - x^2) & \text{when } x < 0 \\ \cos x, & \text{when } x \geq 0 \end{cases}$

continuous at x = 0 is-

(A) 1 (B) 2
(C) 4 (D) None of these

Q.32 If $f(x) = \dfrac{(a+x)^2 \sin(a+x) - a^2 \sin a}{x}$, $x \neq 0$; then the value of f(0) so that f is continuous at x = 0 is-

(A) $a^2 \cos a + a \sin a$ (B) $a^2 \cos a + 2a \sin a$
(C) $2a^2 \cos a + a \sin a$ (D) None of these

Q.33 Let f(x) = |x| + |x–1|, then-

(A) f(x) is continuous at x = 0 and x = 1
(B) f(x) is continuous at x = 0 but not at x = 1
(C) f(x) is continuous at x= 1 but not at x = 0
(D) None of these

Q.34 Consider the following statements:

I. A function f is continuous at a point $x_0 \in$ Dom (f) if $\lim_{x \to x_0} f(x) = f(x_0)$.

II. f is continuous in [a, b] if f is continuous in (a, b) and f(a) = f(b).

III. A constant function is continuous in an interval.

Out of these correct statements are

(A) I and II (B) II and III
(C) I and III (D) All the above

Q.35 If $f(x) = \begin{cases} x + 2, & \text{when } x < 1 \\ 4x - 1, & \text{when } 1 \leq x \leq 3 \\ x^2 + 5, & \text{when } x > 3 \end{cases}$, then correct statement is-

(A) $\lim_{x \to 1} f(x) = \lim_{x \to 3} f(x)$
(B) f(x) is continuous at x = 3
(C) f(x) is continuous at x = 1
(D) f(x) is continuous at x = 1 and 3

Q.36 Let f(x) and ϕ(x) be defined by f(x) = [x] and

$\phi(x) = \begin{cases} 0, & x \in I \\ x^2, & x \in R - I \end{cases}$ [.] = G.I.F.

(A) $\lim_{x \to 1} \phi(x)$ exist but ϕ is not continuous at x = 1
(B) $\lim_{x \to 1} f(x)$ does not exist and f is continuous at x =1
(C) ϕ is continuous for all x
(D) None of these

Q.37 $f(x) = \begin{cases} \dfrac{x - 4}{|x - 4|} + a, & x < 4 \\ a + b, & x = 4 \\ \dfrac{x - 4}{|x - 4|} + b, & x > 4 \end{cases}$ is continuous at x = 4, if-

(A) a = 0, b = 0 (B) a =1, b =1
(C) a = 1, b = –1 (D) a = –1, b = 1

Q.38 The function $f(x) = \frac{\cos x - \sin x}{\cos 2x}$ is continuous everywhere then f (π/4) =

(A) 1 (B) –1

(C) $\sqrt{2}$ (D) $1/\sqrt{2}$

Q.39 If $f(x) = \frac{\tan\left(\frac{\pi}{4} - x\right)}{\cot 2x}$, $x \neq \pi/4$ is every where continuous, then f(π/4) equals-

(A) 0 (B) 1 (C) –1 (D) 1/2

Question based on **Continuity from left and right**

Q.40 If $f(x) = \begin{cases} \frac{x}{e^{1/x} + 1}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then -

(A) $\lim_{x \to 0^+} f(x) = 1$

(B) $\lim_{x \to 0^-} f(x) = 1$

(C) f(x) is continuous at x = 0

(D) None of these

Q.41 If f(x) = [x], where [x] = greatest integer ≤ x, then at x = 1, f is-

(A) continuous (B) left continuous

(C) right continuous (D) None of these

Question based on **Continuity of a function in an interval**

Q.42 If $f(x) = \begin{cases} \frac{\sqrt{1+px} - \sqrt{1-px}}{x}, & -1 \le x < 0 \\ \frac{2x+1}{x-2}, & 0 \le x \le 1 \end{cases}$ is continuous in the interval [–1,1] then p equals -

(A) –1 (B) 1

(C) 1/2 (D) –1/2

Q.43 If $f(x) = \begin{cases} \frac{x^2}{a}, & 0 \le x < 1 \\ a, & 1 \le x < \sqrt{2} \\ \frac{(2b^2 - 4b)}{x^2}, & \sqrt{2} \le x < \infty \end{cases}$ is continuous in the interval [0, ∞), then values of a and b are respectively -

(A) 1, –1 (B) –1, 1+ $\sqrt{2}$

(C) –1, 1 (D) None of these

Q.44 Which of the following function is not continuous in the interval (0, π)

(A) $x \sin \frac{1}{x}$

(B) $\begin{cases} 1, & 0 < x \le \frac{3\pi}{4} \\ 2\sin\left(\frac{2x}{9}\right), & \frac{3\pi}{4} < x < \pi \end{cases}$

(C) tan x

(D) None of these

Question based on **Continuous and discontinuous function**

Q.45 Function f(x) = |x| is-

(A) discontinuous at x = 0

(B) discontinuous at x = 1

(C) continuous at all point

(D) discontinuous at all points

Q.46 Point of discontinuity for sec x is -

(A) x = –π/2 (B) x = 3 π/2

(C) x = –5 π /2 (D) All of these

Q.47 Function $f(x) = \frac{1}{\log |x|}$ is discontinuous at -

(A) one point

(B) two points

(C) three points

(D) infinite number of points

Q.48 If f(x) = x – [x], then f is discontinuous at -

(A) every natural number

(B) every integer

(C) origin

(D) Nowhere

Q.49 Which one is the discontinuous function at any point -

(A) sin x (B) x^2

(C) 1/(1– 2x) (D) $1/(1+ x^2)$

Q.50 The point of discontinuity of cosec x is -

(A) x = π (B) x = π /2

(C) x = 3 π /2 (D) None of these

Q.51 In the following, continuous function is-

(A) tan x (B) sec x

(C) sin 1/x (D) None of these

Q.52 In the following, discontinuous function is-

(A) sin x (B) cos x

(C) tan x (D) sinh x

Q.53 Which of the following functions is every where continuous-

(A) x + |x| (B) x– |x|

(C) x |x| (D) All above

Q.54 Which of the following functions is discontinuous at x = a-

(A) tan (x – a) (B) sin (x – a)

(C) cosec (x – a) (D) sec (x – a)

Q.55 If f(x) is continuous and g(x) is discontinuous function, then f(x) + g(x) is-

(A) continuous function

(B) discontinuous function

(C) constant function

(D) identity function

Q.56 Function f(x) = |x–2| –2| x–4| is discontinuous at

(A) x = 2, 4 (B) x = 2

(C) Nowhere (D) Except x = 2, 4

Q.57 Function f(x) = |sin x| + |cos x| + |x| is discontinuous at-

(A) x = 0 (B) x = π/2

(C) x = π (D) No where

Q.58 Function f(x) = 1+ |sin x| is-

(A) continuous only at x = 0

(B) discontinuous at all points

(C) continuous at all points

(D) None of these

Q.59 If function is f(x) = |x| + |x – 1| + |x – 2|, then it is -

(A) discontinuous at x = 0

(B) discontinuous at x = 0, 1

(C) discontinuous at x = 0, 1, 2

(D) everywhere continuous

Q.60 Function $f(x) = \frac{x^3 - 1}{x^2 - 3x + 2}$ is discontinuous at -

(A) x = 1 (B) x = 2

(C) x = 1, 2 (D) No where

Q.61 If $f(x) = \frac{1}{(1 - x)}$ and g(x) = f [f{f(x)}], then g(x) is discontinuous at -

(A) x = 3 (B) x = 2

(C) x = 0 (D) x = 4

Q.62 The function $f(x) = \frac{|3x - 4|}{3x - 4}$ is discontinuous at

(A) x = 4 (B) x = 3/4

(C) x = 4/3 (D) No where

Q.63 The function $f(x) = \left(\frac{\pi}{2} - x\right)$ tan x is discontinuous at-

(A) x = π (B) x = 0

(C) $x = \frac{\pi}{2}$ (D) None of these

Q.64 Which of the following function has finite number of points of discontinuity-

(A) sin [πx] (B) |x|/x

(C) tan x (D) x + [x]

Q.65 The points of discontinuity of

$f(x) = \tan\left(\frac{\pi x}{x + 1}\right)$ other than x = –1 are-

(A) x = π (B) x = 0

(C) $x = \frac{2m - 1}{2m + 1}$

(D) $x = \frac{2m + 1}{1 - 2m}$, m is any integer

Q.66 In the following continuous function is-

(A) [x] (B) x – [x]

(C) sin [x] (D) $e^x + e^{-x}$

Q.67 In the following, discontinuous function is-

(A) $\sin^2 x + \cos^2 x$ (B) $e^x + e^{-x}$

(C) e^{x2} (D) $e^{1/x}$

Q.68 If f(x) is continuous function and g(x) is discontinuous function, then correct statement is -

(A) f(x) + g(x) is a continuous function
(B) f(x) – g(x) is a continuous function
(C) f(x) + g(x) is a discontinuous function
(D) f(x) g(x) is a continuous function

Question based on **Differentiability of function**

Q.69 Which of the following functions is not differentiable at x = 0-

(A) $x|x|$ (B) x^3
(C) e^{-x} (D) $x + |x|$

Q.70 Which of the following is differentiable function-

(A) $x^2 \sin\frac{1}{x}$ (B) $x|x|$
(C) cosh x (D) all above

Q.71 The function f(x) = sin |x| is-

(A) continuous for all x
(B) continuous only at certain points
(C) differentiable at all points
(D) None of these

Q.72 If f(x) = |x–3|, then f is-

(A) discontinuous at x = 2
(B) not differentiable at x = 2
(C) differentiable at x = 3
(D) continuous but not differentiable at x = 3

Q.73 If $f(x) = \frac{|x-1|}{x-1}$, $x \neq 1$, and f(1) = 1, then the correct statement is-

(A) discontinuous at x = 1
(B) continuous at x = 1
(C) differentiable at x = 1
(D) discontinuous for x > 1

Q.74 If $f(x) = \begin{cases} x+1, & x > 0 \\ 0, & x = 1 \\ 7-3x, & x < 1 \end{cases}$, then f '(0) equals-

(A) 1 (B) 2 (C) 0 (D) –3

Q.75 The function f(x) = |x| + |x – 1| is not differential at -

(A) x = 0,1 (B) x = 0, –1
(C) x = –1, 1 (D) x = 1, 2

Q.76 If $f(x) = \begin{cases} e^{1/x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then which one is correct-

(A) f(x) is differentiable at x = 0
(B) f(x) is discontinuous at x = 0
(C) f(x) is continuous no where
(D) None of these

Q.77 Function [x] is not differentiable at -

(A) every rational number
(B) every integer
(C) origin
(D) every where

Q.78 If $f(x) = \begin{cases} |x-3|, & \text{when } x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & \text{when } x < 1 \end{cases}$, then correct statement is-

(A) f is discontinuous at x = 1
(B) f is discontinuous at x = 3
(C) f is differentiable at x = 1
(D) f is differentiable at x = 3

Q.79 Function $f(x) = \frac{|x|}{x}$ is-

(A) continuous every where
(B) differentiable every where
(C) differentiable every where except at x = 0
(D) None of these

Q.80 Let f(x) = |x– a| + | x – b|, then-

(A) f(x) is continuous for all $x \in R$
(B) f(x) is differential for $\forall\, x \in R$
(C) f(x) is continuous except at x = a and b
(D) None of these

Q.81 Function f(x) = |x – 1| + |x – 2| is differentiable in [0, 3], except at-

(A) x = 0 and x = 3 (B) x = 1
(C) x = 2 (D) x = 1 and x = 2

Q.82 If $f(x) = \begin{cases} 1, & \text{when } x < 0 \\ 1 + \sin x, & \text{when } 0 \le x \le \pi/2 \end{cases}$, then at x = 0, f ' (x) equals-

(A) 1 (B) 0

(C) ∞ (D) Does not exist

Q.83 If $f(x) = \begin{cases} \dfrac{x}{1 + e^{1/x}}, & x \ne 0 \\ 0, & x = 0 \end{cases}$, then the function f(x) is differentiable for -

(A) $x \in R_+$ (B) $x \in R$

(C) $x \in R_0$ (D) None of these

Q.84 If $f(x) = \begin{cases} x^{\alpha} \sin \dfrac{1}{x}, & x \ne 0 \\ 0, & x = 0 \end{cases}$ is differentiable at x = 0, then-

(A) $\alpha > 0$ (B) $\alpha > 1$

(C) $\alpha \ge 1$ (D) $\alpha \ge 0$

Q.85 If $f(x) = \begin{cases} e^{x}, & x \le 0 \\ |1 - x|, & x > 0 \end{cases}$, then f(x) is-

(A) continuous at x = 0

(B) differentiable at x = 0

(C) differentiable at x = 1

(D) differentiable both at x = 0 and 1

Q.86 The function f(x) = x – | x | is not differentiable at

(A) x = 1 (B) x = –1

(C) x = 0 (D) Nowhere

Q.87 Which of the following function is not differentiable at x = 1

(A) $\sin^{-1}x$ (B) tan x

(C) a^x (D) cosh x

Q.88 If $f(x) = \begin{cases} \dfrac{x - 1}{2x^2 - 7x + 5}, & x \ne 1 \\ -\dfrac{1}{3} & x = 1 \end{cases}$, then f '(1) equals -

(A) $\frac{2}{9}$ (B) $-\frac{2}{9}$

(C) 0 (D) Does not exist

Q.89 If $f(x) = \begin{cases} \dfrac{\sin x^2}{x}, & x \ne 0 \\ 0 & x = 0 \end{cases}$, then at x = 0, f(x) is

(A) continuous and differentiable

(B) neither continuous nor differentiable

(C) continuous but not differentiable

(D) None of these

Q.90 Function f(x) = 1 + | sin x | is–

(A) continuous no where

(B) differentiable no where

(C) everywhere continuous

(D) None of these

Q.91 Function $f(x) = \begin{cases} x^2, & x \le 0 \\ 1, & 0 < x \le 1 \\ 1/x, & x > 1 \end{cases}$ is–

(A) differentiable at x = 0, 1

(B) differentiable only at x = 0

(C) differentiable at only x = 1

(D) Not differentiable at x = 0, 1

LEVEL- 2

Q.1 If [.] denotes G.I.F. then, in the following, continuous function is-

(A) $\cos [x]$ (B) $\sin \pi[x]$

(C) $\sin \frac{\pi}{2} [x]$ (D) All above

Q.2 If $f(x) = \frac{1 - \cos(1 - \cos x)}{x^4}$, $(x \neq 0)$ is continuous everywhere, then f(0) equals-

(A) 1/8 (B) 1/2

(C) 1/4 (D) None of these

Q.3 For function $f(x) = \begin{cases} \left(1 + \frac{4x}{5}\right)^{1/x}, & x \neq 0 \\ e^{4/5}, & x = 0 \end{cases}$, the correct statement is-

(A) f(0 + 0) and f(0 – 0) do not exist

(B) $f(0 + 0) \neq f(0 - 0)$

(C) f(x) continuous at x = 0

(D) $\lim_{x \to 0} f(x) \neq f(0)$

Q.4 If $f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x}, & x < 0 \\ c, & x = 0 \\ \frac{\sqrt{x + bx^2} - \sqrt{x}}{bx\sqrt{x}}, & x > 0 \end{cases}$, is continuous at x = 0, then

(A) a = 3/2, c = 1/2, b is any real number

(B) a = –3/2, c =1/2, b is R – {0}

(C) a = 3/2, c = –1/2, b ∈ R – {0}

(D) None of these

Q.5 Function $f(x) = 4x^3 + 3x^2 + e^{\cos x} + |x-3| + \log(a^x - 1) + x^{1/3}$ (a >1) is discontinuous at-

(A) x = 0 (B) x = 1

(C) x = 2 (D) $x = \frac{\pi}{2}$

Q.6 If $f(x) = \frac{\sqrt{a^2 - ax + x^2} - \sqrt{a^2 + ax + x^2}}{\sqrt{a + x} - \sqrt{a - x}}$ is continuous for all values of x, then f(0) is equal to-

(A) $a\sqrt{a}$ (B) $\sqrt{a}$

(C) $-\sqrt{a}$ (D) $-a\sqrt{a}$

Q.7 Function $f(x) = \begin{cases} \frac{(b^2 - a^2)}{2}, & 0 \le x \le a \\ \frac{b^2}{2} - \frac{x^2}{6} - \frac{a^3}{3x}, & a < x \le b \\ \frac{1}{3}\left(\frac{b^3 - a^3}{x}\right), & x > b \end{cases}$, is

(A) continue at x = a

(B) continue at x = b

(C) discontinue on both x = a, x = b

(D) continue at both x = a, x = b

Q.8 The function $f(x) = \begin{cases} \frac{e^{1/x} - 1}{e^{1/x} + 1}, & x \neq 0 \\ 0, & x = 0 \end{cases}$,

(A) is continuous at x = 0

(B) is not continuous at x = 0

(C) is continuous at x = 2

(D) None of these

Q.9 If function $f(x) = \left(\frac{\sin x}{\sin \alpha}\right)^{1/x - \alpha}$ where, $\alpha \neq m\pi$ $(m \in I)$ is continuous then -

(A) $f(\alpha) = e^{\tan \alpha}$ (B) $f(\alpha) = e^{\cot \alpha}$

(C) $f(\alpha) = e^{2\cot \alpha}$ (D) $f(\alpha) = \cot \alpha$

Q.10 If $f(x) = \begin{cases} -2\sin x, & x \le -\pi/2 \\ a\sin x + b, & -\pi/2 < x < \pi/2 \\ \cos x, & x \ge \pi/2 \end{cases}$, is a continuous function for every value x, then-

(A) a = b = 1 (B) a = b = –1

(C) a = 1,b = –1 (D) a = –1, b = 1

Q.11 If function $f(x) = x - |x - x^2|, -1 \le x \le 1$ then f is-

(A) continuous at x = 0
(B) continuous at x = 1
(C) continuous at x = –1
(D) everywhere continuous

Q.12 $f(x) = 1 + 2^{1/x}$ is-

(A) continuous everywhere
(B) continuous nowhere
(C) discontinuous at x = 0
(D) None of these

Q.13 Let [.] denotes G.I.F. and f(x) = [x] + [–x] and m is any integer, then correct statement is -

(A) $\lim_{x \to m} f(x)$ does not exist
(B) f(x) is continuous at x = m
(C) $\lim_{x \to m} f(x)$ exists
(D) None of these

Q.14 If $f(x) = (\tan x \cot \alpha)^{1/(x-\alpha)}$ is continuous at $x = \alpha$, then the value of $f(\alpha)$ is -

(A) $e^{2 \sin 2\alpha}$ (B) $e^{2 \operatorname{cosec} 2\alpha}$
(C) $e^{\operatorname{cosec} 2\alpha}$ (D) $e^{\sin 2\alpha}$

Q.15 Let [.] denotes G.I.F. for the function

$f(x) = \frac{\tan(\pi[x - \pi])}{1 + [x]^2}$ the wrong statement is -

(A) f(x) is discontinuous at x = 0
(B) f(x) is continuous for all values of x
(C) f(x) is continuous at x = 0
(D) f(x) is a constant function

Q.16 The point of discontinuity of the function $f(x) = \frac{1 + \cos 5x}{1 - \cos 4x}$ is-

(A) x = 0 (B) $x = \pi$
(C) $x = \pi/2$ (D) All the above

Q.17 Let $f(x) = \frac{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}{x}$. The value which should be assigned to f at x = 0 so that it is continuous everywhere is-

(A) 1 (B) 2 (C) –2 (D) 1/2

Q.18 If the function

$$f(x) = \begin{cases} \frac{\sin (k+1)x + \sin x}{x}, & \text{when } x < 0 \\ 1/2, & \text{when } x = 0 \\ \frac{(x + 2x^2)^{1/2}}{2x^{3/2}}, & \text{when } x > 0 \end{cases}$$ is

continuous at x = 0, then the value of k is-

(A) 1/2 (B) –1/2
(C) –3/2 (D) 1

Q.19 If $f(x) = \begin{cases} 3, & x < 0 \\ 2x + 1, & x \ge 0 \end{cases}$ then -

(A) both f(x) and f(| x |) are differentiable at x = 0
(B) f(|x|) is differentiable but f(x) is not differentiable at x = 0
(C) f(x) is differentiable but f(|x|) is not differentiable at x = 0
(D) both f(x) and f(|x|) are not differentiable at x = 0

Q.20 The number of points in the interval (0, 2) where the derivative of the function $f(x) = |x - 1/2| + |x - 1| + \tan x$ does not exist is-

(A) 1 (B) 2 (C) 3 (D) 4

Q.21 Function $f(x) = \sin(\pi[x])$ is-

(A) differentiable every where
(B) differentiable no where
(C) not differentiable at x = 1 and –1
(D) None of these

Q.22 Function $f(x) = \begin{cases} x \tan^{-1}(1/x), & x \ne 0 \\ 0, & x = 0 \end{cases}$ at x = 0 is-

(A) discontinuous (B) continuous
(C) differentiable (D) None of these

Q.23 Function $f(x) = \frac{\cos x - \sin x}{\sin 4x}$ is not defined at $x = \frac{\pi}{4}$. The value which should be assigned to f at $x = \frac{\pi}{4}$, so that it is continuous there, is-

(A) 0 (B) $\frac{1}{2\sqrt{2}}$ (C) $-\frac{1}{\sqrt{2}}$ (D) None

Q.24 Let f(x) = max {2 sin x, 1 – cos x), x ∈ (0, π). Then set of points of non-differentiability is -

(A) φ (B) {π/2}

(C) $\{\pi - \cos^{-1} 3/5\}$ (D) $\{\cos^{-1} 3/5\}$

Q.25 If $f(x) = \begin{cases} x\dfrac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then correct statement is-

(A) f is continuous at all points except x = 0

(B) f is continuous at every point but not differentiable

(C) f is differentiable at every point

(D) f is differentiable only at the origin

Q.26 Consider the following statements-

(I) If a function is differentiable at some point then it must be continuous at that point

(II) If a function continuous at some point then it is not necessary that it is differentiable at that point.

(III) Differentiability of a function at some point is the necessary and sufficient condition for continuity at that point.

From above, correct statements are-

(A) I, II, III (B) I, III

(C) I, II (D) II, III

Q.27 State which of the following is a false statement -

(A) If f(x) is continuous at x = a then $f(a) = \lim_{x \to a} f(x)$

(B) If $\lim_{x \to a} f(x)$ exists, then f(x) is continuous at x = a

(C) If f (x) is differentiable at x = a, then it is continuous at x = a

(D) If f(x) is continuous at x= a, then $\lim_{x \to a} f(x)$ exists

LEVEL- 3

Q.1 If the derivative of the function -

$$f(x)=\begin{cases} ax^2+b, & x<-1 \\ bx^2+ax+4, & x\ge -1 \end{cases}$$

is everywhere continuous, then

(A) a = 2, b = 3 (B) a = 3, b = 2

(C) a = –2, b = –3 (D) a = –3, b = –2

Q.2 The value of f(0), so that the function $f(x)=\dfrac{(27-2x)^{1/3}-3}{9-3(243+5x)^{1/5}}$, $(x \neq 0)$ is continuous, is given by -

(A) 2/3 (B) 6 (C) 2 (D) 4

Q.3 If $f(x)=\begin{cases} |x-4|, & \text{for } x\ge 1 \\ \left(x^3/2\right)-x^2+3x+(1/2), & \text{for } x<1 \end{cases}$, then

(A) f(x) is continuous at x = 1 and at x = 4

(B) f(x) is differentiable at x = 4

(C) f(x) is continuous and differentiable at x = 1

(D) f(x) is only continuous at x = 1

Q.4 Let f(x) = |x| and g(x) = $|x^3|$, then –

(A) f(x) & g(x) both are continuous at x = 0

(B) f(x) & g(x) both are differentiable at x = 0

(C) f(x) is differentiable but g(x) is not differentiable at x = 0

(D) f(x) & g(x) both are not differentiable at x = 0

Q.5 Let $f(x)=\begin{cases} x^n \sin \dfrac{1}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}$. Then f(x) is continuous but not differentiable at x = 0 if -

(A) n ∈ (0, 1] (B) n ∈ [0, ∞)

(C) n ∈ (–∞, 0) (D) n = 0

Q.6 If $f(x) = a|\sin x| + be^{|x|} + c|x|^3$ and if f(x) is differentiable at x = 0,
then -

(A) a = b = c = 0

(B) a = 0, b = 0; c ∈ R

(C) b = c = 0; a ∈ R

(D) c = 0, a = 0 ; b ∈ R

Q.7 The set of points where function $f(x)=\sqrt{1-e^{-x^2}}$ is differentiable is -

(A) (–∞, ∞) (B) (–∞, 0) ∪ (0, ∞)

(C) (–1, ∞) (D) none of these

Q.8 Let $f(x)=\begin{cases} \sin 2x, & 0<x\le \pi/6 \\ ax+b, & \pi/6<x<1 \end{cases}$; If f(x) and $f\,'$(x) are continuous, then -

(A) a = 1, b = $\dfrac{1}{\sqrt{2}}+\dfrac{\pi}{6}$

(B) a = $\dfrac{1}{\sqrt{2}}$, b = $\dfrac{1}{\sqrt{2}}$

(C) a = 1, b = $\dfrac{\sqrt{3}}{2}-\dfrac{\pi}{6}$

(D) none of these

Q.9 Let $f(x)=\lim\limits_{n\to\infty}(\sin x)^{2n}$; then f is -

(A) discontinuous at x = 3π/2

(B) discontinuous at x = π/2

(C) discontinuous at x = – π/2

(D) All the above

Q.10 Let [.] denotes G.I.F. and if function f(x) = $\left(\dfrac{x}{2}-1\right)$ then in the interval [0, π]

(A) tan [f(x)] is discontinuous but 1/f(x) is continuous

(B) tan [f(x)] is continuous but $\dfrac{1}{f(x)}$ is discontinuous

(C) tan [f(x)] and f^{-1}(x) is continuous

(D) tan [f(x)] and 1/f(x) both are discontinuous

Q.11 The function $f(x)=\dfrac{4-x^2}{4x-x^3}$ is equal to -

(A) discontinuous at only one point

(B) discontinuous exactly at two points

(C) discontinuous exactly at three points

(D) none of these

Q.12 The function $f(x) = \sin^{-1}(\cos x)$ is -
(A) discontinuous at x = 0
(B) continuous at x = 0
(C) differentiable at x = 0
(D) none of these

Q.13 The function $f(x) = e^{-|x|}$ is -
(A) continuous everywhere but not differentiable at x = 0
(B) continuous and differentiable everywhere
(C) not continuous at x = 0
(D) none of these

Q.14 If x + 4 |y| = 6 y, then y as a function of x is -
(A) continuous at x = 0 (B) derivable at x = 0
(C) $\frac{dy}{dx} = \frac{1}{2}$ for all x (D) none of these

Q.15 Let $f(x + y) = f(x) + f(y)$ and $f(x) = x^2 g(x)$ for all x, y, ∈ R, where g(x) is continuous function. Then $f'(x)$ is equal to -
(A) g' (B) g(x)
(C) $f(x)$ (D) none of these

Q.16 Let $f(x + y) = f(x) f(y)$ for all x, y, ∈ R, Suppose that $f(3) = 3$ and $f'(0) = 11$ then $f'(3)$ is equal to-
(A) 22 (B) 44
(C) 28 (D) none of these

➤ Statement type Questions

All questions are Assertion & Reason type questions. Each of these questions contains two statements: Statement-1 (Assertion) and Statement-2 (Reason). Answer these questions from the following four option.
(A) Statement-I and Statement-II are true Statement-II is the correct explanation of Statement-I
(B) Statement-I Statement-II are true but Statement-II is not the correct explanation of Statement-I.
(C) Statement-I is true but Statement-II is false
(D) Statement-I is false but Statement-II is true.

Q.17 **Statement-1 :**
$f(x) = \frac{1}{x - [x]}$ is discontinuous for integral values of x
Statement-2 : For integral values of x, f(x) is undefined.

Q.18 **Statement-1 :**
If $f(x) = \frac{(e^{kx} - 1)\sin kx}{4x^2}$ $(x \neq 0)$ and f(0) = 9 is continuous at x = 0 then k = ± 6.
Statement-2 : For continuous function
$\lim_{x \to 0} f(x) = f(0)$

Q.19 **Statement I:**
$y = \frac{x}{1 + |x|}$, x ∈ R, f(x) is differentiable every where.
Statement II :
$f(x) = \frac{x}{1+|x|}$, x ∈ R then $f'(x) = \begin{cases} \frac{1}{(1+x)^2}, & x \geq 0 \\ \frac{1}{(1-x)^2}, & x < 0 \end{cases}$

Q.20 **Statement-1 :** If $f(x) = \lim_{n \to \infty} (\sin x)^{2n}$, then the set of points discontinuities of f is $\left\{(2n+1)\frac{\pi}{2},\ n \in I\right\}$
Statement-2 : Since – 1 < sin x < 1, as n → ∞, $(\sin x)^{2n} \to 0$, $\sin x = \pm 1 \Rightarrow \pm (1)^{2n} \to 1$, n → ∞

Q.21 **Statement I :**
f(x) = |x – 2| is differentiable at x = 2.
Statement II :
f(x) = |x – 2| is continuous at x = 2.

Q.22 **Statement-1 :** The function
$y = \sin^{-1}(\cos x)$ is not differentiable at x = nπ, n ∈ Z is particular at x = π
Statement-2 : $\frac{dy}{dx} = \frac{-\sin x}{|\sin x|}$ so the function is not differentiable at the points where sin x = 0.

Q.23 **Statement-1 :**
The function $f(x) = |x^3|$ is differentiable at x = 0
Statement-2 : at x = 0, $f'(x) = 0$

Q.24 **Statement I :** f(x) = sinx and g(x) = sgn(x) then f(x) g(x) is differentiable at x = 1.
Statement II : Product of two differentiable function is differentiable function

Passage Based Questions

Let $f(x) = \begin{cases} \dfrac{a(1 - x\sin x) + b\cos x + 5}{x^2} & , x < 0 \\ 3 & , x = 0 \\ \left\{1 + \left(\dfrac{cx + dx^3}{x^2}\right)\right\}^{1/x} & , x > 0 \end{cases}$

If f is continuous at x = 0

On the basis of above information, answer the following questions : -

Q.25 The value of *a* is -

(A) – 1 (B) ln 3 (C) 0 (D) – 4

Q.26 The value of b is -

(A) – 1 (B) ln 3 (C) 0 (D) – 4

Q.27 The value of c is

(A) 2 (B) 3

(C) 0 (D) none of these

Q.28 The value of e^d is -

(A) 0 (B) 1 (C) 2 (D) 3

Column Matching Questions

Match the entry in Column I with the entry in Column II.

Q.29

Column-I	Column-II
(A) $f(x) = x^2 \sin(1/x), x \neq 0$, $f(0) = 0$	(P) continuous but not derivable
(B) $f(x) = \dfrac{1}{1 - e^{-1/x}}, x \neq 0$, and $f(0) = 0$	(Q) f is differentiable f ' is not continuous
(C) $f(x) = x \sin 1/x, x \neq 0$, $f(0) = 0$	(R) f is not continuous
(D) $f(x) = x^3 \sin 1/x, x \neq 0$, $f(0) = 0$	(S) f ' is continuous but not derivable

Q.30

Column I	Column II
(A) $f(x) = \lvert x^3 \rvert$ is	(P) continuous in (–1, 1)
(B) $f(x) = \sqrt{\lvert x \rvert}$	(Q) differentiable in (–1, 1)
(C) $f(x) = \lvert \sin^{-1} x \rvert$ is	(R) differentiable in (0, 1)
(D) $f(x) = \lvert x \rvert$ is	(S) not differentiable atleast at one point in (–1, 1)

LEVEL- 4

(Question asked in previous AIEEE and IIT-JEE)

SECTION –A

Q.1 If $f(x) = \begin{cases} x & x \in Q \\ -x & x \notin Q \end{cases}$, then f is continuous at-

[AIEEE-2002]

(A) only at zero
(B) only at 0, 1
(C) all real numbers
(D) all rational numbers

Q.2 If for all values of x & y; f(x + y) = f(x) .f(y) and f(5) = 2 f ′(0) = 3, then f ′ (5) is-

[AIEEE- 2002]

(A) 3 (B) 4
(C) 5 (D) 6

Q.3 If $f(x) = \begin{cases} x\,e^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ then f(x) is

[AIEEE- 2003]

(A) discontinuous everywhere
(B) continuous as well as differentiable for all x
(C) continuous for all x but not differentiable at x = 0
(D) neither differentiable nor continuous at x = 0

Q.4 Let $f(x) = \frac{1-\tan x}{4x-\pi}$, $x \neq \frac{\pi}{4}$, $x \in \left[0, \frac{\pi}{2}\right]$. If f(x) is continuous in $\left[0, \frac{\pi}{2}\right]$, then $f\left(\frac{\pi}{4}\right)$ is-

[AIEEE- 2004]

(A) 1 (B) 1/2
(C) – 1/2 (D) –1

Q.5 If f is a real-valued differentiable function satisfying $|f(x) - f(y)| \leq (x-y)^2$, x, y ∈ R and f(0) = 0, then f(1) equals- **[AIEEE-2005]**

(A) –1 (B) 0
(C) 2 (D) 1

Q.6 Suppose f(x) is differentiable at x = 1 and $\lim_{h \to 0} \frac{1}{h} f(1+h) = 5$, then f ′(1) equals -

[AIEEE-2005]

(A) 3 (B) 4
(C) 5 (D) 6

Q.7 The set of points where $f(x) = \frac{x}{1+|x|}$ is differentiable is - **[AIEEE- 2006]**

(A) (–∞, –1) ∪ (–1, ∞)
(B) (–∞, ∞)
(C) (0, ∞)
(D) (–∞, 0) ∪ (0, ∞)

Q.8 The function f : R\{0} → R given by $f(x) = \frac{1}{x} - \frac{2}{e^{2x}-1}$ can be made continuous at x = 0 by defining *f*(0) as - **[AIEEE- 2007]**

(A) 2 (B) –1
(C) 0 (D) 1

Q.9 Let *f* : R → R be a function defined by *f*(x) = Min {x + 1, |x| + 1}. Then which of the following is true? **[AIEEE 2007]**

(A) *f*(x) ≥ 1 for all x ∈ R
(B) *f*(x) is not differentiable at x = 1
(C) *f*(x) is differentiable everywhere
(D) *f*(x) is not differentiable at x = 0

Q.10 Let $f(x) = \begin{cases} (x-1)\sin\frac{1}{x-1}, & \text{if } x \neq 1 \\ 0, & \text{if } x = 1 \end{cases}$

Then which one of the following is true?

[AIEEE 2008]

(A) f is differentiable at x = 0 and at x = 1
(B) f is differentiable at x = 0 but not at x = 1
(C) f is differentiable at x = 1 but not at x = 0
(D) f is neither differentiable at x = 0 nor at x = 1

Statement Based Question : (Q.11 to Q.12)

(A) Statement -1 is true, Statement -2 is true; Statement -2 is a correct explanation for Statement -1

(B) Statement -1 is true, Statement -2 is true; Statement -2 is ***not*** a correct explanation for Statement -1

(C) Statement -1 is true, Statement -2 is false.

(D) Statement -1 is false, Statement -2 is ture.

Q.11 Let $f(x) = x\,|x|$ and $g(x) = \sin x$.

Statement – 1 :

gof is differentiable at $x = 0$ and its derivative is continuous at that point.

Statement – 2 :

gof is twice differentiable at $x = 0$.

[AIEEE 2009]

Q.12 Let $f : R \to R$ be a continuous function defined by $f(x) = \frac{1}{e^x + 2e^{-x}}$

Statement-1 : $f(c) = \frac{1}{3}$, for some $c \in R$.

Statement-2 : $0 < f(x) \le \frac{1}{2\sqrt{2}}$, for all $x \in R$

[AIEEE 2010]

Q.13 The value of p and q for which the function

$$f(x) = \begin{cases} \frac{\sin(p+1)x \sin x}{x} & , \quad x < 0 \\ q & , \quad x = 0 \\ \frac{\sqrt{x + x^2} - \sqrt{x}}{x^{3/2}} & , \quad x > 0 \end{cases}$$

is continuous for all x in R, are :

[AIEEE 2011]

(A) $p = \frac{1}{2}, q = -\frac{3}{2}$ (B) $p = \frac{5}{2}, q = \frac{1}{2}$

(C) $p = -\frac{3}{2}, q = \frac{1}{2}$ (D) $p = \frac{1}{2}, q = \frac{3}{2}$

SECTION-B

Q.1 If $f(x) = \begin{cases} \frac{1 - \cos 4x}{x^2} & , \text{when } x < 0 \\ a & , \text{when } x = 0 \\ \frac{\sqrt{x}}{\sqrt{(16 + \sqrt{x})} - 4} & , \text{when } x > 0 \end{cases}$ is continuous at x = 0, then the value of 'a' will be

[IIT-1990]

(A) 8 (B) – 8

(C) 4 (D) None

Q.2 The following functions are continuous on $(0, \pi)$ **[IIT-1991]**

(A) tan x

(B) $\begin{cases} x \sin x ; & 0 < x \le \pi/2 \\ \frac{\pi}{2} \sin(\pi + x); & \frac{\pi}{2} < x < \pi \end{cases}$

(C) $\begin{cases} 1, & 0 < x \le \frac{3\pi}{4} \\ 2 \sin \frac{2}{9} x, & \frac{3\pi}{4} < x < \pi \end{cases}$

(D) None of these

Q.3 If $f(x) = \begin{cases} x \sin x \text{ , when } 0 < x \le \frac{\pi}{2} \\ \frac{\pi}{2} \sin(\pi + x) \text{ , when } \frac{\pi}{2} < x < \pi \end{cases}$, then -

[IIT-1991]

(A) f(x) is discontinuous at $x = \frac{\pi}{2}$

(B) f(x) is continuous at $x = \frac{\pi}{2}$

(C) f(x) is continuous at x = 0

(D) None of these

Q.4 The function $f(x) = [x] \cos \{(2x - 1)/2\}\pi$, [ ] denotes the greatest integer function, is discontinuous at **[IIT-1995]**

(A) all x

(B) all integer points

(C) no x

(D) x which is not an integer

Q.5 Let f(x) be defined for all x > 0 and be continuous. Let f(x) satisfy $f\left(\frac{x}{y}\right) = f(x) - f(y)$ for all x, y & f(e) = 1. Then- **[IIT Scr.95]**

(A) f(x) is bounded

(B) $f\left(\frac{1}{x}\right) \to 0$ as $x \to 0$

(C) $x f(x) \to 1$ as $x \to 0$

(D) f(x) = log x

Q.6 The function $f(x) = [x]^2 - [x^2]$ (where [y] is the greatest integer less than or equal to y), is discontinuous at - **[IIT-1999]**

(A) All integers

(B) All integers except 0 and 1

(C) All integers except 0

(D) All integers except 1

Q.7 Indicate the correct alternative:

Let [x] denote the greater integer $\le x$ and $f(x) = [\tan^2 x]$, then **[IIT-1993]**

(A) $\lim_{x \to 0} f(x)$ does not exist

(B) f(x) is continuous at x = 0

(C) f(x) is not differentiable at x = 0

(D) $f'(0) = 1$

Q.8 $g(x) = x\, f(x)$, where $f(x) = \begin{cases} x \sin(1/x), & x \ne 0 \\ 0 & x = 0 \end{cases}$

at x = 0 **[IIT-1994]**

(A) g is differentiable but g' is not continuous

(B) both f and g are differentiable

(C) g is differentiable but g' is continuous

(D) None of these

Q.9 Let $f\left(\frac{x+y}{2}\right) = \frac{f(x)+f(y)}{2}$ for all real x and y and $f'(0) = -1$, $f(0) = 1$, then $f'(2) =$ **[IIT-1995]**

(A) 1/2 (B) 1

(C) – 1 (D) – 1/2

Q.10 Let $h(x) = \min\{x, x^2\}$, for every real number of x. Then - **[IIT-1998]**

(A) h is not differentiable at two values of x

(B) h is differentiable for all x

(C) $h'(x) = 0$, for all $x > 1$

(D) None of these

Q.11 The function $f(x) = (x^2 - 1)\,|x^2 - 3x + 2| + \cos(|x|)$ is not differentiable at. **[IIT-1999]**

(A) –1 (B) 0

(C) 1 (D) 2

Q.12 Let $f : R \to R$ is a function which is defined by $f(x) = \max\{x, x^3\}$ set of points on which f (x) is not differentiable is **[IIT Scr. 2001]**

(A) {–1, 1} (B) {–1, 0}

(C) {0, 1} (D) {–1, 0, 1}

Q.13 Find left hand derivative at x = k, $k \in I$.

$f(x) = [x] \sin(\pi x)$ **[IIT Scr. 2001]**

(A) $(-1)^k (k-1)\pi$ (B) $(-1)^{k-1} (k-1)\pi$

(C) $(-1)^k (k-1)k\pi$ (D) $(-1)^{k-1} (k-1)k\pi$

Q.14 Which of the following functions is differentiable at x = 0? **[IIT Scr. 2001]**

(A) $\cos(|x|) + |x|$ (B) $\cos(|x|) - |x|$

(C) $\sin(|x|) + |x|$ (D) $\sin(|x|) - |x|$

Q.15 $f(x) = |\,|x| - 1|$ is not differentiable at x = **[IIT Scr.2005]**

(A) 0 , ±1 (B) ± 1

(C) 0 (D) 1

Q.16 Let $g(x) = \frac{(x-1)^n}{\log \cos^m (x-1)}$; 0 < x < 2, m and n are integers, $m \ne 0$, $n > 0$, and let p be the left hand derivative of |x – 1| at x = 1. If $\lim_{x \to 1^+} g(x) = p$, then **[IIT- 2008]**

(A) n = 1, m = 1 (B) n = 1, m = –1

(C) n = 2, m = 2 (D) n > 2, m = n

Q.17 Let $f : R \to R$ be a function such that

$f(x + y) = f(x) + f(y), \forall x, y \in R$

If $f(x)$ is differentiable at $x = 0$, then **[IIT- 2011]**

(A) $f(x)$ is differentiable only in a finite interval containing zero

(B) $f(x)$ is continuous $\forall\, x \in R$

(C) $f'(x)$ is constant $\forall\, x \in R$

(D) $f(x)$ is differentiable except at finitely many points

Q.18 If

$$f(x) = \begin{cases} -x - \frac{\pi}{2}, & x \le -\frac{\pi}{2} \\ -\cos x, & -\frac{\pi}{2} < x \le 0 \\ x - 1, & 0 < x \le 1 \\ \ln x, & x > 1 \end{cases}$$, then

[IIT- 2011]

(A) f (x) is continuous at $x = -\frac{\pi}{2}$

(B) f (x) is not differentiable at x = 0

(C) f (x) is differentiable at x = 1

(D) f (x) is differentiable at $x = -\frac{3}{2}$

ANSWER KEY

LEVEL-1

Que	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	D	C	A	D	C	A	C	B	D	D	D	D	A	C	D	A	D	C	A	C
Que	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans	B	C	D	B	D	B	D	A	A	A	D	B	A	C	C	A	C	D	D	C
Que	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	C	D	C	C	C	D	C	B	C	A	D	C	D	C	B	C	D	C	D	C
Que	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	C	C	C	B	D	D	D	C	D	D	A	D	A	D	A	B	B	C	C	A
Que	81	82	83	84	85	86	87	88	89	90	91									
Ans.	D	D	C	B	A	C	A	B	A	C	D									

LEVEL-2

Que	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	B	A	C	B	A	C	D	B	B	D	D	C	C	B	A	D	A	C	D	C
Que	21	22	23	24	25	26	27													
Ans	A	B	B	C	B	C	B													

LEVEL-3

Que	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	A	C	A	A	A	B	B	C	D	D	C	B	A	A	D	D	A	A	A	A
Que	21	22	23	24	25	26	27	28												
Ans	D	A	A	A	A	D	C	D												

29. $A \to Q$; $B \to R$; $C \to P$; $D \to S$

30. $A \to P,Q,R$; $B \to P,R,S$; $C \to P,R,S$; $D \to P,R,S$

LEVEL- 4

SECTION-A

Que	1	2	3	4	5	6	7	8	9	10	11	12	13
Ans.	A	D	C	C	B	C	B	D	C	B	C	A	C

SECTION-B

Que	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Ans.	A	C	A	C	D	D	B	A	C	A	D	D	A	D	A	C	B,C	A,B,C,D