Paper Code : 1001CJA102118069

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

CLASSROOM CONTACT PROGRAMME

(Academic Session : 2018 - 2019)

JEE (Main + Advanced) : ENTHUSIAST COURSE : SCORE-II

ANSWER KEY : PAPER-1 **TEST DATE : 10-02-2019**

Test Type : FULL SYLLABUS

Test Pattern : JEE-Advanced

PART-1 : PHYSICS

	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	A,B,C,D	A,B,C	A,C,D	A,B	A,C	B,D	B,C,D	A,B,D	A,D	A,B,C
	Q.	11	12								
	A.	B	A,B,C								
SECTION-III	Q.	1	2	3	4	5	6	7	8		
	A.	4	1	5	1	3	2	8	4		

PART-2 : CHEMISTRY

	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	A,B,C	A,B	A,B,C	A,B,D	A,C,D	B,D	A,C,D	A,C	C	A
	Q.	11	12								
	A.	A,C	A,B,D								
SECTION-III	Q.	1	2	3	4	5	6	7	8		
	A.	9	3	2	4	1	7	3	6		

PART-3 : MATHEMATICS

	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	A,C	A,C	A,C,D	A,B,C	A,B	B,D	A,B	B,C,D	B	C
	Q.	11	12								
	A.	A,C,D	A								
SECTION-III	Q.	1	2	3	4	5	6	7	8		
	A.	7	7	2	1	8	6	4	1		

Paper Code : 1001CJA102118070

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

CLASSROOM CONTACT PROGRAMME

(Academic Session : 2018 - 2019)

JEE (Main + Advanced) : ENTHUSIAST COURSE : SCORE-II

ANSWER KEY : PAPER-2 **TEST DATE : 10-02-2019**

Test Type : FULL SYLLABUS

Test Pattern : JEE-Main

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	2	1	4	3	3	4	2	2	3	2	1	1	4	1	3	2	2	3	2	3
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	2	4	3	2	3	1	4	2	1	4	2	2	3	2	2	1	4	1	2	1
Que.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	3	4	4	3	1	3	3	4	4	2	1	3	4	4	4	2	1	1	3	2
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	2	1	2	2	2	3	2	3	4	2	1	3	2	3	1	3	4	2	2	3
Que.	81	82	83	84	85	86	87	88	89	90										
Ans.	2	4	3	4	1	2	2	1	1	1										

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) — path to success

CLASSROOM CONTACT PROGRAMME
(Academic Session : 2018 - 2019)

JEE (Main + Advanced) : ENTHUSIAST COURSE

SCORE : II

Test Type : FULL SYLLABUS   Test Pattern : JEE-Advanced

TEST DATE : 10 - 02 - 2019

PAPER-1

PART-1 : PHYSICS   SOLUTION

SECTION-I

1. Ans. (A,B,C,D)

Sol. (A) $\vec{F}_{net} = \vec{O} \Rightarrow$ particle is in equilibrium

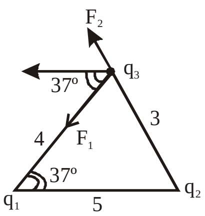
(B) Bottom most point is AOR, so no point has vertical velocity

(C) Because of angular momentum conservation, as 'I' increases 'w' must decrease.

(D) The resultant of centripetal force and tangential force will be skew.

2. Ans. (A,B,C)

Sol.



As per the given condition 'q_1' must be negative & 'q_2' must be positive

$F_1 \cos 53 = F_2 \cos 37°$

solving this we get

$q_2 = \frac{27}{32} \mu C$.

3. Ans. (A,C,D)

Sol. (A) Instanding waves phase difference between 2 points can be only zero or π.

(B) The energy of small element of string at '0' keeps on chaning with time.

(C) It is the property of standing waves.

(D) $y(x,t)$ = A sinkx cos wt.

$$\frac{dy(x,t)}{dt} = -Aw\sin kx \sin wt$$

There will be only 2 points between A & O which will have same speed.

4. Ans. (A,B)

Sol. (A) $\lambda_i = \frac{10u - \left(\frac{u}{2}\right)}{f} = \frac{19u}{2f}$

(B) $f_i = \frac{10u - u}{10u - \frac{u}{2}}.f = \frac{18f}{19}$

(C) $\lambda_r = \frac{10u + u}{f_r} = \frac{11u \times 19}{18f} = \frac{11 \times 19u}{18f}$

(D) $f_r = \frac{18f}{19}$

5. Ans. (A,C)

Sol. $dQ = dW + dU = \frac{3}{2}dW = 3dU$

$ncdT = 3nC_v\,dT$

$\Rightarrow C_V = \frac{5R}{2} = \frac{fR}{2}$;

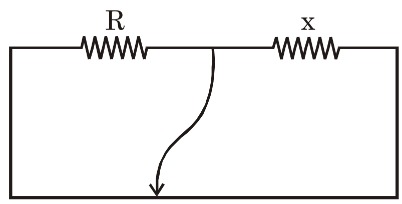
Degree of freedom = 5 Diatomic

6. Ans. (B,D)

Sol. Use Ampere's circuital law.

7. Ans. (B,C,D)

Sol.



$\frac{R_1}{x} = \frac{40}{60} = -\frac{2}{3}$

$\Rightarrow R_1 = \frac{2}{3}x$

Now, $\frac{R_2}{x} = \frac{50}{50} \Rightarrow R_2 = x$

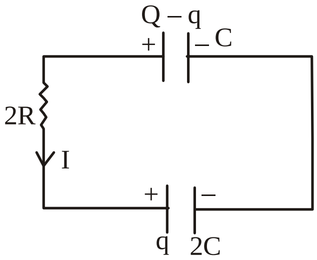
8. Ans. (A,B,D)

Sol. Conceptual

9. Ans. (A,D)

10. Ans. (A,B,C)

Sol. at 't' time



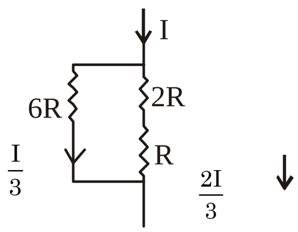
$$\frac{Q-q}{C}-2IR-\frac{q}{2c}=0$$

$$\frac{2Q-3q}{4C}=\frac{dq}{dt}R$$

$$\int_0^q \frac{dq}{2Q-3q}=\int_0^t \frac{dt}{4RC}$$

$$\ln\left(\frac{2Q-3q}{2Q}\right)=-\frac{3t}{4RC}$$

$$q=\frac{2Q}{3}\left(1-e^{-\frac{3t}{4RC}}\right)$$



total power = $2I^2R = P$

Power of resistance = $\frac{4I^2}{9}R=\frac{2}{9}P$

so, heat product in R = $\frac{2}{9}H=\frac{2}{9}\left(\frac{Q^2}{3c}\right)=\frac{2Q^2}{27c}$

11. Ans. (B)

12. Ans. (A,B,C)

Sol. Particle would travel in cycloidal path.

$x = vt - R\cos\omega t$

$y = R(1 - \cos\omega t)$

where, $v=\frac{E_0}{B_0}$, $\omega=\frac{qB_0}{m}$, $R=\frac{mE_0}{qB_0^2}$

$$y_{max}=2R=\frac{2mE_0}{qB_0^2}$$

and radius of curvature at y_{max}

$$=4R=\frac{4mE_0}{qB_0^2}$$

SECTION–III

1. Ans. 4

Sol. $T=2\pi\sqrt{\frac{I}{C}}=2\pi\sqrt{\frac{\frac{2}{5}mR^2}{C}}=4\,\text{sec}$

2. Ans. 1

Sol. $\Delta x=(\mu_{rel}-1)t=\left[\frac{\mu_2}{\mu_1}-1\right]t$

$\mu_1=\frac{4}{3}$

$\mu_2=\frac{3}{2}$

$t = 8\ \mu m$

$\Delta x = 1\mu m$

3. Ans. 5

Sol. Temp. is increased by $\Delta\theta$ then

$\Delta l = l\alpha\Delta\theta$

$\Rightarrow \quad \Delta\theta=\frac{\Delta l}{l\alpha}$

$E_1=(\rho A l)S\Delta\theta=\rho A l\,S\frac{\Delta l}{l\alpha}$

when stretched, Stress = $Y\frac{\Delta l}{l}$

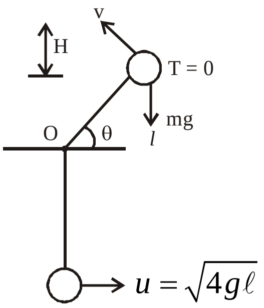
$$E_2=\frac{1}{2}\left(Y\frac{\Delta l}{l}\right)\left(\frac{\Delta l}{l}\right)\times Al=\frac{Y(\Delta l)^2A}{2l}$$

So, $\frac{E_1}{E_2}=\frac{\rho A l S\Delta l\times 2l}{l\times Y(\Delta l)^2A}=\frac{2\rho S l}{\alpha(\Delta l)Y}=500$

4. Ans. 1

Sol. $mg\cos(90-\theta)=\frac{mv^2}{\ell}$ (1)

$\frac{1}{2}mu^2-\frac{1}{2}mv^2=mg\ell(1+\cos\theta)$ (2)



solving, $\sin\theta=\frac{2}{3}$

for projectile motion, $H=\frac{v^2\sin^2(90-\theta)}{2g}$

$H=\frac{5\ell}{27}$

H_{max} for ground $=\ell+\ell\sin\theta+\frac{5\ell}{27}$

$=\frac{50\ell}{27}$

for ℓ = 54cm, H_{max} = 1 m

5. Ans. 3

Sol. For image formed by lens

$\frac{1}{v_1}-\frac{1}{-15}=\frac{1}{10}\Rightarrow v_1=+30cm$

i.e. 20 cm behind mirror For mirror

$\frac{1}{v_2}+\frac{1}{20}=\frac{1}{-20}\Rightarrow v_2=-10$ cm

Overall magnification = $\left(\frac{30}{-15}\right)\times\left(\frac{10}{20}\right)=1$

Length of image = 1× 3 = 3mm

6. Ans. 2

Sol. $d\phi=(2\pi xdx)kxt^2$

$\phi=\frac{2\pi}{3}kx^3t^2$

$E\times 2\pi x=\frac{4\pi kx^3t}{3}$

$E=\frac{2ktx^2}{3}$

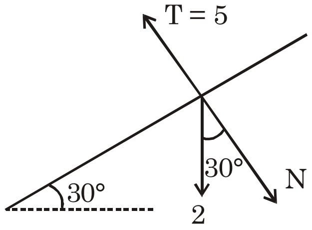
$d\tau=\left(\frac{Q}{\pi R^2}2\pi xdx\right)\frac{2}{3}ktx^2x$

$\tau=\frac{4ktQ}{3R^2}\int_0^R x^4dx$

$=\frac{4ktQR^3}{15}$

7. Ans. 8

Sol.



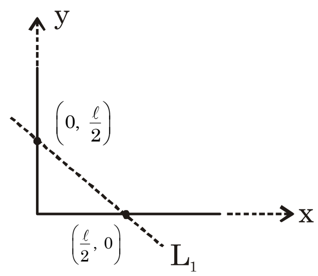
$N+\sqrt{3}=5$

$f_\ell=\mu N$

$\Rightarrow 1=\mu(5-\sqrt{3})\Rightarrow\mu=\frac{1}{5-\sqrt{3}}$

8. Ans. 4

Sol.



Centre of mass must lie on line L_1

$\frac{x}{\frac{\ell}{2}}+\frac{y}{\frac{\ell}{2}}=1$

$2x+2y=\ell$

a + b = 4

PART–2 : CHEMISTRY SOLUTION

SECTION-I

1. Ans. (A,B,C)

$$\underset{0.1-x}{CO_3^{2-}} + H_2O \rightleftharpoons \underset{x-y}{HCO_3^{2-}} + \underset{x+y}{OH^-}, \quad K_{h_1} = \frac{10^{-14}}{10^{-9}} = 10^{-5}$$

$$\underset{x-y}{HCO_3^{-}} + H_2O \rightleftharpoons \underset{y}{H_2CO_3} + \underset{x+y}{OH^-}, \quad K_{h_2} = \frac{10^{-14}}{10^{-5}} = 10^{-9}$$

Neglect y w.r.t. x, then

$$K_{h_1} = 10^{-5} = \frac{x^2}{0.1-x} \approx \frac{x^2}{0.1} \Rightarrow x \approx 10^{-3}$$

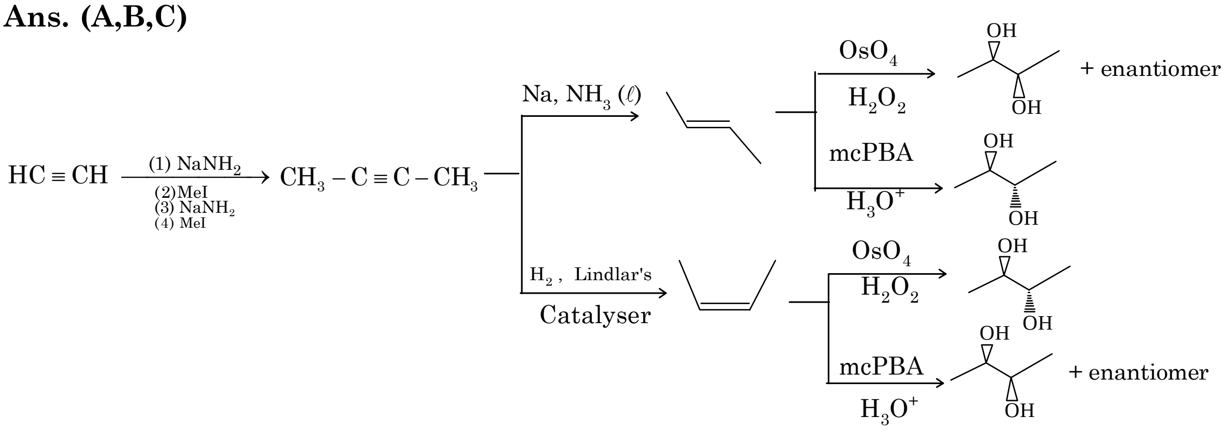
$$y = K_{h_2} = 10^{-9} = [H_2CO_3]$$

$$[OH^-] \approx x = 10^{-3} M \Rightarrow pOH = 3,\ pH=11$$

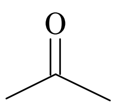
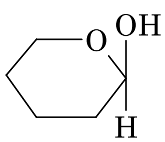
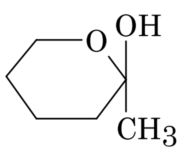
$$[Co_3^{2-}] \approx 0.1M$$

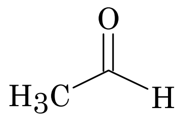
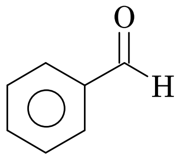
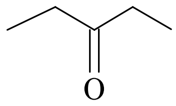
2. Ans. (A,B)

3. Ans. (A,B,C)

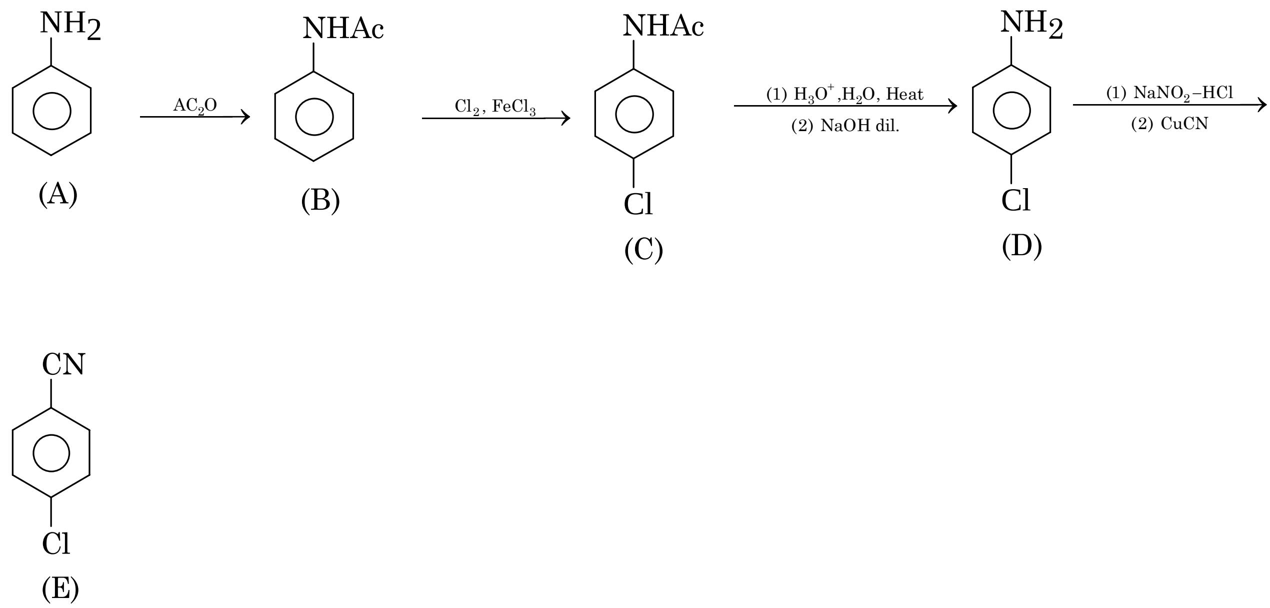


4. Ans. (A,B,D)

Comp.	Fehling Test	Tollen's test
HCHO	Y	Y
	N	N
	Y	Y
	N	N

	Y	Y
	N	Y
	N	N

5. Ans. (A, C, D)



6. Ans. (B,D)

7. Ans. (A,C, D)

$$2NaHCO_3 \xrightarrow{\Delta} Na_2CO_3 + CO_2\uparrow + H_2O\uparrow$$

$$Na_2CO_3.10H_2O \xrightarrow{\Delta} Na_2CO_3 + 10\,H_2O\uparrow$$

$$4NaNO_3 \xrightarrow{\Delta} 2Na_2O + 2N_2\uparrow + 5O_2\uparrow$$

$$CaC_2O_4 \xrightarrow{\Delta} CaO + CO\uparrow + CO_2\uparrow$$

8. Ans. (A, C)

$Fe + dilH_2SO_4 \rightarrow FeSO_4 + H_2$

9. Ans. (C)

At 27°C

$E_{cell} = 0.4 - 2 \times 10^{-6}\ (300)^2 = 0.4 - 0.18 = 0.22$ V

$\Delta G = -\ 0.22$ F

10. Ans. (A)

$$\left[\frac{dE_{cell}}{dT}\right]_P = -4\times10^{-6}\times T = -1.2\times10^{-3}\ V/K$$

$\Delta S = 1 \times F \times (-1.2 \times 10^{-3}) = -115.8$ J/K

11. Ans. (A, C)

$4BF_3 + 3H_2O \rightarrow H_3BO_3 + 3HBF_4$

$SiCl_4 + 4H_2O \rightarrow H_4SiO_4 + 4\ HCl$

$3SiF_4 + 4H_2O \rightarrow H_4SiO_4 + 2H_2SiF_6$

$SOCl_2 + H_2O \rightleftharpoons H_2SO_3 + 2HCl$

12. Ans. (A, B, D)

$PCl_3 + 3H_2O \rightarrow H_3PO_3 + 3HCl$

$PCl_5 + H_2O \rightarrow POCl_2 + 2HCl$

$2NCl_3 + 6H_2O \rightarrow 2NH_3 + 6HClO$

SECTION–III

1. Ans. (9)

Initial mass ratio of H_2 & He = 1 : 4

$\Rightarrow$ Initial molar ratio of H_2 & He = 1 : 2

Final molar ratio of H_2 and He = 8 : 1

we have,

$$\left(\frac{n_{H_2}}{n_{H_e}}\right)_{n-step} = \left(\frac{n_{H_2}}{n_{H_e}}\right)_{initial}\left(\sqrt{\frac{M_{H_e}}{M_{H_2}}}\right)^n$$

$$\frac{8}{1} = \frac{1}{2}.\left(\sqrt{\frac{4}{2}}\right)^n \Rightarrow n = 8$$

$\Rightarrow$ minimum number of containers required = n + 1 = 9

2. Ans. (3)

G = H – TS

As process is reversible adiabatic the entropy stays constant.

$$\Rightarrow dG = dH - SdT$$

$$\Delta G = \Delta H - S\Delta T = nC_{P,m}\Delta T - S\Delta T = 1\times\frac{7}{2}\times R\Delta T - S\Delta T$$

$$= \frac{7}{2}\times 2\times\Delta T - 3\Delta T = 4\times\Delta T$$

for adiabatic process

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$T_2 = T_1\times\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 1000\times\left(\frac{1}{32}\right)^{7/5-1} = 250\,K$$

$$\Delta T = 250 - 1000 = -750$$

$$\Rightarrow \Delta G = 4\times 4T = 4\times -750 = -\ 3000\ \text{cal}$$

$$\Rightarrow |\Delta G| = 3\,kcal$$

3. Ans. (2)

$$\frac{d}{dT}(\ell nK) = \frac{E_a}{RT^2}$$

$$\int d(\ell n K) = \int \frac{E_a}{RT^2}.dT$$

$$\ell nK = \int \frac{0.01T^2 + \frac{20}{7}T}{RT^2}.dT$$

$$\ell nk = \frac{0.01}{R}.T + \frac{20}{7R}\ell nT + C \Rightarrow \ell n\frac{k_2}{k_1} = \frac{0.01}{R}(400-200) + \frac{20}{7R}\ell n\frac{400}{200} = 1 + 1 = 2$$

4. Ans. (4)

$$R-CH(OH)+CH_3COCl \longrightarrow R-CH-O-\overset{\overset{\displaystyle O}{\|}}{C}-CH_3$$

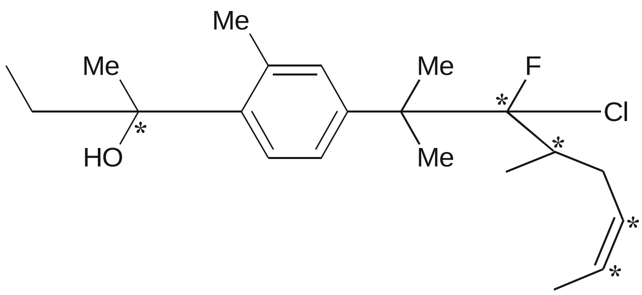
for every acetylation reaction, molecular mass increases by 42

No. of OH group $= \frac{318-150}{42} = 4$

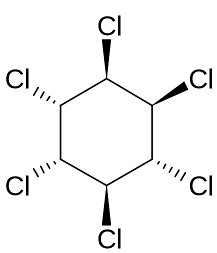
5. Ans. (1)

Only (f)

6. **Ans. (7)**

(a) 

= 5

(b) 

and its enantiomer = 2

7. Ans. (3)

SO_2, SO_3, $N(SiH_3)_3$

8. Ans. (6)

$PbCrO_4$, $BaCrO_4$, $(NH_4)_3PO_4.12MoO_3$, $K_3[Co(NO_2)_6]$AgI, PbI_2

PART-3 : MATHEMATICS SOLUTION

SECTION-I

1. Ans. (A,C)

Sol. $f(x) = 1-(\sin 2x)\sqrt[4]{2\left(\tan^2 x+3\right)+\tan^4 x-5}$

$= 1-(\sin 2x)\sqrt[4]{\tan^4 x+2\tan^2 x+1}$

$=1-(\sin 2x)\sqrt[4]{\left(\tan^2 x+1\right)^2} = 1-(\sin 2x)\sec x$

$= 1-2\sin x \in [1,3)$

2. Ans. (A,C)

Sol. tangents are perpendicular
so they intersect on M(2, 0) lie on directrix

mid point of A(1, 1) and B(4, 2) is $\left(\frac{5}{2},\frac{3}{2}\right)$

slope of axis is = $\frac{\frac{3}{2}-0}{\frac{5}{2}-2}=3$

equation of directrix, $y-0=-\frac{1}{3}(x-2)$

$\Rightarrow 3y+x=2$

AM = AS (S is the focus and M is the foot of perpendicular from A) $\Rightarrow \frac{2}{\sqrt{10}}$

BN = BS (S is the focus and N is the foot of perpendicular from B) $\Rightarrow \frac{8}{\sqrt{10}}$

AS : BS = 1 : 4

$\Rightarrow S\left(\frac{1\times4+4\times1}{5},\frac{1\times2+4\times1}{5}\right)=\left(\frac{8}{5},\frac{6}{5}\right)$

solve equation of directrix and eq. of axis to get foot of directrix. vertex is mid point of foot of directrix and focus.

3. Ans. (A,C,D)

Sol. ${}^{2n}C_n\left(\frac{1}{2}\right)^n\left(\frac{1}{2}\right)^n=\sum_{r=0}^{n}\left(\frac{{}^nC_r}{2^n}\right)^2$ = (C) option

${}^{2n}C_n\left(\frac{1}{2}\right)^n\left(\frac{1}{2}\right)^n=\frac{2n!}{n!n!2^{2n}}$

$=\frac{2^n.n!1.3.5....(2n-1)}{n!n!2^{2n}}=\frac{1.3.5......(2n-1)}{n!2^n}$

$\prod_{r=1}^{n}\left(\frac{2r-1}{2r}\right)$ = (A) option

$\prod_{r=1}^{n}\left(\frac{n+r}{2r}\right)=\frac{(n+1)(n+2)(n+3)....(2n)}{2.4.6......2n}$

$=\frac{2n!}{n!2^n.n!}=\frac{{}^{2n}C_n}{2^n}$

= (B) option wrong

$\frac{\sum_{r=0}^{n}\left({}^nC_r\right)^2}{\left(\sum_{r=0}^{2n}{}^{2n}C_r\right)}=\frac{\left({}^nC_0\right)^2+\left({}^nC_1\right)^2+.....+\left({}^nC_n\right)^2}{{}^{2n}C_0+.....+{}^{2n}C_n}=\frac{{}^{2n}C_n}{2^{2n}}$

= option (D)

4. Ans. (A,B,C)

Sol. $P\left(2^2.5^2\right)=2^2.5^2\left(1-\frac{1}{2}\right)\left(1-\frac{1}{5}\right)=2.5.4$

$\because$ no. of numbers multiple of 2 = 2.5^2
no. of numbers multiple of 5 = $2^2.5$
no. of numbers multiple of both = 2.5

$\therefore P\left(2^2.5^2\right)=2^2.5^2-2.5^2-2^2.5+2.5$

$=2^25^2\left(1-\frac{1}{2}\right)\left(1-\frac{1}{5}\right)$

Similarly check all options.

5. Ans. (A,B)

Sol. Let the equation of circle 's' is given by

$(x-1)^2+(y-1)^2+\lambda(x+y-2)=0$ ... (1)

sine circle 'x' is orthogonal to the circle $x^2+y^2+2x+2y-2=0$ then $\lambda=1$

$\Rightarrow$ equation of 's' is $x^2+y^2-x-y=0$

$\Rightarrow$ now the length of the tangent from (2, 2) is $\sqrt{s}=2$.

and the equation of the director circle is

$2x^2+2y^2-2x-2y-1=0$

6. Ans. (B,D)

Sol. $I_m=\int_0^{m\pi}x|\sin x|e^{\cos 4x}dx$ .... (1)

by using the property

$\int_0^a f(x)dx=\int_0^q f(a-x)dx$

$$I_m = \int_0^{m\pi} (m\pi - x)|\sin x| e^{\cos 4x} dx \quad ....(2)$$

adding eq. (1) & eq. (2) we get

$$2I_m = m\pi \int_0^{m\pi} |\sin x| e^{\cos 4x} dx$$

by using periodic property

$$2I_m = m^2\pi \int_0^{\pi} |\sin x| e^{\cos 4x} dx$$

$$2I_m = 2m^2\pi \int_0^{\pi/2} \sin x\, e^{\cos 4x} dx$$

$$\Rightarrow I_m = m^2\pi \int_0^{\pi/2} \sin x\, e^{\cos 4x} dx \quad ....(3)$$

and

$$J_n = n\int_0^{\pi} |\cos x| e^{\cos 4x} dx = 2n\int_0^{\pi/2} \sin x e^{\cos 4x} dx \quad .... (4)$$

on dividing eq. (3) and (4)

$\dfrac{I_m}{J_n} = \dfrac{m^2\pi}{2n}$ now B,D is correct

7. Ans. (A,B)

Sol. Since $f(x) = \cos x \sin 2x = 2\sin x - 2\sin^3 x$

so $\min f(x) = \min g(t)$ where

$g(t) = 2t - 2t^3$ where $t \in [-1,1]$

Now for maxima/minima, we have

$g'(t) = 2 - 6t^2 = 0 \Rightarrow t = \pm\dfrac{1}{\sqrt{3}}$

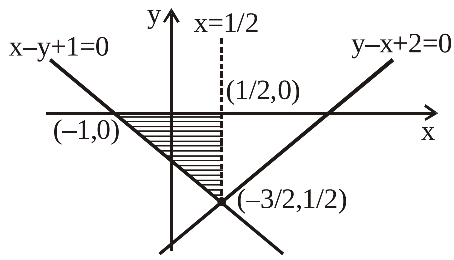
Again $g''(t) = -12t$

$\therefore g''\left(\dfrac{1}{\sqrt{3}}\right) < 0$ & $g'\left(\dfrac{-1}{\sqrt{3}}\right) > 0$

hence $\min g(t) = g\left(\dfrac{-1}{\sqrt{3}}\right) = -\dfrac{4}{3\sqrt{3}} > \dfrac{-7}{9} > \dfrac{-9}{7}$

8. Ans. (B,C,D)

Sol. Domain $x + y + 1 > 0$,
$y - x + 2 > 0$
Squring $x + y + 1 < y - x + 2$



i.e. $x < \dfrac{1}{2}$

Shaded region indicated in the figure

and its area $= \dfrac{1}{2}.\dfrac{3}{2}.\dfrac{3}{2} = \dfrac{9}{8}$

Paragraph for Questions 9 and 10

9. Ans. (B)

Sol. $(AB + BA)^T = (AB)^T + (BA)^T$
$= B^TA^T + A^TB^T = BA + AB$
$(AB - BA)^T = (AB)^T - (BA)^T$
$= B^TA^T - A^TB^T = BA - AB = -(AB - BA)$

10. Ans. (C)

Sol. Since A is skew-symmetric matrix therefore $|A| = 0$

Paragraph for Questions 11 and 12

11. Ans. (A,C,D)

Sol. Using the concept of family of planes let required equation of plane is

$p_1 + \lambda p_2 = 0$, since the plane passes through $(3,2,1)$ therefore

$\Rightarrow 2x - y + z - 2 + \lambda(x + 2y - z - 3)$

$\Rightarrow \lambda = -1$,

required plane is $x - 3y + 2z + 1 = 0$

12. Ans. (A)

Sol. Equation of the plane passes through $(-1,3,2)$

is $a(x+1) + b(y-3) + c(z-2) = 0.....(1)$

(1) is perpendicular to L = 0, (Dr's of L are $(2i - j + k) \times (i + 2j - \hat{k})$) we get

$\dfrac{a}{-1} = \dfrac{b}{3} = \dfrac{c}{5}$ therefore equation of plane is

$(\vec{r} - (-\hat{i} + 3\hat{j} + 2\hat{k})).(\hat{i} - 3\hat{j} - 5\hat{k}) = 0$

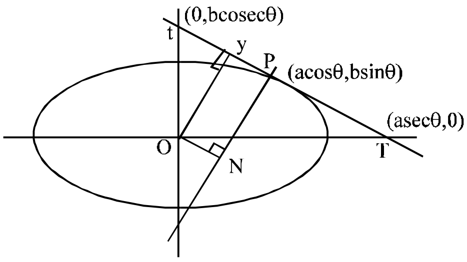
SECTION–III

1. Ans. 7

Sol. Tangent at P : $\dfrac{x\cos\theta}{a} + \dfrac{y\sin\theta}{b} = 1$

T : $(a \sec\theta, 0)$, t : $(0, b\,\text{cosec}\theta)$

hence $Tt = \sqrt{a^2\sec^2\theta + b^2\text{cosec}^2\theta}$



To find Py, we draw normal at P.
now ON = Py
Hence Normal at P

$$\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 - b^2$$

$$ON : \frac{a^2 - b^2}{\sqrt{a^2\sec^2\theta + b^2\operatorname{cosec}^2\theta}} = Py$$

Hence Py. Tt = $a^2 - b^2$]

2. Ans. 7

Sol. $\frac{f(x)}{1+x^2} = 1 + \int_0^x \frac{f^2(t)}{1+t^2}\,dt \Rightarrow f(0) = 1$

differentiate w.r.t. 'x' :

$$\frac{(1+x^2).f'(x) - 2x\,f(x)}{(1+x^2)^2} = \frac{f^2(x)}{1+x^2}$$

$$\frac{dy}{dx} - \left(\frac{2x}{1+x^2}\right)y = y^2$$

or $\frac{1}{y^2}\frac{dy}{dx} - \left(\frac{2x}{1+x^2}\right)\frac{1}{y} = 1$

Let $-\frac{1}{y} = t \Rightarrow \frac{1}{y^2}\frac{dy}{dx} = \frac{dt}{dx}$

$$\frac{dt}{dx} + \left(\frac{2x}{1+x^2}\right)t = 1$$

Solution of above equation is

$$-\frac{1}{y}(1 + x^2) = \frac{x^3}{3} + x + c.$$

Since f(0) = 1, we get c = –1

Hence, $f(x) = \frac{-3(1+x^2)}{x^3+3x-3}$.

Equation of tangent at (1, –6) to y = f(x) is y = 30x – 36.

$$\Delta = \frac{108}{5} \text{ sq. units.}$$

3. Ans. 2

Sol. $P(x) - P(x+1) = 2x - 1 \ \forall\ x \in R$

$P(x) = ax^2 + bx + c$

$ax^2 + bx + c - [a(x+1)^2 + b(x+1) + c] = 2x - 1$

$- [ax^2 + 2ax + a + bx + b]$

$-2ax - (a + b) = 2x - 1$

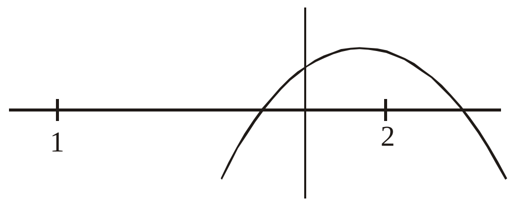
$-2a = 2 \qquad a + b = 1$

$a = -1 \qquad b = 2$

$P(x) = -x^2 + 2x + c$

$\because$ Greatest value of

P(x) in $(-\infty, 0]$ is 2



$\therefore$ P (0) = c = 2

$\therefore$ $P(x) = -x^2 + 2x + 2$

P(1) = –1 + 2 + 2 = 3

P(–1) = –1 –2 + 2 = –1

P(1) + P(–1) = 2

4. Ans. 1

Sol. $S_n = \frac{1}{n}\left(\frac{1}{\sqrt{4n^2}} + \frac{1}{\sqrt{4n^2-1^\alpha}} + \frac{1}{\sqrt{4n^2-2^\alpha}} + ... + \frac{1}{\sqrt{4n^2-(n-1)^\alpha}}\right)$

$$L = \lim_{n\to\infty}(n.S_n)$$

$$= \lim_{n\to\infty}\left(\frac{1}{\sqrt{4n^2}} + \frac{1}{\sqrt{4n^2-1}} + \frac{1}{\sqrt{4n^2-2}} + ... + \frac{1}{\sqrt{4n^2-(n-1)}}\right)$$

By sandwich Theorem $L = \frac{1}{2}$

$$M = \lim_{n\to\infty}(n.S_n)$$

$$= \lim_{n\to\infty}\left(\frac{1}{\sqrt{4n^2}} + \frac{1}{\sqrt{4n^2-1^2}} + \frac{1}{\sqrt{4n^2-2^2}} + ... + \frac{1}{\sqrt{4n^2-(n-1)^2}}\right)$$

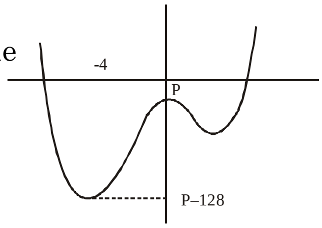
$$= \lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{1}{\sqrt{4n^2-r^2}} = \int_0^1\frac{dx}{\sqrt{4-x^2}} = \left(\sin^{-1}\frac{x}{2}\right)_0^1$$

$$= \frac{\pi}{6}$$

$$\therefore L + M = \frac{1}{2} + \frac{\pi}{6} = \frac{3+\pi}{6} \Rightarrow [L + M] = 1$$

5. Ans. 8

Sol. Graph of f(x) is



so for differentiable

$P-128 \geq 0$

$P \geq 128$

$\lambda = 128$

6. Ans. 6

Sol. $\because$ (x, 1), (y, 2), (z, 1), (2, 3) $\in$ A × B

$\Rightarrow$ A = {x, y, z, 2}

B = {1, 2, 3}

Number of onto map = $^4C_2.3! = 36$

7. Ans. 4

Sol. $\left(\left(\vec{\ell}\times\left(\vec{m}\times\vec{\ell}\right)\right)\times\left(\vec{n}\times\vec{m}\right)\right).\vec{n}$

$\left(\left(\vec{\ell}.\vec{\ell}\right)\vec{m}\times\left(\vec{n}\times\vec{m}\right)\right).\vec{n}$

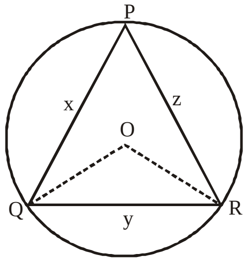
$\left|\vec{\ell}\right|^2\left(\left|\vec{m}\right|^2\vec{n}\right).\vec{n} = \left|\vec{\ell}\right|^2\left|\vec{m}\right|^2\left|\vec{n}\right|^2 = 36$

$9\times\frac{1}{4}\times\left|\vec{n}\right|^2 = 36$

$\left|\vec{n}\right| = 4$

8. Ans. 1

Sol. $x^2+y^2+z^2=5$



$\angle QPR = 30^o$

$\frac{1}{2}xz.\frac{1}{2} = \frac{\sqrt{3}}{4} \Rightarrow xz = \sqrt{3}$

$\cos\angle QPR = \frac{x^2+z^2-y^2}{2xz}$

$\frac{\sqrt{3}}{2} = \frac{5-2y^2}{2\times\sqrt{3}} \Rightarrow 2y^2 = 2 \Rightarrow y = 1$

from sine rule $= \frac{y}{\sin 30^o} = 2R \Rightarrow R = 1$

JEE (Main + Advanced) : ENTHUSIAST COURSE

SCORE : II

Test Type : FULL SYLLABUS Test Pattern : JEE-Main

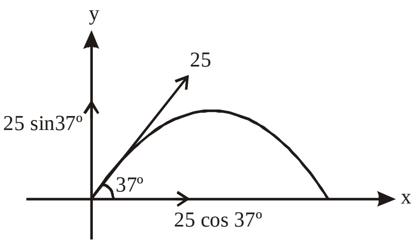
TEST DATE : 10 - 02 - 2019

PAPER-2

SOLUTION

1. Ans. (2)

Sol.



$$x = 25 \times \frac{4}{5} \times 2 = 40\,m$$

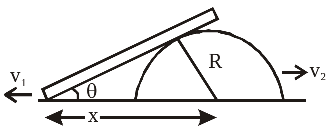
$$u_y = 25 \times \frac{3}{5} = 15$$

$$y = 15 \times 2 - \frac{1}{2}10(4) = 10m$$

$$\vec{r} = 40\hat{i} + 10\hat{j}$$

2. Ans. (1)

Sol.



$$\sin\theta = \frac{R}{x}$$

$$x = R\,\text{cosec}\theta$$

$$\frac{dx}{dt} = -R\,\text{cosec}\theta\cot\theta\frac{d\theta}{dt}$$

$$\frac{dx}{dt} = (v_1 + v_2)$$

$$\therefore\ (v_1 + v_2) = R\,\text{cosec}\theta\cot\theta\frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = \left(\frac{(v_1 + v_2)\sin^2\theta}{R\cos\theta}\right)$$

3. Ans. (4)

Sol. $m_1 = m_n$

$$m_2 = Am_n$$

$$v_1 = v \qquad v_2 = 0$$

$$v_1 = \frac{(m_n - Am_n)}{m_n + Am_n}v + \frac{2Am_n}{m_A + Am_n} \times 0$$

$$v_1 = \left(\frac{1-A}{1+A}\right)v$$

$$v_f = \frac{1}{2}m_n v_1^2$$

$$v_i = \frac{1}{2}m_n v^2$$

$$\frac{v_f}{v_i} = \left(\frac{1-A}{1+A}\right)^2$$

4. Ans. (3)

Sol. $v = \sqrt{Rg\tan\theta}$

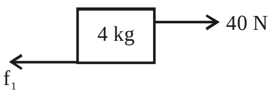
$$80 \times {}^5\!/_{18} = \sqrt{200 \times 10\tan\theta}$$

$$\tan\theta = \frac{20}{81}$$

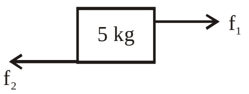
$$\theta = \tan^{-1}\left(\frac{20}{81}\right)$$

5. Ans. (3)

6. Ans. (4)



Sol.



$40 - 27 = 9a$

$a = \frac{13}{9} m/sec^2$

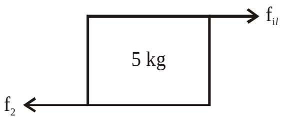
$40 - f_1 = 4\left(\frac{13}{9}\right)$

$f_1 = 34.22N$ but $f_{1\ell} = 20N$

(Limiting friction force)

$\therefore\ f_1 > f_{1\ell}$

$\therefore$ 4 kg will slide on 5 kg.



$20 - 27 = -ve$

$\because f_{1\ell} < f_2$

$\therefore$ 5 kg will not slide or move

$\therefore$ 4 kg will move with acceleration

$\frac{40-20}{4} = 5m/sec^2$

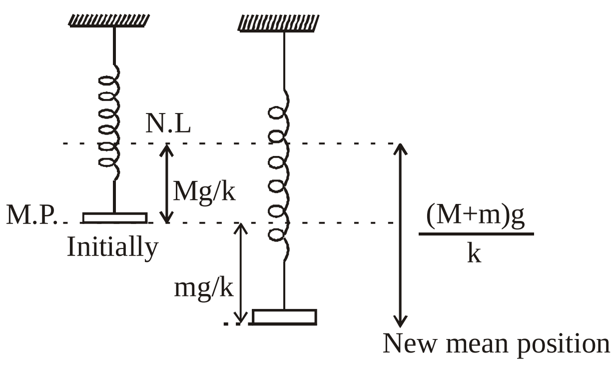
and 5 kg will not move.

7. Ans. (2)

Sol. By momentum conservation

mv = (M+m)v'

$v' = \frac{mv}{M+m}$



$v'^2 = \omega^2\left(A^2 - x^2\right)$

$x = \frac{mg}{k}$

$\omega^2 = \frac{k}{M+m}$

$v'^2 = \omega^2 (A^2 - x^2)$

$\frac{m^2v_0^2}{(M+m)^2} = \frac{k}{M+m}\left(A^2 - \frac{m^2g^2}{k^2}\right)$

$A^2 = \frac{m^2v_0^2}{k(M+m)} + \frac{m^2g^2}{k^2}$

$A = \frac{m}{k}\sqrt{\frac{v_0^2k}{M+m} + g^2}$

8. Ans. (2)

Sol. F.B.D or pulley : 2g – T = 2a

T – 1g = 1a

$T = \frac{4g}{3}$

$T' = 2T = \frac{8g}{3}$

Taking torque about A, $\tau_A = 0$

$\frac{8g}{3} \times L = M'g \times \frac{1}{2}L \Rightarrow \frac{16}{3} = M'$

9. Ans. (3)

Sol. By conservation of mechanical energy

$\frac{1}{2}I\ \omega^2 = \left(mg\frac{\ell}{2} + mg\frac{\ell}{2}\right)$

$\frac{1}{2} \times \left(\frac{m\ell^2}{3} + m\left(\frac{\ell}{2}\right)^2\right)\omega^2 = mg\ell$

$\frac{1}{2} \times \left(\frac{7m\ell^2}{12}\right)\omega^2 = mg\ell$

$\omega^2 = \frac{24g}{7\ell} = \frac{24 \times 10}{7 \times 3} = \frac{80}{7}$ ....(1)

By conservation of angular momentum about the hinge

$$mv\cos 60^\circ \times \frac{\ell}{2} = \left[\frac{m\ell^2}{3} + m\left(\frac{\ell}{2}\right)^2\right] \times \omega$$

$$mv\frac{\ell}{4} = \frac{7m\ell^2}{12} \times \omega$$

$$\omega = \frac{3v}{4\ell} = \frac{3\times v}{4\times 3} \quad ....(2)$$

by (1) and (2) $v = 4\sqrt{35}\ m/s$

10. Ans. (2)

Sol. $v_P = \sqrt{2g\times h}$

$(Range)_P = v_P \times t = v_P = \sqrt{2gh} \times \sqrt{\frac{2\times 3h}{g}}$

$= 2\sqrt{3}h \quad ....(i)$

Bernoulli's theorem between surface and Q

$2\rho g \times 2h + 4\rho g \times 2h = \frac{1}{2}4\rho v_Q^2 + 4\rho gh$

$v_Q = \sqrt{4gh} = 2\sqrt{gh}$

$(Range)_Q = \sqrt{4gh} \times \sqrt{\frac{2\times h}{g}}$

$\Rightarrow 2\sqrt{2}h$

$$\frac{R_P}{R_Q} = \frac{2\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}}$$

11. Ans. (1)

Sol. $x = 15\sin\omega t - 20\sin^3\omega t$

$= 15\sin\omega t - 5(4\sin^3\omega t)$

$(\because 4\sin^3\theta = 3\sin\theta - \sin 3\theta)$

$= 15\sin\omega t - 5(3\sin\omega t - \sin 3\omega t)$

$= 15\sin\omega t - 15\sin\omega t + 5\sin 3\omega t = 5\sin 3\omega t$

$\Rightarrow$ motion is SHM with angular frequency 3ω

So $a_{max} = 45\omega^2$

12. Ans. (1)

Sol. $\frac{h_1}{h_2} = \frac{T_1}{T_2}\cdot\frac{\rho_2}{\rho_1} = \frac{50}{25}\frac{0.3}{0.4} = \frac{3}{2}$

13. Ans. (4)

Sol. $V_{A_1} = \frac{-kp}{\sqrt{2}r^2}$

$V_{A_2} = \frac{kp}{\sqrt{2}r^2}$

$\therefore V_{net} = 0$

14. Ans. (1)

15. Ans. (3)

Sol. Apply kirchoff law

for larger loop

$iR_1 + iR_3 = v_1 + v_2$

$i = \frac{v_1 + v_2}{R_1 + R_3}$ .... (1)

and for smaller loop

$i = \frac{v_1}{R_1}$ .... (2)

solving $\frac{v_1}{v_2} = \frac{R_1}{R_3}$

16. Ans. (2)

Sol. $\frac{V}{2R} = \frac{V}{R}e^{-t/\tau}$

$\frac{t}{\tau} = \ell n2$

$t = \tau\ell n2$

17. Ans. (2)

18. Ans. (3)

Sol. The volume of the cubic domain is 10^{-12}cc its mass is volume density = 8 g/cm^3

It is given that Avagadro number (6×10^{23}) of iron atoms have a mass of 56g. Hence, the number of atoms in the domain is

$$N = \frac{8\times10^{-12}\times6\times10^{23}}{56} = 0.82\times10^{11}$$

The maximum possible dipole moment m_{max} is achived for the (unrealistic) case when all the atomic moments are perfectly aligned. Thus.

$m_{max} = (8.2\times10^{10}) \quad (9.1\times10^{-23})$

$= 7.8\times10^{-12}$

$M_{max} = m_{max}$ / Domain volume

19. Ans. (2)

Sol. Time constant = $\frac{L}{Req}$

$100\times10^{-6} = 40\times 10^{-3}/Req$

$\left[\frac{600R}{600+R}\right]+200=400$

$R = 300\ \Omega$

20. Ans. (3)

Sol. $\cos\phi=\frac{R}{z}=\frac{R}{\sqrt{X_c^2+R^2}}$

$=\frac{300}{500}=0.6$

$P = V_{rms}\, i_{rms} \cos\phi$

$=\frac{V_{rms}^2}{z}\cos\phi$

$=\frac{(200)^2}{500}\times0.6$

$= 4.8$ W

21. Ans. (2)

Sol. For damped oscillation $\tau = \frac{2L}{R}$

half time = $\tau \times \ln 2$

$=\frac{2\times12\times10^{-3}}{1.5}\times0.693$

$=0.0111 = 11.1$ ms

22. Ans. (4)

23. Ans. (3)

Sol. $E_{ka} = 13.6\,(z-1)^2\left(1-\frac{1}{4}\right) = 13.6\,(z-1)^2\times\frac{3}{4}$

For P, z = 10

For Q, z = 18

So number of elements between P & Q = 7

24. Ans. (2)

Sol. $M_\infty=\frac{f_0}{f_e}$ and tube length = $f_0 + f_e$

$\frac{f_0}{f_e}=5$, $f_0 + f_e = 36 \Rightarrow 6f_e = 36$

$f_e = 6$cm .... (1)

$f_0 = 30$ cm .... (2)

tube length for image at D

$L=f_0+\frac{Df_e}{D+f_e}=30+4.8$

= 34.8 cm

25. Ans. (3)

Sol. $2\,\mu t \cos r = n\lambda$

$2 \times 1.5 \times t \cos60 = 1 \times 5 \times 10^{-7}$

$t=\frac{10}{3}\times10^{-7}$m

26. Ans. (1)

Sol. Most of the light is diffracted between the two first order minima.

$b \sin\theta = \pm\lambda$

$\sin\theta=\pm\frac{\lambda}{b}=\pm2.25\times10^{-3}$

So angular divergence = 4.5×10^{-3} rad

27. Ans. (4)

Sol. VSD = (0.4)°

20 × MSD = 25 VSD

VSD = $\frac{4}{5}$MSD

L.C. = MSD – VSD

$=\frac{5}{4}VSD-VSD = \frac{VSD}{4}=0.1$

28. Ans. (2)

Sol. $\lambda=\frac{12400}{E\ (eV)}$, $\lambda=\frac{12400}{2} = 6100$ Å

29. Ans. (1)

Sol. The action of transisters is controlled by motions of charge carrier.

30. Ans. (4)

Sol. carrier frequency should be larger than modulating signal frequency

31. Ans.(2)

$\frac{r_A}{rSO_2}=\frac{\eta_A}{\eta_{SO_2}}=\frac{\eta_A \text{ is container}}{\eta_{SO_2}\text{ is container}}\sqrt{\frac{M_{SO_2}}{M_A}}$

$\frac{1}{1}=\frac{1}{2}\sqrt{\frac{64}{M_A}}$

$4=\frac{64}{M_A}$ $\quad M_A=\frac{64}{4}=16$ so gas is CH_4

32. Ans.(2)

If backward reaction is endothermic means forward reaction is exothermic hence on addition of Cl_2 reaction will go forward & temp. will increase.

33. Ans.(3)

$$\Delta x.\Delta p = \frac{h}{4\pi}$$

$$\Delta x = \frac{1}{2}\lambda = \frac{1}{2}.\frac{h}{p}$$

$$\frac{1}{2}.\frac{h}{p}.\Delta P = \frac{h}{4\pi}$$

$$\frac{\Delta P}{P} = \frac{1}{2\pi}$$

$$\frac{m\Delta V}{mV} = \frac{1}{2\pi}$$

% uncertainity

$$= \frac{1}{2\pi}\times 100 = 15.9\%$$

$$\therefore \frac{\Delta V}{V} = \frac{1}{2\pi}$$

34. Ans.(2)

$$P_g + \frac{3}{2}Cl_2 \rightarrow PCl_{3(g)}\Delta H = -320\,KJ/ml^{-1}$$

$$-320 = \left[315 + \frac{3}{2}\times 240\right] - \left[3\,BE\,P-Cl\right]$$

$$\therefore 3P-Cl = 995; \qquad BE\,P-Cl = 331.66$$

$$P_{(g)} + \frac{5}{2}Cl_{2g} \rightarrow PCl_{5(g)}\Delta H = -400\,KJ$$

$$-400 = \left[315 + \frac{5}{2}\times 240\right] - \left[2\,BE_{\substack{P-Cl\\ axial}} + 995\right]$$

$$2\times BE_{P-Cl} = 315 + 600 + 400 - 995$$

$$BE_{P-Cl} = 160\,KJ/mol$$

35. Ans.(2)

$$f_1 = e^{-\frac{E_a}{R\times 200}}$$

$$f_2 = e^{-\frac{E_a}{R\times 400}}$$

$$\frac{f_2}{f_1} = e^{1/2}$$

$$E_a = 200R \;\; = 400\,cal$$

36. Ans.(1)

$$dq = idt$$

$$= \int_0^{20}(2+t)\,dt \Rightarrow 240 \text{ coulomb}$$

$$\Rightarrow \frac{24}{9650}F$$

at cathode $2H^+ + 2e^- \longrightarrow H_2$

$$\frac{24}{9650}F \qquad\qquad \frac{12}{9650}mol$$

37. Ans.(4)

In aqueous medium $\left[\Delta H_f^0\right]_{H^+_{(aq)}} = 0$

38. Ans.(1)

$$E_{cell} = E^0_{cell} \frac{-0.06}{n}\log Q$$

$$0.3 = E^0_{cell}\frac{-0.06}{2}\log 10$$

$$E^0_{cell} = 0.33\text{volt}$$

At equilibrium

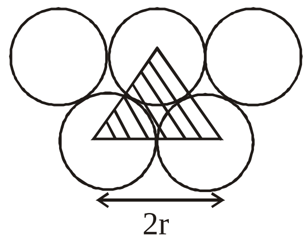
$\Delta G = 0 \qquad E_{cell} = 0$

Q = k

0= 0.33 – 0.03 log k

k = 10^{11}

39. Ans.(2)



Effective area occupied by each NH_3 molecule = $\frac{\sqrt{3}}{4}\times(2r)^2\times 2$

$= 2\sqrt{3}\,r^2\;nm^2$

Number of ammonia molecule required

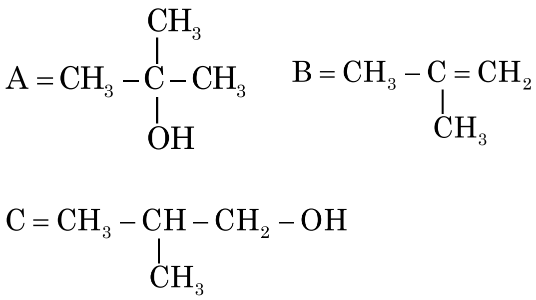
$$= \frac{20\times 10^3}{2\sqrt{3}\times\left(r\times 10^{-9}\right)^2} = \frac{10^{22}}{\sqrt{3}\,r^2}$$

40. Ans.(1)

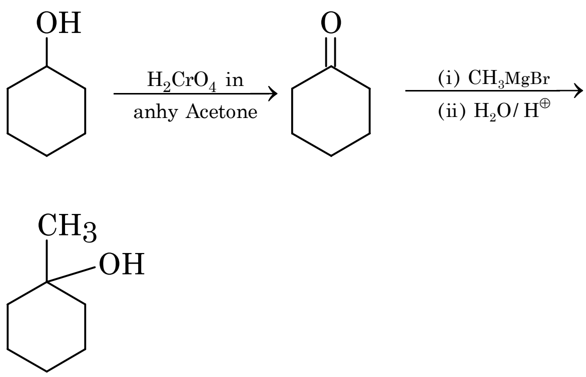
PH of salt of weak acid and weak base. remains independent of concentration.

Thus $\frac{x_1}{x_2} = 1$

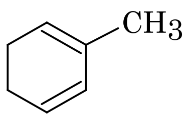
41. Ans.(3)



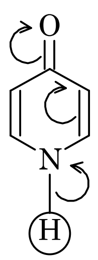
42. Ans.(4)



43. Ans.(4)

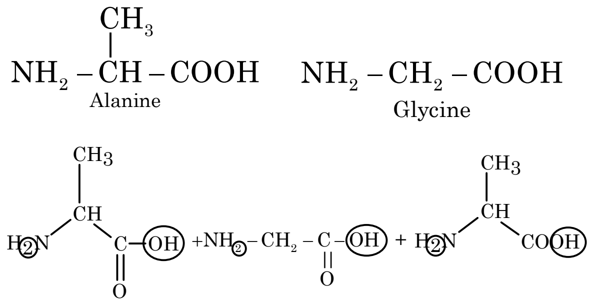
 represent the resonance and have maximum no. of α-H thats why its most stable.

44. Ans.(3)

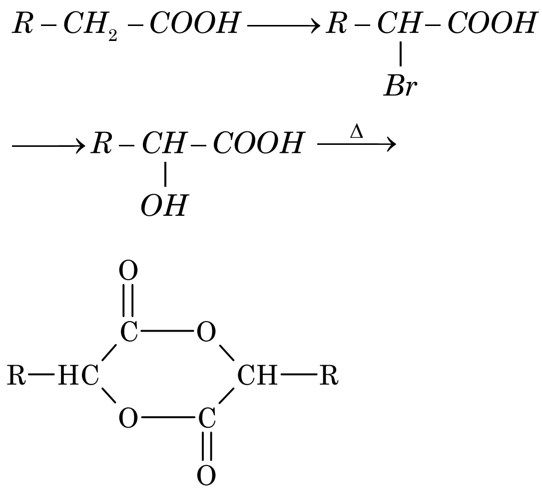
 this circle acidic H migrate w.r.t π-bond & shows tautomerism.

45. Ans.(1)

46. Ans.(3)

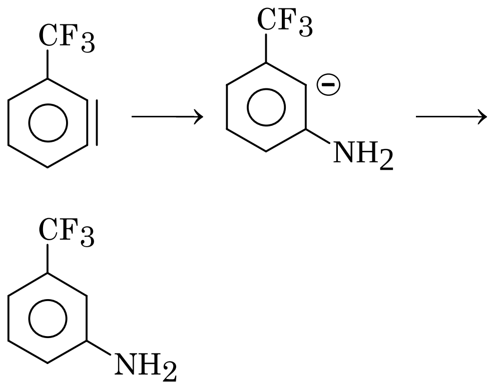


47. Ans.(3)



48. Ans.(4)

49. Ans.(4)



50. Ans.(2)

51. Ans.(1)

52. Ans.(3)

53. Ans.(4)

54. Ans.(4)

55. Ans.(4)

56. Ans.(2)

2nd I.P $\rightarrow$ Cr $\rightarrow 3d^5 4s^0$

Mn $\rightarrow 3d^5 4s^1$

v $\rightarrow 3d^3 4s^1$

Ti $\rightarrow 3d^2 4s^1$

57. Ans.(1)

58. Ans.(1)

59. Ans.(3)

60. Ans.(2)

61. Ans. (2)

Sol. From the given data

$f_1 = a_1 x_1 + a_2 x_2 + a_3 x_3$

$f_2 = b_1 x_1 + b_2 x_2 + b_3 x_3$

$f_3 = c_1 x_1 + c_2 x_2 + c_3 x_3$

Since $f_1 = f_2 = f_3 = 0$, so this becomes in form of homogeneous linear equation

$\therefore$ $a_1 x_1 + a_2 x_2 + a_3 x_3 = 0$

$b_1 x_1 + b_2 x_2 + b_3 x_3 = 0$

$c_1 x_1 + c_2 x_2 + c_3 x_3 = 0$

$$\text{so, } \Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$$

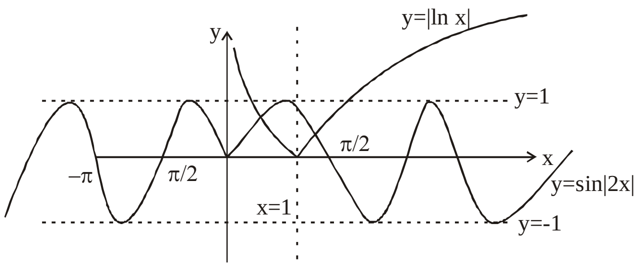
62. Ans. (1)

Sol. Correct result is $(\sim p \vee \sim q) \Rightarrow (r \wedge s)$

So, $\sim(p \wedge q) \Rightarrow (r \wedge s)$.

63. Ans. (2)

Sol. from the graphs of $y = f(|x|)$ and $y = |g(x)|$.



so total no. of solutions will be two in $x \in (0, \pi)$

64. Ans. (2)

Sol. Prob that student is passed in mathematics P(M) = 1/4

Prob that students is passed in physics P(P) = 1/5

Prob. that student is passed in both the subjects $P(m \cap P) = 3/20$

$$\text{required probability} = \frac{P(M \cap \bar{P})}{P(\bar{P})}$$

$$= \frac{P(M) - P(M \cap P)}{1 - P(P)}$$

$$= \frac{\frac{1}{4} - \frac{3}{20}}{1 - \frac{1}{5}} = \frac{1/10}{4/5} = \frac{1}{8}$$

65. Ans. (2)

Sol. Since $|z_1| = 12$

$|z_2 - \sqrt{3} - i| = 5$

$= |z_2 - (\sqrt{3} + i)| = 5 \geq |z_2| - |\sqrt{3} + i|$

$5 \geq |z_2| - 2$

$7 \geq |z_2|$

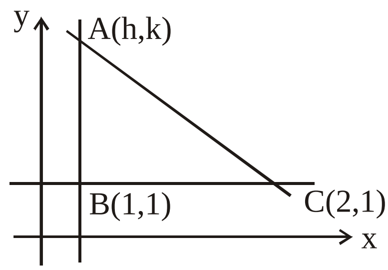
Now $\big||z_1| - |z_2|\big| \leq |z_1 - z_2| \leq |z_1| + |z_2|$

$|12 - 7| \leq |z_1 - z_2| \leq 12 + 7$

$5 \leq |z_1 - z_2| \leq 19$

66. Ans. (3)

Sol.



$\therefore \frac{1}{2} \times (1) \times (k - 1) = \pm 2$

$k - 1 = \pm 4$

$k = 1 + 4$ $\quad$ $k = 1 - 4$

$k = 5$ $\quad$ $k = -3$

$\therefore$ values of $h^2 + k^2 = 1^2 + 5^2 = 26$

or

$h^2 + k^2 = 1^2 + 9 = 10$

67. Ans. (2)

Sol. $3x + 4y + z - 3 = 0$ .... (1)

$3x + 4y + z - \frac{5}{2} = 0$ ... (2)

Let equation of plane is

$3x + 4y + z + k = 0$ ... (3)

Given that

$$\Rightarrow \frac{\left|\frac{k+3}{\sqrt{26}}\right|}{\left|\frac{k + \frac{5}{2}}{\sqrt{26}}\right|} = \frac{2}{3} \quad \Rightarrow 3|k+3| = 2\left|k + \frac{5}{2}\right|$$

$\Rightarrow 3k + 9 = -2k - 5$ (Taking -ve sign)

$$\Rightarrow k = \frac{-14}{5}$$

Equation required plane is

$$3x + 4y + z - \frac{14}{5} = 0$$

$15x + 20y + 5z - 14 = 0$

68. Ans. (3)

Sol. Let $p(x) = x^2 - 4x + 3$

$$\frac{dp}{dx} = 2x - 4 = 0 \Rightarrow x = 2$$

$p(2) = 3$, $p(5) = 8$, $p(-5) = 48$

$$(f(x))_{max} = f(-5) = \log_{\sqrt{48}}(25 + 20 + 3)$$

$$= \log_{\sqrt{48}} 48 = 2$$

69. Ans. (4)

Sol. $\tan^{-1}\frac{3}{5} + \tan^{-1}\frac{3}{29} + \tan^{-1}\frac{3}{71} + \tan^{-1}\frac{3}{131} + ...$ n terms

$$= \tan^{-1}\left(\frac{4-1}{1+4\times 1}\right) + \tan^{-1}\left(\frac{7-4}{1+7\times 4}\right) +$$

$$\tan^{-1}\left(\frac{10-7}{1+10\times 7}\right) + ... \tan^{-1}\left(\frac{3n+1-(3n-2)}{1+(3n+1)(3n-2)}\right)$$

$$= \tan^{-1}4 - \tan^{-1}1 + \tan^{-1}7 - \tan^{-1}4 + \tan^{-1}10$$

$$- \tan^{-1}7 + ... \tan^{-1}(3n+1) - \tan^{-1}(3n-2)$$

$$= \tan^{-1}(3n+1) - \tan^{-1}1 = \tan^{-1}\left(\frac{3n+1-1}{1+(3n+1)}\right)$$

$$= \tan^{-1}\left(\frac{3n}{2+3n}\right)$$

70. Ans. (2)

Sol. $A = \int_0^1 (2x - x)dx + \int_1^4 (2\sqrt{x} - x)dx = \frac{7}{3}$

71. Ans. (1)

Sol. $\frac{dy}{dx} + \frac{3y}{x} = 1$

I.F. $= e^{\int \frac{3dx}{x}} = e^{3\log x} = x^3$

solution of differential equation

$$y.x^3 = \int x^3\, dx \Rightarrow yx^3 = \frac{x^4}{4} + c$$

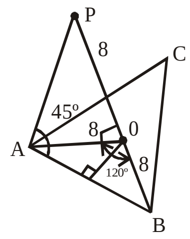
put x = 2, y = 1, we get c = 4

$$yx^3 = \frac{x^4}{4} + 4$$

Put $x = 1$, $y(1) = \frac{1}{4} + 4 = \frac{17}{4}$

72. Ans. (3)

Sol.



Area of ΔABC

= 3 × Area of triangle OAB

$$= 3 \times \frac{1}{2} \times 8 \times 8 \times \sin 120^o$$

$$= 3 \times 4 \times 8 \times \frac{\sqrt{3}}{2} = 48\sqrt{3}$$

73. Ans. (2)

Sol. $r = a_1 + \lambda b_1$ and $r = a_2 + \lambda b_2$

then $S.D. = \left|\frac{(a_2 - a_1).(b_1 \times b_2)}{|b_1 \times b_2|}\right| \Rightarrow \ell = \frac{6}{\sqrt{5}}$

then $[\ell] = 2$

74. Ans. (3)

Sol. $|adjA| = |A|^{n-1}$

$$\Rightarrow |adj(adj\ A)| = |adj\ A|^{n-1} = |A|^{(n-1)^2}$$

$$\Rightarrow (n-1)^2 = P^2 \Rightarrow n - 1 = P$$

$$\Rightarrow n = P + 1$$

75. Ans. (1)

Sol. $A_n = \int_0^{\pi/2} \frac{\sin(2n-1)x}{\sin x} dx$

$$A_{n+1} - A_n = \int_0^{\pi/2} \frac{(\sin(2n+1)x - \sin(2n-1)x)}{\sin x} dx$$

$$= \int_0^{\pi/2} 2\cos 2nx\, dx = 0$$

$$\Rightarrow A_{n+1} = A_n$$

$$\Rightarrow A_1 = A_2 = A_3 = ... = A_{100} = ...A_{1000}$$

76. Ans. (3)

Sol. $a+8d=b$ ... (1)

$a+12d=br$ ... (2)

$a+14d=br^2$ ... (3)

$\Rightarrow$ (2) – (1) $\Rightarrow 4d=b(r-1)$

(3) – (2) $\Rightarrow 2d=br(r-1)$

$\Rightarrow 2=\frac{1}{r} \quad \Rightarrow r=\frac{1}{2}$

also $\frac{b}{1-r}=80 \Rightarrow b=40 \;\; d=-5 \Rightarrow a=80$

77. Ans. (4)

Sol. Let point of intersection be (h, k) then equation of PQ is $hx+2ky=4$, it touches $x^2+y^2=1$

$\Rightarrow \frac{16}{4k^2}=1+\frac{h^2}{4k^2}\left(\because c^2=a^2\left(1+m^2\right)\right)$

$\Rightarrow \frac{x^2}{16}+\frac{y^2}{4}=1 \Rightarrow e=\frac{\sqrt{3}}{2}$

78. Ans. (2)

Sol. $\lim_{x\to 0}\frac{1}{2}\left(\frac{2\cos 5x\cos 2x-2\cos 7x\cos 10x}{\ell n^2\left(1+\sin 2x\right)}\right)$

$\lim_{x\to 0}\frac{1}{2}\left(\frac{\cos 7x+\cos 3x-\cos 17x-\cos 3x}{\sin^2 2x}\right)$

$\lim_{x\to 0}\frac{1}{2}\left(\frac{\cos 7x-\cos 17x}{\sin^2 2x}\right)$

$\lim_{x\to 0}\frac{1}{2}\frac{2\sin 12x\sin 5x}{\sin^2 2x}$

$=\frac{12.5}{2.2}=15$

79. Ans. (2)

Sol. $\frac{x^2}{9}-\frac{y^2}{4}=1$

Condition of tangency $c=\pm\sqrt{a^2m^2-b^2}$

$2=\pm\sqrt{9m^2-4}$

$4=9m^2-4$

$9m^2=8$

$m=\pm\frac{2\sqrt{2}}{3}$

80. Ans. (3)

Sol. $\vec{x}\times\vec{z}+\vec{y}=\vec{0}$ (Given)

Taking cross product with $\vec{x}$ both sides,

$\vec{x}\times\left(\vec{x}\times\vec{z}+\vec{y}\right)=\vec{x}\times\vec{0}$

$\vec{x}\times\left(\vec{x}\times\vec{z}\right)+\vec{x}\times\vec{y}=\vec{0}$

$\left(\vec{x}.\vec{z}\right)\vec{x}-\left(\vec{x}.\vec{x}\right)\vec{z}+\vec{x}\times\vec{y}=\vec{0}$

$\Rightarrow 2\vec{z}=\vec{x}\times\vec{y}$

Now $\vec{x}\times\vec{y}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 0\\ 1 & 1 & 1\end{vmatrix}=-\hat{i}-\hat{j}+2\hat{k}$

$\therefore 2\vec{z}=-\hat{i}-\hat{j}+2\hat{k}$

$2\left|\vec{z}\right|=\sqrt{6}$

$\left|\vec{z}\right|=\sqrt{\frac{3}{2}} \;\Rightarrow\; \left|\vec{z}\right|^2=\frac{3}{2}$

81. Ans. (2)

Sol. $\because f^{-1}(2)=3, \quad f^{-1}(-3)=6$

$\Rightarrow f(3)=2, \quad f(6)=-3$

$3a+b=2$

$+6a+b=-3$

Solving eq. (1) & (2)

$a=-\frac{5}{3}, b=7$

$\therefore a+b=-\frac{5}{3}+7=\frac{-5+21}{3}=\frac{16}{3}$

82. Ans. (4)

Sol. $y^2=x^3$

$\frac{dy}{dx}=\frac{3x^2}{2y}$

slope of tangent at $\left(\alpha^2,\alpha^3\right)$ = slope of normal at $\left(\beta^2,\beta^3\right)$

$$\frac{3\alpha^4}{2\alpha^3}=\frac{-1}{\frac{3\beta^4}{2\beta^3}}$$

$$\frac{3}{2}\alpha=\frac{-2}{3\beta}$$

$$\alpha\beta=\frac{-4}{9}$$

83. Ans. (3)

Sol. $(1+a)^x={}^xC_0+{}^xC_1a+{}^xC_2a^2+......+{}^xC_ra^r+....+{}^xC_xa^x...(1)$

$(1+a)^y={}^yC_0+{}^yC_1a+{}^yC_2a^2+......+{}^yC_ra^r+.....+{}^yC_ya^y...(2)$

multiplying eq.(1) and (2) and equating the coefficient of a^r both sides,

$${}^xC_r\,{}^yC_0+{}^xC_{r-1}{}^yC_1+{}^xC_{r-2}{}^yC_2+....+{}^xC_0{}^yC_r$$

= coefficient of a^r in $(1+a)^{x+y}={}^{x+y}C_r$

84. Ans. (4)

Sol. $np=4,\ npq=2$

$$q=\frac{1}{2},\ p=1-q=\frac{1}{2}$$

$$\therefore n=8$$

$$P(x=1)={}^8C_1(1/2)^1\left(\frac{1}{2}\right)^7=8\times\frac{1}{2^8}=\frac{1}{2^5}=\frac{1}{32}$$

85. Ans. (1)

Sol. Let E_1 be the event that one of first n urns is chosen

E_2 be the event that (n + 1)th urn is chosen

A be the event that two balls drawn are black.

then

$$P(E_1)=\frac{n}{n+1}{}'P(E_2)=\frac{1}{n+1},P\left(A/E_1\right)=\frac{{}^6C_2}{{}^{10}C_2}=\frac{1}{3}$$

$$P\left(A/E_2\right)=\frac{{}^5C_2}{{}^{10}C_2}=\frac{2}{9}$$

Now $P\left(E_2/A\right)=\dfrac{\frac{1}{n+1}.\frac{2}{9}}{\frac{n}{n+1}.\frac{1}{3}+\frac{1}{n+1}.\frac{2}{9}}$

$$\frac{1}{16}=\frac{2}{3n+2}$$

$$3n+2=32\Rightarrow n=10$$

86. Ans. (2)

Sol. Let the number is abc

such that $b=\dfrac{a+c}{2}$

$2b=a+c$

$even=a+c$.

a, c should be both even or both odd.

No. of ways $\Rightarrow 5.5+4.5$

$=25+20$ = 45

87. Ans. (2)

Sol. Put x = 2

$$\Rightarrow 2f(2)+2f\left(\frac{1}{2}\right)-2f(1)=4$$

$$\Rightarrow f(2)+f\left(\frac{1}{2}\right)-f(1)=2\ ...\ (1)$$

Put x = 1

$$\Rightarrow f(1)=-1...\ (2)$$

Put $=\dfrac{1}{2}$

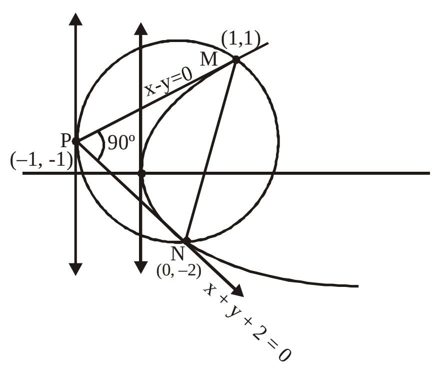
$$2f\left(\frac{1}{2}\right)+\frac{1}{2}f(2)-2f(1)=\frac{5}{2}\ ...\ (3)$$

solving (1), (2), (3)

$$f(2)=1,\ f\left(\frac{1}{2}\right)=0$$

88. Ans. (1)

Sol.



MN will be focal chord of parabola. A circle with MN as diameter will touch directrix at P(–1, –1).

Equation of circle with MN as diameter.

$$(x-1)x+(y-1)(y+2)=0$$

$$\Rightarrow x^2+y^2-x+y-2=0$$

Equation of tangent at P(–1, –1) will be

$$x(-1)+y(-1)-\frac{(x-1)}{2}+\frac{(y-1)}{2}-2=0$$

$$\Rightarrow -3x-y-4=0$$

$$\Rightarrow 3x+y+4=0$$

89. Ans. (1)

Sol. Difference of roots in both equation is same i.e., $\cos 2\theta$

$$\Rightarrow \quad 16-8=4b^2-4b$$

$$\Rightarrow \quad 2=b^2-b$$

$$\Rightarrow \quad b^2-b-2=0$$

$$\Rightarrow \quad (b-2)(b+1)=0 \Rightarrow b=2,-1$$

90. Ans. (1)

Sol. $I=\int_0^1 \sin 8\pi x.g(x)\,dx$

$$=\int_0^{1/2} \sin 8\pi x.g(x)\,dx+\int_{\frac{1}{2}}^{1} \sin 8\pi x.g(x)\,dx.$$

$$=I_1+I_2$$

$$I_2=\int_{\frac{1}{2}}^{1} \sin 8\pi x.g(x)\,dx$$

Put $x=\frac{1}{2}+t \qquad dx=dt$

$$I_2=\int_0^{1/2} \sin 8\pi\left(t+\frac{1}{2}\right).g\left(t+\frac{1}{2}\right)dt$$

$$=\int_0^{1/2} \sin 8\pi t\left(2-g(t)\right)dt$$

$$=2\int_0^{1/2} \sin 8\pi t.\,dx-\int_0^{1/2}\left(\sin 8\pi t\right)g(t)\,dt$$

$$\Rightarrow I=2\int_0^{1/2} \sin 8\pi t\,dt$$

$$=2\left[-\frac{\cos 8\pi t}{8\pi}\right]_0^{\frac{1}{2}}=0$$