ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)
path to success

Paper Code : 1001CJA102118065

CLASSROOM CONTACT PROGRAMME

(Academic Session : 2018 - 2019)

JEE (Main + Advanced) : ENTHUSIAST COURSE : SCORE-II

ANSWER KEY : PAPER-1 TEST DATE : 27-01-2019

Test Type : FULL SYLLABUS — Test Pattern : JEE-Advanced

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B,C	A,B,D	A,D	A,C	A,D	A,C	B,C,D	B,D	B,C	A,B,C
SECTION-III	Q.	1	2	3	4	5	6	7	8		
	A.	9	6	2	9	3	4	2	2		
SECTION-IV	Q.1	A	B	C	D	Q.2	A	B	C	D	
		P,R	P,Q	S	P,T		R,S	R,S	P,Q,T	P,S	

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C,D	A,D	A,C	C,D	A,B	A,B	A,C	A,B,C	A,C,D	C,D
SECTION-III	Q.	1	2	3	4	5	6	7	8		
	A.	8	4	2	7	3	1	4	1		
SECTION-IV	Q.1	A	B	C	D	Q.2	A	B	C	D	
		P,R	S,T	R,S	P,Q		R	R	Q	P	

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,B,C	B,C,D	A,B,C	A,C,D	B,C,D	C,D	B,C,D	B,D	C,D	A,C,D
SECTION-III	Q.	1	2	3	4	5	6	7	8		
	A.	5	1	1	1	6	6	3	0		
SECTION-IV	Q.1	A	B	C	D	Q.2	A	B	C	D	
		P,S,T	Q,R,S,T	Q,R,T	Q,R,T		P,Q,T	P,R	P,Q,R,T	P,Q,R,T	

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JEE (Main + Advanced) : ENTHUSIAST COURSE : SCORE-II

ANSWER KEY : PAPER-2 TEST DATE : 27-01-2019

Test Type : FULL SYLLABUS — Test Pattern : JEE-Main

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	4	2	2	3	3	4	2	3	4	1	4	1	1	3	3	2	3	2	3	4
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	3	3	4	2	3	1	3	1	1	2	3	2	1	4	3	3	3	2	2	3
Que.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	1	1	3	2	3	3	1	4	1	2	2	3	4	3	4	2	4	4	3	2
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	3	1	4	3	1	4	3	4	1	3	3	2	2	1	4	4	2	2	4	1
Que.	81	82	83	84	85	86	87	88	89	90										
Ans.	2	2	4	3	1	4	4	1	2	1										

JEE (Main + Advanced) : ENTHUSIAST COURSE

SCORE : II

Test Type : FULL SYLLABUS   Test Pattern : JEE-Advanced

TEST DATE : 27 - 01 - 2019

PAPER-1

PART-1 : PHYSICS   SOLUTION

SECTION-I

1. Ans. (B,C)

Sol. Power delivered = F × v for max. power = kx × v

$$\frac{dp}{dx} = 0 \Rightarrow x = \frac{x_0}{\sqrt{2}}; \quad \frac{1}{2}mv^2 = \frac{1}{2}k(x_0^2 - x^2)$$

$$\Rightarrow v = \sqrt{\frac{k}{m}(x_0^2 - x^2)} \quad \therefore P = kx\sqrt{\frac{k}{m}(x_0^2 - x^2)}$$

2. Ans. (A,B,D)

Sol.

$$0 \le x \le a; \; V_x = \left[-\int_0^x E_x dx\right] + V_{(0)} = 0 \;\; (\text{as } E_x = 0)$$

$$x \ge a; \; V_x = -\int_a^x E_x dx + V_{(a)} = \left[-\int_a^x \frac{\sigma}{\varepsilon_0} dx\right] + V_{(a)} = -\frac{\sigma}{\varepsilon_0}(x-a)$$

$$x \le 0; \; V_x = -\int_a^x E_x dx + V_{(0)} = -\left(-\frac{\sigma}{\varepsilon_0}\cdot x\right) + V_{(0)} = \frac{\sigma}{\varepsilon_0}\cdot x$$

3. Ans. (A,D)

Sol. (A) $\frac{\mu_0 K}{2}$ hence K = σV

(D) $F = q\vec{u} \times \vec{B}$ hence upwards

4. Ans. (A,C)

Sol. $K_{max} = \frac{hc}{\lambda} - \phi; \quad Iev = \frac{hc}{400} - 1.9$

So hc = 2.9 × 400 ev nm

Next $K_{max} = \frac{hc}{500} - 1.9 = 0.42\,ev$

For longest $\lambda = \frac{hc}{\lambda_{max}} = 1.9$

5. Ans. (A,D)

Sol.

$$\frac{2\pi}{\lambda}\left(\frac{dy}{D+v}\right) = 4\pi + \frac{\pi}{3}$$

$$\frac{2\pi}{\lambda}\left(\frac{dy}{D+v}\right) = 6\pi - \frac{\pi}{3}$$

$$\frac{dy}{D} = 3\lambda$$

$$\frac{dy}{D} = 3\lambda$$

$$\frac{2\pi \cdot 3D}{dx}\left(\frac{dy}{D+v}\right) = \frac{13\pi}{3}$$

$$\frac{2\pi \cdot 3D}{dx}\left(\frac{dy}{D+v'}\right) = \frac{17\pi}{3}$$

$$\frac{6D}{D+v} = \frac{13}{3}$$

$$\frac{6D}{D+v'} = \frac{17}{3}$$

$$18D = 13D + 13v$$

$$18D = 17D + 17v'$$

$$\frac{5D}{13} = v$$

$$v' = \frac{D}{17}$$

6. Ans. (A,C)

Sol. $10 \log\left(\frac{I}{I_0}\right) = 60$

$\Rightarrow \quad I = I_0 \times 10^6 = 10^{-12} \times 10^6$

$I = 10^{-6}$ watt/m^2

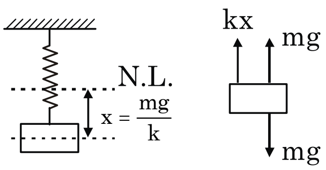
$P = IA = 10^{-6} \times 2 = 2\mu w$

$E = P.t. = 2 \times 10^{-6} \times 10 \times 60 \times 60$

$= 72 \times 10^{-3}$ Joule (where I → intensity, P → power, E → Energy)

7. Ans. (B,C,D)

Sol.



Now as lift starts descending by acceleration 'g' of downward, in the frame of lift

Net force = –kx

$$\frac{F}{m}=\frac{-k}{m}\cdot x$$

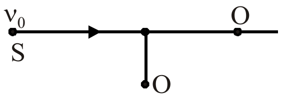
$$t=2\pi\sqrt{\frac{m}{k}};$$

Minimum potential energy is at the mean position = 0 when x = 0.

8. Ans. (B,D)

Sol. conceptual

9. Ans. (B, C)

Sol. 

When source is nearest than frequency heard.

$$f'=\frac{fC}{C-V_s\cos\theta}$$

after crossing this point frequency obtained corresponding to wave emitted from this point is

$f' = f_0$

10. Ans. (A,B,C)

Sol. Using dimensional analysis

$T \propto \rho^a r^b S^c$

$$=\left(ML^{-3}\right)^a (L)^b\left(\frac{MLT^{-2}}{L}\right)^c$$

$= M^{a+c} L^{-3a+b} T^{-2c}$

$-2c = 1$

$\Rightarrow c = -1/2$

$a + c = 0$

$\Rightarrow a = 1/2$

$-3a + b = 0$

$\Rightarrow b = 3/2$

$T \propto \rho^{1/2}$

$T \propto \rho^{3/2}$

$T \propto S^{-1/2}$

SECTION-III

1. Ans. 9

Sol. Case-I : $v^2 = u^2 + 2as$

$$0=\left(5\sqrt{5}\right)^2-2gh$$

$$h=\frac{25\times5}{20}=\frac{25}{4}m$$

Case-II : $v^2 = u^2 + 2as$

$0 = (13)^2 - 2(g + a)h$

$$\frac{169\times4}{2\times25}=g+a$$

$$g+a=\frac{338}{25}$$

$$a=\frac{338}{25}-g=\frac{88}{25}$$

$$a=\frac{88}{25}$$

Case-III : $v^2 = u^2 + 2as$

$$u^2=2(g-a)h=2\times\left(10-\frac{88}{25}\right)\frac{25}{4}=\frac{2\times162\times25}{25\times4}=81$$

u = 9 m/s

2. Ans. 6

Sol. $V_L^2+R_R^2==150^2\Rightarrow V_L^2=150^2-120^2$

$\Rightarrow V_L = 90\,V$

Also $A_C^2+V_R^2=130^2\Rightarrow V_C=50\,V$

$$V=\sqrt{V_R^2+(V_L-V_C)^2}=\sqrt{120^2+40^2}=40\sqrt{10}$$

Power factor

$$\cos\phi=\frac{R}{2}=\frac{V_R}{V}=\frac{120}{40\sqrt{10}}=\frac{3}{\sqrt{10}}$$

3. Ans. 2

Sol. Initially let no. of molecules of A = N_A and that of B = N_B.

$$\frac{\lambda N_A}{\lambda N_A+2\lambda N_B}=\frac{25}{100}$$

$$\therefore\quad 100\lambda N_A=25\lambda N_A+50\lambda N_B\Rightarrow N_A=\frac{2}{3}N_B$$

Now $N_A'=N_Ae^{-\lambda t}=\frac{2}{3}N_Be^{-\lambda t}$

$N_B'=N_Be^{-2\lambda t}$

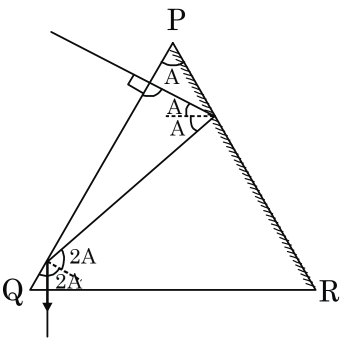
$$\therefore\ \frac{\frac{2}{3}\lambda N_Be^{-\lambda t}}{\frac{2}{3}\lambda N_Be^{-\lambda t}+2\lambda N_Be^{-2\lambda t}}=\frac{75}{100}=\frac{3}{4}\Rightarrow\frac{1}{1+3e^{-\lambda t}}=\frac{3}{4}$$

$\Rightarrow\ 1 = 9e^{-\lambda t}\ \Rightarrow\ e^{\lambda t} = 9$

$$\text{So,}\quad t=\frac{\ln(9)}{\lambda}hr=\frac{2\ln(3)}{\ln(3)}hr=2hr$$

4. Ans. 9

Sol.



$$\frac{\pi}{2}-\frac{A}{2}+\frac{\pi}{2}-2A=\frac{\pi}{2}$$

$$\frac{5A}{2}=\frac{\pi}{2}$$

$$A=\frac{\pi}{5}=36°$$

5. Ans. 3

Sol. Initial charge on capacitor = CE

Initial potential energy of capacitor

$= CE^2/2$

Now $C = \varepsilon_0 A/d$

And new capacitance: $C' = \dfrac{\varepsilon_0 A}{d/2}$, in series with $\dfrac{\varepsilon_0 kA}{d/2}$

$$\Rightarrow \quad C' = \frac{2\varepsilon_0 A}{d} \text{ series } \frac{4\varepsilon_0 A}{d}$$

$$= \frac{\varepsilon_0 A}{d}\left(\frac{2\times 4}{2+4}\right)=\frac{4\varepsilon_0 A}{3d}=\frac{4}{3}C$$

$$\Rightarrow \quad \text{New charge} = \frac{4}{3}CE$$

$$\text{New energy} = \frac{1}{2}\times\frac{4}{3}CE^2=\frac{2}{3}CE^2$$

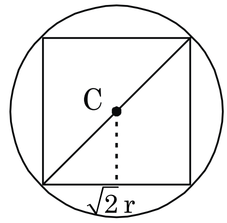
Now $W_{but} = \Delta H + \Delta U$

$$\Rightarrow \quad E\left(\frac{4}{3}CE-CE\right)=\Delta H+\frac{2}{3}CE^2-\frac{1}{2}CE^2$$

$$\Rightarrow \quad \Delta H = \frac{1}{6}CE^2$$

6. Ans. 4

Sol.



I = moment of inertia of system about centre of mass of system

$$=4\left[\frac{1}{12}m(\sqrt{2}r)^2+m\left(\frac{r}{\sqrt{2}}\right)^2\right]=\frac{8}{3}mr^2$$

considering translatory motion of body

$$\frac{4mg\sin\theta-f_s}{4m}=a \quad ....(i)$$

consider rotational motion about C

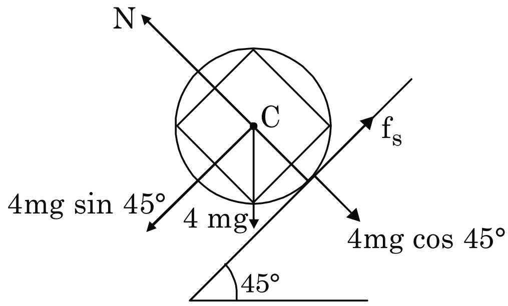
$$f_s \times r = I\alpha$$

$$\alpha=\frac{3f_s r}{8mr^2}=\frac{3f_s}{8mr}$$

using condition of rolling without slipping

$$a = \alpha r$$

$$\Rightarrow \quad \frac{4mg\sin\theta-f_s}{4m}=\frac{3f_s}{8m}$$



$$\Rightarrow \quad 8mg\sin\theta=5f_s=\frac{8mg\sin\theta}{5}\le\mu_s N$$

$$\Rightarrow \quad \mu_s \ge \frac{2}{5}\tan\theta=0.4$$

7. Ans. 2

Sol. $\mu_1 = 4, \ \mu_2 = 9$

$h = 5$

Let plate is x distance below upper plate.

Viscous force $F = \mu_1\dfrac{Av}{x}+\mu_2\dfrac{Av}{(h-x)}$

For minimum force

$$\frac{dF}{dx}=Av\left[-\frac{\mu_1}{x^2}+\frac{\mu_1}{(h-x)^2}\right]=0$$

Solving $x=\dfrac{\sqrt{\mu_1}h}{\sqrt{\mu_1}+\sqrt{\mu_2}}$

8. **Ans. 2**

Sol. Actual current is $i=\frac{E}{R}$

Measured current is

$$i=\frac{E}{\frac{R}{10}+\frac{(10R)(R)}{11R}}=0.99\frac{E}{R}$$

$\therefore$ % error = –1.0%

Actual voltage across R is E

and measured voltage is

$$(i)\left(\frac{10R\times R}{11R}\right)=0.9E$$

$\therefore$ % error is –10%

SECTION - IV

1. **Ans. (A) P,R; (B) P,Q; (C) S; (D) P,T**

Sol. $\vec{F}_m=q(\vec{v}\times\vec{B})$ and $\vec{F}_e=q\vec{E}$

2. **Ans. (A) R,S; (B) R,S; (C) P,Q, T; (D) P,S**

Sol. For D :

$$V_{eq}=\frac{\frac{1.5}{2}+\frac{V}{1}}{\frac{1}{2}+\frac{1}{1}}=\frac{\left(V+\frac{3}{4}\right)}{\frac{3}{2}}=\frac{2V}{3}+\frac{1}{2}$$

$$i=\frac{\frac{2V}{3}+\frac{1}{2}}{\frac{2}{3}+4}$$

Similar for C

A, B are wheatstone so no change.

PART–2 : CHEMISTRY SOLUTION

SECTION-I

1. **Ans. (C,D)**

Sol. $\underset{1\text{Mole}}{2NaOH}+\underset{\frac{1}{2}\text{Mole}}{CO_2}\longrightarrow Na_2CO_3+H_2O$

$n_{CO}+n_{CO_2}=1$

$n_{CO}=\frac{1}{2}$

$CO+\frac{1}{2}O_2\longrightarrow CO_2$

$\frac{1}{2}$mole $\qquad \frac{1}{2}$mole

More NaOH required = 1 mole

So $\frac{1}{2}M$ NaOH = 2 litre

$$n = M \times V = \frac{1}{2}\times 2=1\,\text{mole}$$

Similarly 1 mole of KOH is required = 56 gm = 1 mole.

2. **Ans. (A,D)**

Sol. $CrO_5\longrightarrow Cr\longrightarrow +6$

$$CrO_5\begin{cases}Cr^{+6}\to Cr^{3+}\\ O^{-1}\begin{cases}O_2\\ O^{2-}\end{cases}\end{cases}$$

$$2CrO_5+3H_2SO_4\longrightarrow Cr_2(SO_4)_3+3H_2O+\frac{7}{2}O_2$$

1 mole .... $\frac{7}{4}$ mole O_2.

3. Ans. (A, C)

Sol. $\frac{r_{gas}}{r_{CH_4}} = \sqrt{\frac{M_{CH_4}}{M_{Gas}}}$

$\frac{4}{3} = \sqrt{\frac{16}{M_{gas}}}$

$M_{gas} = 9$

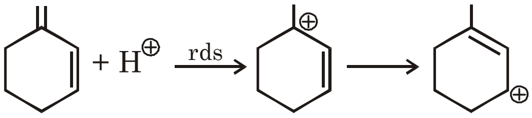
Density = $\frac{9}{22.4} = 0.4017$ g/litre

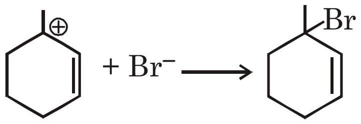
Because vapour are triatomic ; atomic weight = $\frac{9}{3} = 3$

(Note that this is atomic weight, not atomic number)

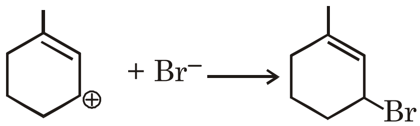
Vapour density = $\frac{MM}{2} = \frac{9}{2} = 4.5$

4. Ans. (C, D)

Sol. (1) 

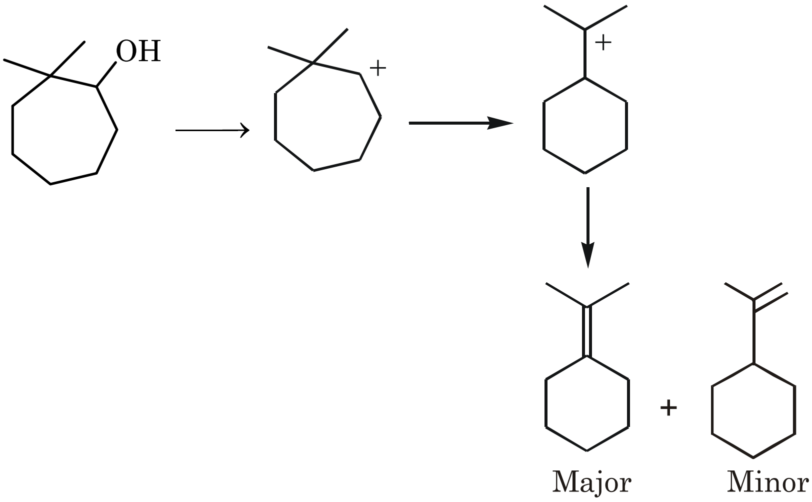
(2) 

(KCP) (1, 2-addition)

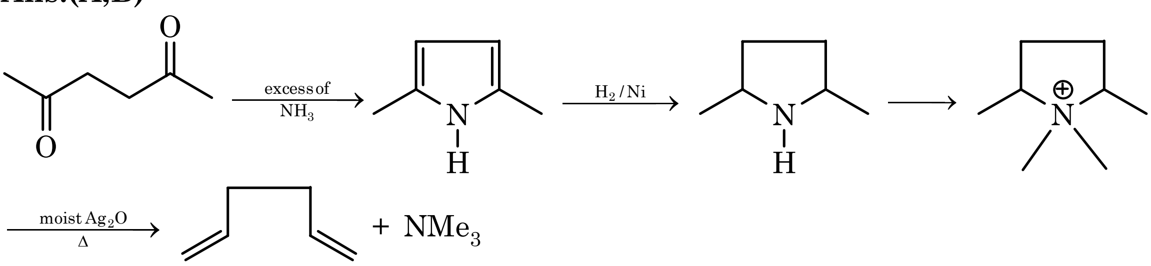
(3) 

(TCP) (1,4–addition)

5. Ans. (A,B)

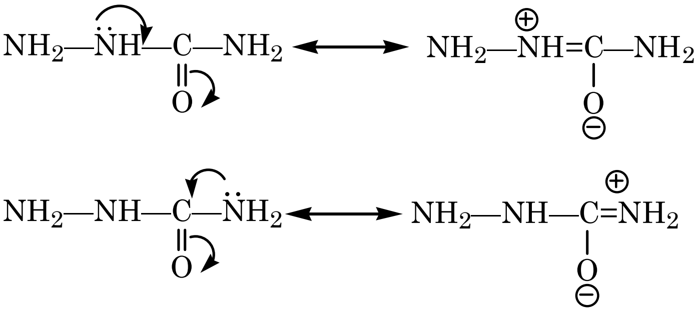
Sol. 

6. Ans.(A,B)

Sol. 

7. Ans. (A,C)

Sol. In NH_2—NH—$\underset{\underset{O}{||}}{C}$—$NH_2$ the resonating structures are



Hence the lone pair of electrons on nitrogen of (y) and (z) are not available on nitrogen, so only the lone pair of electrons on nitrogen of (x) nitrogen end is free.

Hence x–nitrogen end attacks, while y and z nitrogen ends do not attack.

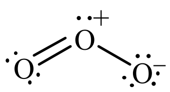
So choice (B) is correct while (A), (C) and (D) are incorrect.

8. Ans. (A,B,C)

Sol. (A), (B) and (C) are correct.

(A) is correct because O_3 in unstable and dark blue gas which is diamagnetic due to absence of unpaired electrons.

(B) O_3 is bent molecule, sp^2 hybridised.



sp^2 hybridised

(C) $2Hg + O_3 \longrightarrow Hg_2O + O_2$

Mercurous oxide is formed which stricks to glass. It is called tailing of mercury.

(D) is wrong because KOH reacts with O_3

$$2KOH + 5O_3 \longrightarrow \underset{\substack{\text{(orange solid)} \\ \text{(Potassium ozonide)}}}{2KO_3} + 5O_2 + H_2O$$

9. Ans.(A,C,D)

Sol. An element can be said to be a catalyst if it can take part in the reaction and can come out unaffected.

This is possible if the element can :

(i) Show variable oxidation state.

(ii) Its surface can take part in bond formation.

(iii) The oxidation states should be easily interconvertible.

These properties are well satisfied by transition elements.

(C) transition elements show variable oxidation state.

(A) transition elements have partially filled d–orbitals, hence their surface will have unsatisfied valencies.

(D) transition elements have low oxidation potential and low reduction potential.

10. Ans. (C,D)

Sol. Bond strength of CO < CO^+.

Bond length $\propto$ 1/bond order.

Unpaired electrons : O_2 = 2, NO = 1 and CO = 0

Number of ABMO electrons : O_2 = 6, N_2^- = 5 and Be_2 = 4.

SECTION-III

1. Ans. (8)

Sol. $t_{298\ K} = t_{308\ K}$

$$\frac{2.303}{K_{298}}\log\frac{100}{86.5} = \frac{2.303}{K_{308}}\log\frac{100}{75}$$

$$\frac{K_{308}}{K_{298}} = 2$$

$$\log\frac{K_{308}}{K_{298}} = \frac{E_a}{2.3\times\frac{25}{3}}\left[\frac{1}{300}-\frac{1}{310}\right]$$

E_a = 53.475 kJ/mole

2. Ans. (4)

Sol. $pH = pK_a + \log\frac{[Salt]}{[Acid]}$ $pK_b = -\log 10^{-10} = 10$

$= pK_a + \log\frac{[HX]}{[X^-]} = 14 - pK_b + \log 1 = 14 - 10 + 0$; pH = 4

3. Ans. (2)

Sol. $HA \rightleftharpoons H^+ + A^-$

$C(1-\alpha)$ $C\alpha$ $C\alpha$; $\alpha = \frac{\wedge_{eq}}{\wedge_{eq}^{\infty}} \Rightarrow 10/100 = 0.1$

$[H^+] = 0.1 \times 0.1 \Rightarrow 0.01$

pH = 2

4. Ans. (7)

Sol. $3C(s) + 3H_2(g) \longrightarrow \Delta(g)$; $\Delta_f H(obs) = 55$

$\Delta_f H$ (The retied) = [3 × 715) + (3 × 2 × 220)] – [(355 × 3) + (6 × 410)]

= [2145 + 1320] – [1065 + 2460]

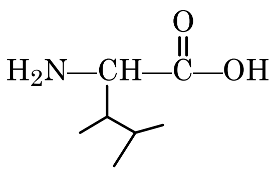
= 3465 – 3525 = –60 kJ/mole.

Ring strain energy = $\Delta_f H$ (Theoretical) – $\Delta_f H$ (observed)

= –60 – 55 = –115 kJ/mole

|Ring strain energy| = 115 kJ/mole.

5. Ans. (3)

Sol.  → Isoleucine → Essential amino acids.

$H_2N—CH—COOH$ with $(CH_2)_4—NH_2$ on CH → Lysine → Essential amino acids.

$H_2N—CH_2—COOH$ → Glycine → Non essential amino acids.

$H_2N—CH—COOH$ with $CH_2—C_6H_5$ on CH → Phenyl alanine → Essential amino acids.

6. Ans. (1)

7. Ans. (4)

Sol. Siderite ⇒ $FeCO_3$

Limonite ⇒ $FeO.nH_2O$

Zincite ⇒ ZnO

Dolomite ⇒ $MgCO_3.CaCO_3$

Calamine ⇒ $ZnCO_3$

Malachite ⇒ $CuCO_3.Cu(OH)_2$

8. Ans. (1)

Sol. (i) $[Cu(C_2H_5NH_2)_4]^{2+}$ ⇒ Blue

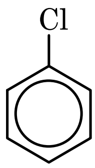
(ii) $[Cu(CN)_4]^{-3}$ ⇒ Colourless

(iii) $Cu_2[Fe(CN)_4]$ ⇒ Chocolate Brown

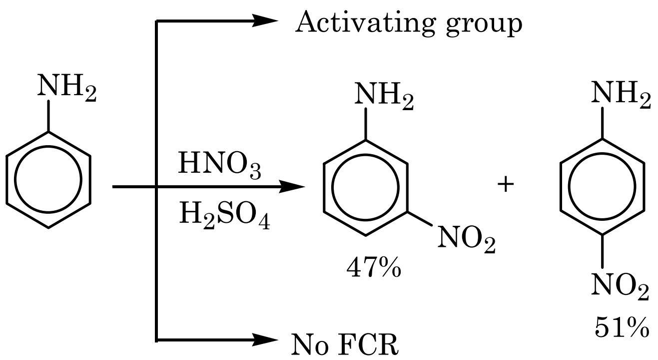
(iv) $\underset{(CuI_3)}{Cu_2I_2+I_2} \longrightarrow$ Brown

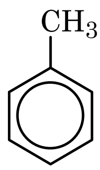
SECTION-IV

1. Ans. (A)-(P,R); (B)-(S,T); (C)-(R,S); (D)-(P,Q)

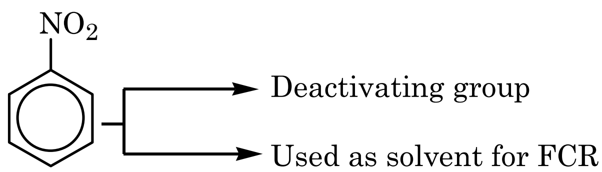
Sol. (A) 

⇒ Deactivating group.

(B) 

(C) 

⇒ Activating group

(D) 

2. Ans. (A)-(R); (B)-(R); (C)-(Q); (D)-(P)

Sol. (A) & (B) enantiomeric pair + 1 meso.

(C) No Stereo.

(D) Unsymmetric ligands ⇒ G.I. + O.I.

PART-3 : MATHEMATICS SOLUTION

SECTION-I

1. Ans. (A,B,C)

Sol. (A) $\int_0^{\pi} \cos^5 x\, dx = 0$

$[\because \cos^5(\pi - x) = -\cos^5 x]$

(B) $\int_{-3}^{-1} (x+1)(x+2)(x+3)\, dx$

Put $x + 2 = t$

$\Rightarrow \int_{-1}^{1} (t-1)(t)(t+1)\, dt$

$\Rightarrow \int_{-1}^{1} t(t^2-1)\, dt = 0$

(C) Let $I = \int_0^{\pi} \frac{\sin 4x}{\sin x} dt$

$I = \int_0^{\pi} \frac{\sin 4(\pi - x)}{\sin(\pi - x)} dx = \int_0^{\pi} \frac{-\sin 4x}{\sin x} dx$

$\Rightarrow I = 0$

(D) $I = \int_0^{\pi} [\cos x]\, dx = \int_0^{\pi} [-\cos x]\, dx$

$2I = \int_0^{\pi} \left([\cos x] + [-\cos x]\right) dx = -\pi$

$\Rightarrow I = -\frac{\pi}{2}$

2. Ans. (B,C,D)

Sol. $f(x) + 2f(1-x) = x^2 + 1 \quad \forall\, x \in R$

$x \to 1 - x$

$f(1-x) + 2f(x) = (1-x)^2 + 1 \quad \forall\, x \in R$

$\Rightarrow 4f(x) + 2f(1-x) = 2(1-x)^2 + 2$ ... (1)

$f(x) + 2f(1-x) = x^2 + 1$ ... (2)

Subtracting Eq. (2) from Eq. (1)

$\Rightarrow 3f(x) = 2[x^2 - 2x + 1] - x^2 + 1$

$= x^2 - 4x + 3$

$f(x) = \frac{1}{3}(x^2 - 4x + 3)$

$f(x) = \frac{1}{3}((x-2)^2 - 1)$

$\Rightarrow$ Range of $f(x) = \left[-\frac{1}{3}, \infty\right)$

3. Ans. (A,B,C)

Sol. Let $S(-1, 2)$ and $S'(1, 2)$

$\Rightarrow$ given equation will be $PS + PS' = k$

$SS' = 2$

When $SS' < k$ then locus is ellipse

When $SS' = k$ then locus is line segment

When $SS' > k$ then no locus

4. Ans. (A,C,D)

Sol. $\bar{r} = \hat{i} + 2\hat{j} + \hat{k} + \lambda(\hat{i} - \hat{j} + \hat{k})$

$\bar{r} = \bar{a} + \lambda \vec{n}_1$ (let)

$\bar{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu(2\hat{i} + \hat{j} + 2\hat{k})$

$\bar{r} = \bar{b} + \lambda \vec{n}_2$ (let)

AB will be line of shortest distance

DR's of $\overline{AB}$ will be $< \vec{n}_1 \times \vec{n}_2 >$

$$< \vec{n}_1 \times \vec{n}_2 > = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = -3\hat{i} + (0)\hat{j} + 3\hat{k}$$

$\Rightarrow$ DR's of AB will be <–1, 0, 1>

$$AB = \frac{\left|(\bar{b} - \bar{a}).(\bar{n}_1 \times \bar{n}_2)\right|}{|\bar{n}_1 \times \bar{n}_2|} = \frac{3}{\sqrt{2}}$$

Let A be $(1 + \lambda, 2 - \lambda, 1 + \lambda)$ and B be $(2 + 2\mu, \mu - 1, 2\mu - 1)$

$\Rightarrow$ DR's of AB will be

$< 2\mu - \lambda + 1, \mu + \lambda - 3, -\lambda + 2\mu - 2 >$

$\Rightarrow \frac{2\mu - \lambda + 1}{-1} = \frac{\mu + \lambda - 3}{0} = \frac{-\lambda + 2\mu - 2}{1}$

$\Rightarrow \mu + \lambda = 3, \lambda - 2\mu - 1 = -\lambda + 2\mu - 2$

$\Rightarrow \mu + \lambda = 3$ and $2\lambda - 4\mu = -1$

$\Rightarrow \lambda = \frac{11}{6}$ and $\mu = \frac{7}{6}$

$\Rightarrow A\left(\frac{17}{6}, \frac{1}{6}, \frac{17}{6}\right)$ and $B\left(\frac{26}{6}, \frac{1}{6}, \frac{8}{6}\right)$

Normal vector to the plane OAB

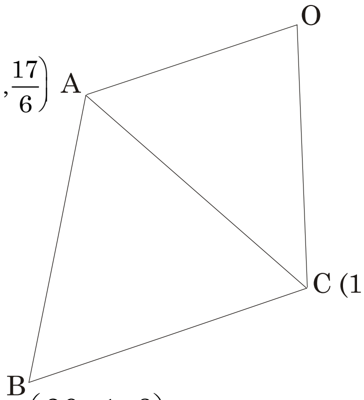
$$= \frac{1}{36} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 17 & 1 & 17 \\ 26 & 1 & 8 \end{vmatrix}$$

$= \frac{1}{36}\left[-9\hat{i} + 306\hat{j} - 9\hat{k}\right]$

$= \frac{1}{4}\left[\hat{i} - 34\hat{j} + \hat{k}\right]$

$\Rightarrow$ Equation of plane is $\bar{r}.(\hat{i} - 34\hat{j} + \hat{k}) = 0$

(D) $\left(\frac{17}{6},\frac{1}{6},\frac{17}{6}\right)$ A



$\left(\frac{26}{6},\frac{1}{6},\frac{8}{6}\right)$

Volume of Tetrahedron = $\frac{1}{6}\left\|\begin{vmatrix} 1 & 0 & 1 \\ \frac{17}{6} & \frac{1}{6} & \frac{17}{6} \\ \frac{26}{6} & \frac{1}{6} & \frac{8}{6}\end{vmatrix}\right\|$

$=\frac{1}{216}\left\|\begin{vmatrix} 1 & 0 & 1 \\ 17 & 1 & 17 \\ 26 & 1 & 8\end{vmatrix}\right\| = \frac{1}{216}\left\|\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 17 \\ 18 & 1 & 8\end{vmatrix}\right\|$

$= \frac{18}{216}=\frac{1}{12}$

5. Ans. (B,C,D)

Sol. $f(x)-2\frac{\sin^2 x}{\cos^5 x}\underbrace{\int_0^{\pi/4}\cos t.f(t)\,dt}=\frac{\sin^2 x}{\cos^5 x}$

$f(x)-2\frac{\sin^2 x}{\cos^5 x}(k)=\frac{\sin^2 x}{\cos^5 x}$

where $k=\int_0^{\pi/4}\cos t.f(t)\,dt$

and $f(x)=(2k+1)\frac{\sin^2 x}{\cos^5 x}$

$\Rightarrow$ $k=\int_0^{\pi/4}(2k+1)\frac{\sin^2 t}{\cos^4 t}dt$

$k = (2k+1)\int_0^{\pi/4}\tan^2 t.\sec^2 t\ dt$

$k = (2k+1)\left(\frac{1}{3}\right)\Rightarrow k=1$

$\Rightarrow$ $f(x)=\frac{3\sin^2 x}{\cos^5 x}\Rightarrow$ period of f(x) is 2π

$\lim_{x\to 0}\frac{f(x)}{x^2}=\lim_{x\to 0}\frac{3\sin^2 x}{x^2\cos^5 x}=3$

Point on $y = \frac{3\sin^2 x}{\cos^5 x}$ whose abscissa is π

$\Rightarrow$ y = 0 $\Rightarrow (\pi, 0)$

$\frac{dy}{dx}=3\left[\frac{\cos^5 x(2\sin x\cos x)-\sin^2 x\cos^4 x(-\sin x)}{\cos^{10} x}\right]$

$\frac{dy}{dx}$ at $(x = \pi) = 0$

$\Rightarrow$ Normal at this point will be vertical

$\Rightarrow$ Equation of normal is $x = \pi$

$f(x)=0\Rightarrow\frac{3\sin^2 x}{\cos^5 x}=0\Rightarrow\sin^2 x=0$

$\Rightarrow$ $x = n\pi$ $\Rightarrow$ no solution in (0, 3)

6. Ans. (C,D)

Sol. $(1+x^2)\frac{dy}{dx}+2xy=2$

$\frac{dy}{dx}+\left(\frac{2x}{1+x^2}\right)y=\frac{2}{1+x^2}$

I.F. = $e^{\int\frac{2x}{1+x^2}dx} = 1+x^2$

$\Rightarrow$ $y\times(1+x^2)=\int 2\,dx + c$

$y\times(1+x^2) = 2x + c$

put x = 0

2 = 0 + c

$y=\frac{2(1+x)}{1+x^2}$

$\Rightarrow$ f(x) is neither even nor odd

$f'(x)=\frac{-2(x^2+2x-1)}{(1+x^2)^2}$

$\Rightarrow$ f′(x) is neither increasing nor decreasing in $(-\infty, 0)$

$f'(1)=\frac{-2(2)}{4}=-1$

$\Rightarrow$ slope of normal = 1

$\Rightarrow$ normal at x = 1 is equally inclined with co-ordinate axes.

Area bounded by f(x) and x axis between x = –1 and x = 1

$=\int_{-1}^{1}\frac{2(1+x)}{1+x^2}dx$

$$= 2\int_{-1}^{1}\frac{dx}{1+x^2}+\int_{-1}^{1}\frac{2x}{1+x^2}dx$$

$$= 4\int_{0}^{1}\frac{dx}{1+x^2}+0$$

$$= 4[\tan^{-1}x]_0^1 = \pi$$

7. Ans. (B,C,D)

Sol. $^3C_2\, p^2\,(1-p) + {}^3C_3\, p^3$

$= {}^5C_3\, p^3\,(1-p)^2 + {}^5C_4\, p^4\,(1-p) + {}^5C_5\, p^5$

$\Rightarrow 3\,p^2(1-p) + p^3$

$= 10\,p^3\,(1-p)^2 + 5\,p^4\,(1-p) + p^5$

$\Rightarrow p = 0$ and

$10\,p\,(1 + p^2 - 2p) + 5p^2\,(1-p) + p^3 - 3(1-p) - p = 0$

$\Rightarrow p = 0$ and

$6\,p^3 - 15p^2 + 12p - 3 = 0$

$2p^3 - 5p^2 + 4p - 1 = 0$

$(p-1)^2\,(2p-1) = 0$

$\Rightarrow \quad p = 0, 1$ or $\frac{1}{2}$

8. Ans. (B,D)

Sol. $z = e^{i\pi/13} \Rightarrow z^{13} = -1$

$z^{13} + 1 = 0 \Rightarrow \frac{1+z^{13}}{1+z} = 0$

$\Rightarrow 1 - z + z^2 - z^3 + \ldots\ldots\ldots + z^{12} = 0 \quad \ldots (1)$

$1 - z^{13} = 2$

$\Rightarrow \frac{1-z^{13}}{1-z} = \frac{2}{1-z}$

$\Rightarrow 1 + z + z^2 + z^3 + \ldots\ldots. + z^{12} = \frac{2}{1-z} \quad \ldots (2)$

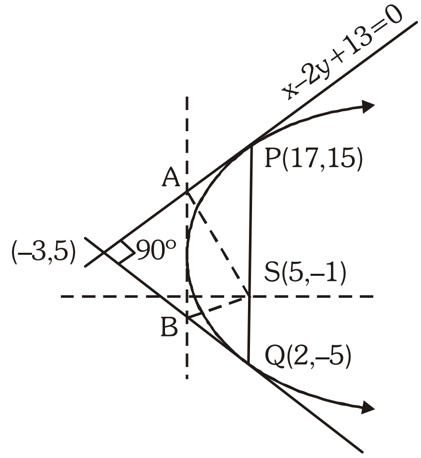
from (1) & (2)

$1 + z^2 + z^4 + z^6 + z^8 + z^{12} = \frac{1}{1-z}$ (B option)

$\& \; z + z^3 + z^5 + \ldots\ldots\ldots + z^{11} = \frac{1}{1-z}$ (D option)

9. Ans. (C,D)

Sol. Point (–3, 5) will lie on directrix of parabola. PQ will be a focal chord and equation PQ is $4x - 3y - 23 = 0$



Focus S will be foot of perpendicular drawn from (–3, 5) on $4x - 3y - 23 = 0$

$\Rightarrow \frac{x+3}{4} = \frac{y-5}{-3} = \frac{-(-12-15-23)}{25}$

$\frac{x+3}{4} = \frac{y-5}{-3} = 2$

$\Rightarrow x = 5 \;\&\; y = -1$

$\Rightarrow$ focus S is (5, –1)

Foot of perpendicular from focus on a tangent lies on tangent at vertex

$\Rightarrow$ Foot of perpendicular from focus on given tangent are A(1, 7) & B(1, –3)

$\Rightarrow$ tangent at vertex is $x = 1$

$\Rightarrow$ directrix of parabola $x = -3$

$\Rightarrow$ Length of L.R. = 2(8) = 16

10. Ans. (A,C,D)

Sol. $AB = \begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}\begin{bmatrix} a \\ b \\ 1 \end{bmatrix}$

$AB = \begin{bmatrix} -3a-7b-5 \\ 2a+4b+3 \\ a+2b+2 \end{bmatrix} = \lambda\begin{bmatrix} a \\ b \\ 1 \end{bmatrix}$

$\Rightarrow \quad (3+\lambda)a + 7b + 5 = 0$

$2a + (4-\lambda)b + 3 = 0$

$a + 2b + 2 - \lambda = 0$

$\Rightarrow \begin{vmatrix} 3+\lambda & 7 & 5 \\ 2 & 4-\lambda & 3 \\ 1 & 2 & 2-\lambda \end{vmatrix} = 0$

$\Rightarrow \quad \lambda = 1$

$4a + 7b + 5 = 0,\; 2a + 3b + 3 = 0$

$\& \quad a + 2b + 1 = 0$

$\Rightarrow \quad a = -3$ and $b = 1$

SECTION-III

1. Ans. 5

Sol. $A_1H_{11} = A_2H_{10} = A_3H_9 = A_4H_8 = A_5H_7 = ab$

a, G_1, G_2, G_3, G_4, ..., G_{10}, G_{11}, b are in G.P.

$\Rightarrow \quad G_2G_{10} = G_4G_8 = (G_6)^2 = ab$

$$\prod_{k=1}^{5}\left(A_K G_{12-2K} H_{12-K}\right)$$

$= (A_1H_{11})\ (A_2H_{10})\ (A_3H_9)\ (A_4H_8)\ (A_5H_7)\ (G_{10}G_8G_6G_4G_2)$

$= (ab)^5\ (ab)^2\ \sqrt{ab}$

$= (ab)^7\sqrt{ab} = (3^2)^7 \cdot 3 = 3^{15} = p^q$

$\Rightarrow \quad \left[\frac{q}{p}\right] = 5$

2. Ans. 1

Sol. Case-1

$x^2 + 2(K - 3)x + 7 - 3K = 0$

& $x^2 + 2(K - 1)x + 3K - 5 = 0$ has both roots common :

$$\frac{1}{1} = \frac{2(K-3)}{2(k-1)} = \frac{7-3K}{3K-5}$$

$\Rightarrow$ No values of K

Case-2

Each equation has equal roots

$\Rightarrow 4(K - 3)^2 - 4(7 - 3K) = 0$

and $4(K - 1)^2 - 4(3K - 5) = 0$

$\Rightarrow K^2 - 3K + 2 = 0$ and $K^2 - 5K + 6 = 0$

$(K - 2)\ (K - 1) = 0$ and $(K - 2)\ (K - 3) = 0$

$\Rightarrow \quad K = 2$

3. Ans. 1

Sol. $\lim\limits_{x\to 0}\frac{1}{x^3}\left(\frac{(1-bx)-(1-ax)\sqrt{1+x}}{\sqrt{1+x}(1-bx)}\right)$

$$\Rightarrow \lim_{x\to 0}\frac{1}{x^3}\left[\frac{(1-bx)-(1-ax)(1+\frac{x}{2}-\frac{1}{8}x^2+\frac{1}{16}x^3+\ldots)}{\sqrt{1+x}(1-bx)}\right]$$

$$\Rightarrow \lim_{x\to 0}\frac{\left(a-b-\frac{1}{2}\right)x+\left(\frac{a}{2}+\frac{1}{8}\right)x^2+\left(-\frac{1}{16}-\frac{a}{8}\right)x^3+\ldots}{x^3\sqrt{1+x}(1-bx)}$$

$\Rightarrow a - b - \frac{1}{2} = 0 \ \ \& \ \ \frac{a}{2}+\frac{1}{8} = 0$

and $\ell = -\frac{1}{16}-\frac{a}{8}$

$\Rightarrow a - b = \frac{1}{2} \ \ \& \ \ a = -\frac{1}{4}$

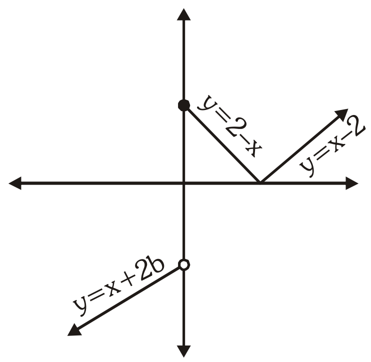
$b = -\frac{3}{4} \ \ \& \ \ a = -\frac{1}{4}$

and $\ell = -\frac{1}{16}+\frac{1}{32} = -\frac{1}{32}$

$\Rightarrow \frac{1}{a}-\frac{3}{b}-32\ell = -4 + 4 + 1 = 1$

4. Ans. 1

Sol. $f(g(x)) = \begin{cases} g^2(x)+1, & g(x) < 0 \\ (g(x)-1)^3+2a, & g(x) \ge 0 \end{cases}$



$$f(g(x)) = \begin{cases} (x+2b)^2+1, & g(x) < 0 \Rightarrow x < 0 \\ ((2-x)-1)^3+2a, & 0 \le x \le 2 \\ ((x-2)-1)^3+2a, & x > 2 \end{cases}$$

$$fog(x)) = \begin{cases} (x+2b)^2+1, & x < 0 \\ (1-x)^3+2a, & 0 \le x \le 2 \\ (x-3)^3+2a, & x > 2 \end{cases}$$

fog(x) is continuous $\forall\ x \in R$ if it is continuous at x = 0

$\Rightarrow f(0^-) = f(0^+) = f(0)$

$\Rightarrow 4b^2 + 1 = 1 + 2a \Rightarrow a = 2b^2$

fog(x) is differetiable at x = 0

$\Rightarrow 2(2b) = -3$

$\Rightarrow b = -\frac{3}{4}$

$a = 2b^2 \Rightarrow a = \frac{9}{8}$

$\Rightarrow a - b = \frac{9}{8}+\frac{6}{8} = \frac{15}{8}$

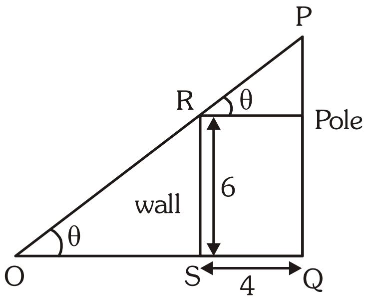
$\Rightarrow [a - b] = 1$

5. Ans. 6

Sol. Let length of wire be y.

$y = 6 \operatorname{cosec}\theta + 4 \sec\theta$

$\frac{dy}{d\theta} = -6 \operatorname{cosec}\theta \cot\theta + 4 \sec\theta \tan\theta = 0$



$\Rightarrow -6\dfrac{\cos\theta}{\sin^2\theta}+\dfrac{4\sin\theta}{\cos^2\theta}=0$

$\Rightarrow \tan^3\theta = \dfrac{3}{2}$

$\tan\theta = \left(\dfrac{3}{2}\right)^{1/3}$

$\dfrac{d^2y}{d\theta^2} = 6\text{ cosec}^3\theta + 6\text{ cosec}^2\theta\cot\theta + 4\sec^3\theta + 4\sec\theta\tan^2\theta$

θ will be acute

$\Rightarrow \dfrac{d^2y}{d\theta^2} > 0 \text{ for } \tan\theta = \left(\dfrac{3}{2}\right)^{1/3}$

⇒ minimum length of wire

$y = 6\text{ cosec}\theta + 4\sec\theta$ at $\tan\theta = \dfrac{3^{1/3}}{2^{1/3}}$

$y = \sqrt{2^{2/3}+3^{2/3}}\left[\dfrac{6}{3^{1/3}}+\dfrac{4}{2^{1/3}}\right]$

$= \sqrt{2^{2/3}+3^{2/3}}\left[3^{2/3}.2+2^{2/3}.2\right]$

$= 2\left[2^{2/3}+3^{2/3}\right]^{3/2} = 2\left[a^{2/3}+b^{2/3}\right]^{p/q}$

$\Rightarrow a + b = 5, p = 3 \ \& \ q = 2$

$\Rightarrow a + b + p - q = 5 + 3 - 2 = 6$

6. Ans. 6

Sol. $\lim\limits_{x\to 0}\left(3-\dfrac{P(x)}{x}\right)=27$

⇒ P(x) has no constant term

let $P(x) = ax^4 + bx^3 + cx^2 + dx$

$\Rightarrow 3 - d = 27 \Rightarrow d = -24$

$P(x) = ax^4 + bx^3 + cx^2 - 24x$

$P'(2) = 0, p(1) = -9, p'''(2) = 0$

$P'(x) = 4ax^3 + 3bx^2 + 2cx - 24$

$P''(x) = 12ax^2 + 6bx + 2c$

$P'''(x) = 24ax + 6b$

$a + b + c - 24 = -9$

$\Rightarrow a + b + c = 15 \quad \ldots (1)$

$P'(2) = 0$

$\Rightarrow 4a(8) + 3b(4) + 2c(2) - 24 = 0$

$\Rightarrow 8a + 3b + c = 6 \quad \ldots (2)$

$P'''(2) = 0$

$\Rightarrow 24a(2) + 6(b) = 0$

$\Rightarrow 8a + b = 0 \quad \ldots (3)$

Solving (1), (2) & (3)

$a = 1, b = -8, c = 22$

$\Rightarrow P(x) = x^4 - 8x^3 + 22x^2 - 24x$

$P'(x) = 4x^3 - 24x^2 + 44x - 24$

$= 4[x^3 - 6x^2 + 11x - 6]$

$P'(x) = 4[(x-1)(x-2)(x-3)]$

$P''(x) = 4[3x^2 - 12x + 11] > 0 \quad \forall x \in [3, 4]$

$\Rightarrow P'(x) = 4[(4-1)(4-2)(4-3)]$

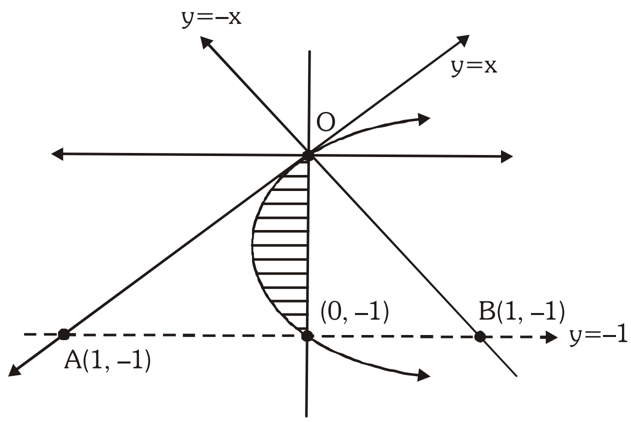
$= 4[(3)(2)(1)]$

$= 24 = 4M$

$\Rightarrow M = 6$

7. Ans. 3

Sol. Tangent to parabola at origin is $x - y = 0$.



$y^2 + y = x$

$\left(y+\dfrac{1}{2}\right)^2 = x + \dfrac{1}{4}$

Let A be the shaded area

$A = \int_{-1}^{0} -(y^2+y)\,dx = -\left[\dfrac{y^3}{3}+\dfrac{y^2}{2}\right]_{-1}^{0}$

$= -\left[-\left(-\dfrac{1}{3}+\dfrac{1}{2}\right)\right] = \dfrac{1}{6}$

Area of OAB $= \dfrac{1}{2}\times 1\times 2 = 1$

$\dfrac{A_1}{A_2} = \dfrac{\frac{1}{2}-\frac{1}{6}}{\frac{1}{2}+\frac{1}{6}}$ or $\dfrac{\frac{1}{2}+\frac{1}{6}}{\frac{1}{2}-\frac{1}{6}}$

$\Rightarrow \quad \frac{A_1}{A_2} = \frac{1}{2}$ or $\frac{2}{1}$

$\Rightarrow \quad \frac{p}{q} = \frac{1}{2}$ or $\frac{2}{1}$

$\Rightarrow \quad p + q = 3$

8. Ans. 0

Sol. $2 \sin 2x + \sin x (1 - 2\cos 2x) = 4$

$4 \sin x \cos x + \sin x (1 - 2(2\cos^2 x - 1)) = 4$

$$4 \cos x + 1 - 4\cos^2 x + 2 = \frac{4}{\sin x}$$

$$-[4\cos^2 x - 4\cos x] + 3 = \frac{4}{\sin x}$$

$$-[(2\cos x)^2 - 1] + 3 = \frac{4}{\sin x}$$

$$4 - (2\cos x - 1)^2 = \frac{4}{\sin x}$$

$\Rightarrow$ L.H.S. ≤ 4 and R.H.S. ≥ 4

and equality holds when $\cos x = \frac{1}{2}$ and $\sin x = 1$ which is not possible.

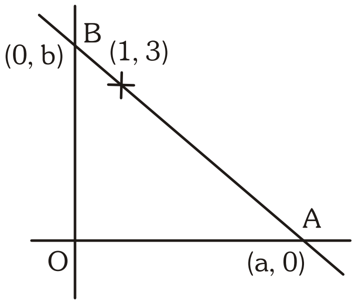
$\Rightarrow$ No solution

SECTION - IV

1. Ans. (A) → (P,S,T) ; (B) → (Q,R,S,T) ; (C) → (Q,R,T) ; (D) → (Q,R,T)

Sol. (A) $\frac{x}{a} + \frac{y}{b} = 1 \Rightarrow \frac{1}{a} + \frac{3}{b} = 1$

apply A.M. $\ge$ G.M. in $\frac{1}{a}$ and $\frac{3}{b}$



$\Rightarrow \quad \frac{\frac{1}{a} + \frac{3}{b}}{2} \ge \left(\frac{3}{ab}\right)^{1/2} \Rightarrow \sqrt{ab} \ge 2\sqrt{3}$

$\Rightarrow \quad ab \ge 12$

Area of $\Delta OAB = \frac{1}{2} ab \ge 6$

$\Rightarrow$ least value of area is 6

condition for area to be least.

$\frac{1}{a} = \frac{3}{b}$

slope of straight line $= -\frac{b}{a} = -3$

(B) Common chord of circles,

$3x + 4y + p + q = 0$

will pass through centre of circle

$x^2 + y^2 + 2x + 6y + p = 0$

$\Rightarrow \quad -3 - 12 + p + q = 0$

$\Rightarrow \quad p + q = 15$

(C) $\sum_{i=1}^{5} \left((SN_i)(S'N_i')\right) = 169$

$5(b^2) = 169$

$b = \frac{13}{\sqrt{5}} = 5.81...$

(D) $(PQ)(P'Q) = (\text{semi minor axis})^2 = 9$

2. Ans. (A) → (P,Q,T) ; (B) → (P,R) ; (C) → (P,Q,R,T) ; (D) → (P,Q,R,T)

Sol. (A) $2^{2019} + 2^{2018} + 2^{2017} + ... + 2 + 1$

$= [2^{2020} - 1] = 2[2^3]^{673} - 1$

$= 2[9 - 1]^{673} - 1$

$= 2[9\lambda - 1] - 1$

$= 9(2\lambda) - 3$

$\Rightarrow$ remainder is 6

(B) Probability that problem will not solved

$= \left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right)\left(1 - \frac{1}{4}\right)$

$= \frac{2}{3} \cdot \frac{4}{5} \cdot \frac{3}{4}$

$= \frac{2}{5}$

$\Rightarrow$ odds against the given event are $2 : 3$

$\Rightarrow \quad a : b = 2 : 3$

$\Rightarrow \quad a = 2$ & $b = 3$

$\Rightarrow \quad 2b + a = 6 + 2 = 8$

(C) $\{1, 2, 3, 4, 5, 6\}$

$$p = \frac{{}^4C_2 + {}^3C_1 \times {}^2C_1}{{}^6C_2} = \frac{12}{15} = \frac{4}{5}$$

$15\,p = 12$

(D) $3^4 - {}^3C_1 (2)^4 + {}^3C_2 (1)^4$

$= 81 - 3 \times 16 + 3$

$= 81 - 48 + 3$

$= 84 - 48 = 36$

JEE (Main + Advanced) : ENTHUSIAST COURSE
SCORE : II

Test Type : FULL SYLLABUS   Test Pattern : JEE-Main

TEST DATE : 27 - 01 - 2019

PAPER-2

SOLUTION

1. Ans. (4)

Sol. Let acceleration of A be a_1 downwards and that of B is a_2 upwards and let T be the tension in the string. Then equations of motion of A and B gives

$m_1g - T = m_1a_1$ ..... (i)

$2T - m_2g = m_2a_2$ ..... (ii)

Equilibrium of pulley C gives

$2T - T = 0$

$\Rightarrow T = 0$

Substituting in (i) and (ii)

$a_1 = g\downarrow$ and $a_2 = +g\downarrow$

2. Ans. (2)

Sol. $a_t = \alpha t$

$$v = \frac{\alpha t^2}{2}$$

$$a_{total} = \sqrt{a_r^2 + a_t^2}$$

$$\tan 45° = \frac{a_t}{a_r} = 1$$

$$\frac{\alpha t \times 4 \times 2}{\alpha^2 t^4} = 1$$

at $t = 2s$

$$\frac{\alpha(2)\times 4\times 2}{\alpha^2(2)^4} = 1$$

$\alpha = 1\ m/s^3$

3. Ans. (2)

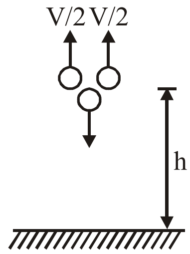
Sol. At equilibrium $\frac{dU}{dr} = 0$

$$\frac{-12A}{r^{13}} + \frac{6B}{r^7} = 0$$

$$r = \left(\frac{2A}{B}\right)^{1/6}$$

4. Ans. (3)

Sol. $h = V(10) + \frac{1}{2}g(10)^2$



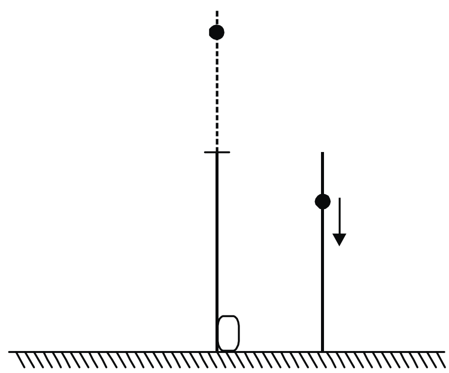
$$h = -\frac{V}{2}(20) + \frac{1}{2}g(20)^2$$

adding we get

$h = 1250$ m

5. Ans. (3)

Sol.



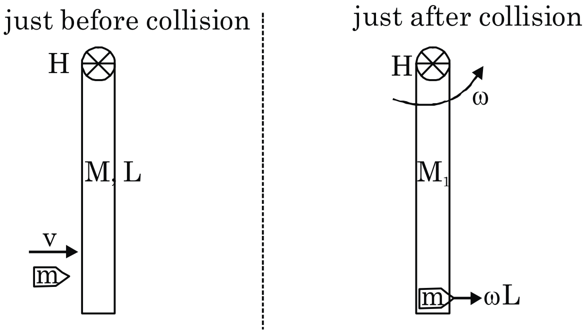
$$F_T = \lambda v^2 = \frac{m}{L}\left(\sqrt{2\times g\frac{\ell}{2}}\right)^2$$

$F_T = mg$

$$F_{Net} = mg + \frac{mg}{2} = \frac{3mg}{2}$$

6. Ans. (4)

Sol.



$(L_i)_H = (L_f)_H$

$$mvL(\hat{v}) = m(\omega L)(L)\hat{v} + \frac{ML^2\omega}{3}\hat{v}$$

$$mv^2 = \left(\frac{3m+M}{3}\right)\omega v^2$$

$$\omega = \frac{3mv}{(3m+M)L}$$

7. Ans. (2)

Sol. $$f_0 = \frac{1}{2\pi}\sqrt{\frac{mg\ell/2}{m\ell^2/3}}$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{3}{2}\frac{g}{\ell}} \quad .... (i)$$

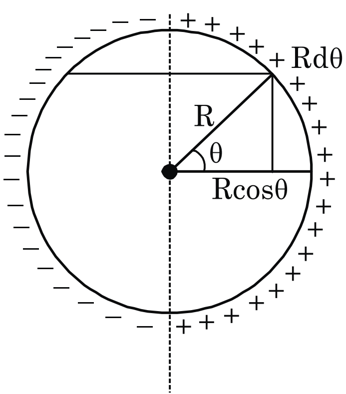
$$f' = \frac{1}{2\pi}\sqrt{\frac{(m/2)g(\ell/4)}{\left(\frac{m}{2}\right)\frac{(\ell/2)^2}{3}}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{2}\times\frac{1}{4}\times 2\times 4\times 3\cdot\frac{g}{\ell}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{3g}{\ell}} = \sqrt{2}f_0$$

8. Ans. (3)

Sol.



$dq = \lambda R d\theta$

$dq = \lambda_0 \cos\theta\, Rd\theta$

$dp = dq.2R\cos\theta$

$$= \frac{\lambda_0}{\pi/2}\cos\theta R d\theta 2R\cos\theta$$

$$p = \int dp = \int_{-\pi/2}^{\pi/2} 2R^2\lambda_0\cos^2\theta\, d\theta$$

$$= \frac{2R^2\lambda_0}{2}\int_{-\pi/2}^{\pi/2}\cos 2\theta + 1\, d\theta$$

$$= R^2\lambda_0\left[\frac{\sin 2\theta}{2} + \theta\right]_{-\pi/2}^{\pi/2}$$

$= R^2\lambda_0[\pi] = R^2\lambda_0\pi.$

9. Ans. (4)

Sol. $(Te)_i = (Te)_\infty$

$$-\frac{GMm}{R} + \frac{1}{2}m(3v_e)^2 = \frac{1}{2}m(v_\infty)^2$$

$$-\frac{GMm}{R} + \frac{1}{2}m\,9\left(\frac{2GM}{R}\right) = \frac{1}{2}m(v_\infty)^2$$

$$\frac{8GMm}{R} = \frac{1}{2}m(v_\infty)^2$$

$$\sqrt{\frac{16GM}{R}} = v_\infty = \sqrt{8}v_e = 2\sqrt{2}v_e$$

10. Ans. (1)

Sol. Balance point = 40 cm

$$\therefore \quad \frac{P}{40} = \frac{Q}{60}$$

$$P = \frac{2Q}{3} \quad ..... (i)$$

when Q shunt with 10Ω resistance

$$P = Q' \quad \left(Q' = \frac{10Q}{10+Q}\right)$$

$$\frac{2Q}{3} = \frac{10Q}{10+Q}$$

$Q = 5\Omega$

$$P = \frac{10}{3}\Omega$$

11. Ans. (4)

Sol. $$U_i = \frac{1}{2}C_0E^2 \quad .... (i)$$

On inserting a dielectric new capacitance is

$C = kC_0$ ..... (ii)

and new potential difference is {V = Ed}

$E' = f_e'$ where f_e is the electrostatic field. But on inserting the dielectric, the new field f_e' becomes $\frac{1}{k}$ times the original field.

So $f_e' = \frac{f_e}{k}$

$\Rightarrow E' = \frac{E}{k}$ ...... (iii)

So $U_f = \frac{1}{2}CE'^2$

$\Rightarrow U_f = \frac{1}{2}(kC_0)\left(\frac{E}{k}\right)^2$

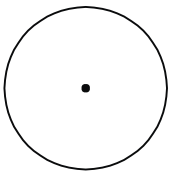
$\Rightarrow U_f = \left(\frac{1}{2}C_0E^2\right)\frac{1}{k}$

$\Rightarrow \Delta U = \frac{1}{2}C_0E^2\left(\frac{1}{k}-1\right)$

12. Ans. (1)

Sol. $B = Kt^2$

$\int \vec{E}\cdot\vec{dl} = \frac{d\phi}{dt} = \frac{AdB}{dt}$



$\Rightarrow E\cdot 2\pi r = \pi r^2 \cdot 2Kt$

$\Rightarrow E = Krt$

Ring starts moving when torque due to electric field becomes greater than torque due to limiting friction

$QEr = \mu Mgr$

$\Rightarrow Q . Krt . r = \mu Mg\ r$

$\Rightarrow t = \frac{\mu Mg}{QKr}$

13. Ans. (1)

Sol. Current sensitivity is given by $\phi/i = NAB/k$

$(\phi/i)_1 = (30 \times 3.6 \times 10^{-3} \times 0.25)/k$

$(\phi/i)_2 = (42 \times 1.8 \times 10^{-3} \times 0.5)/k$

$\therefore (\phi/i)_1 / (\phi/i)_2 = 5/7$ Ans.

14. Ans. (3)

Sol. $E = (10.2\text{ eV})(z-1)^2$

$\frac{hc}{\lambda} = (10.2\,\text{eV})(z-1)^2$

$\frac{\lambda_2}{\lambda_1} = \frac{(z_1-1)^2}{(z_2-1)^2}$

$\frac{\lambda_2}{\alpha} = \frac{(56)^2}{(28)^2} \Rightarrow \lambda_2 = 4\alpha$

15. Ans. (3)

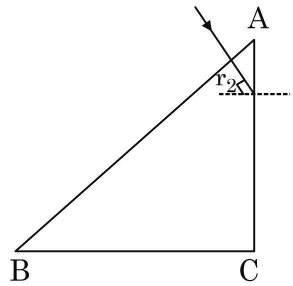
Sol. Energy released = (B. E. of product – BE of reactant)

$= (80 \times 7 + 120 \times 8 - 200 \times 6.5)$

= 220 MeV.

16. Ans. (2)

Sol.



$A = 90° - \theta$

$r_2 = A = 90° - \theta > \theta_C$

$\cos\theta > \sin\theta_C = \frac{6/5}{3/2} = \frac{4}{5}$ (θ_C is critical angle)

$\Rightarrow \theta < \cos^{-1}\frac{4}{5} = 37°$

17. Ans. (3)

Sol. In a closed tube, the first resonance takes place, when the length of air column = $\lambda/4$

The wavelength of sound in air

$\lambda = \frac{v}{n} = \frac{340}{340}$

= 1m [As v = 340 m/s and n = 340]

Here length of first resonance can be given as

$\frac{\lambda}{4} = 0.25\text{m} = 25\text{cm}$

The length of second resonance

$\frac{3\lambda}{4} = \frac{3}{4}\text{m} = 75\text{cm}$

The length of third resonance

$\frac{5\lambda}{4} = \frac{5}{4}\text{m} = 125\text{cm}$

As the length of the tube is only 120 cm, hence third resonance is not possible.

Therefore, the minimum height of water required for resonance

= 120 – 75 = 45 cm

18. Ans. (2)

Sol. From snell's low

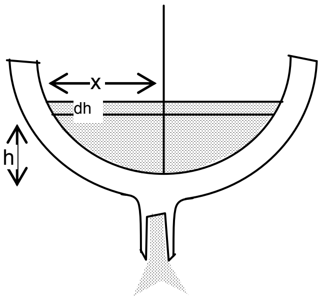
$1 \sin 45° = \sqrt{2}\sin\gamma$

$\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \sin\gamma$

$\gamma = 30°$

19. Ans. (3)

Sol. Hint $Q = av = \pi x^2\left(\frac{dh}{dt}\right)$



$dt\ a\sqrt{2gh} = -\pi x^2 dh$

$x^2 = R^2 - (R-h)^2$

20. Ans. (4)

Sol. From Stefan-Boltzmann law

Power $\propto T^4$

From Wein's law

$T \propto \frac{1}{\lambda_m}$

$\therefore$ Power $\propto \frac{1}{\lambda_m^4}$

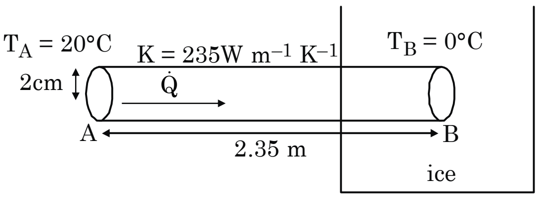
$P = \frac{K}{\lambda_0^4}$

$P_f = \frac{K}{\left(\frac{3\lambda_0}{4}\right)^4} = \left(\frac{4}{3}\right)^4 P$

$P_f = \frac{256}{81}P$

21. Ans. (3)

Sol.



$\dot{Q} = \frac{dQ}{dt} = \frac{kA\Delta T}{L} = \left(\frac{dm}{dt}\right)_{ice} L_f$

$\frac{235 \times \pi \times 2 \times 2 \times 10^{-4} \times 20}{2.35} = \left(\frac{dm}{dt}\right)_{ice} \times \frac{1}{3} \times 10^6$

$\frac{dm}{dt} = 2.4\pi \times 10^{-6}\ kg/sec.$

22. Ans. (3)

Sol. The strain in the wire $\frac{\Delta \ell}{\ell} = \frac{2.0mm}{2.0m} = 10^{-3}$

The stress in the wire = Y × strain

$= 2.0 \times 10^{11}\ Nm^{-2} \times 10^{-3}$

$= 2.0 \times 10^{8}\ Nm^{-2}$.

The volume of the wire

$= (4 \times 10^{-6}\ m^2) \times (2.0\ m)$

$= 8.0 \times 10^{-6}\ m^3$.

The elastic potential energy stored

$= \frac{1}{2} \times$ stress × strain × volume

$= \frac{1}{2} \times 2.0 \times 10^8\ N\ m^{-2} \times 10^{-3} \times 8.0 \times 10^{-6}\ m^3$

$= 0.8$ J.

23. Ans. (4)

Sol. $0.6 = \frac{\text{work done}}{Q_{input}} = \frac{Q_{input} - Q_{reject}}{Q_{input}} = 1 - \frac{Q_r}{Q_i}$

$0.6 = 1 - \frac{20}{Q_i}$

$Q_i = 50$

$W = Q_i - Q_r = 30$ J

24. Ans. (2)

Sol. In this case, vernier constant

$= \frac{1mm}{10} = 0.1mm$

Zero error = 6 × 0.1 = +0.6 mm

Correction = –0.6 mm

Actual length = (4.3 + 2 × 0.01) + correction

= 4.32 – 0.06 = 4.26 cm

25. Ans. (3)

Sol. In this case we use

$B_H = B_E \cos\theta$

$\Rightarrow B_E = \frac{B_H}{\cos\theta} = \frac{40 \times 10^{-6}}{\sqrt{3}/2}$

$\Rightarrow B_E = 46\mu T$

26. Ans. (1)

Sol. Damping force $F = -b\frac{dx}{dt}$

$= -bx_0\left[\omega_0 e^{-\omega_0 t} - \omega_0(1+\omega_0 t)e^{-\omega_0 t}\right]$

$= bx_0\omega_0^2 te^{-\omega_0 t}$

F is maximum when

$$\frac{dF}{dt} = 0$$

$$\Rightarrow \; bx_0\omega_0^2\left[e^{-\omega_0 t} - \omega_0 t e^{-\omega_0 t}\right] = 0$$

$$t = \frac{1}{\omega_0} = \frac{1}{100}\text{sec.} = 0.01\text{s}$$

27. Ans. (3)

Sol. Position of minima

$a \sin\theta = n\lambda$

$a \sin\theta = \lambda_1$

$$\sin\theta_1 = \frac{\lambda_1}{a} \qquad \text{..... (i)}$$

For maxima

$$a\sin\theta = \frac{3\lambda_2}{2} \; \text{[approximately]} \qquad \text{..... (ii)}$$

$$\sin\theta_1 = \frac{3}{2}\frac{\lambda_2}{a}$$

$$\lambda_1 = \frac{3}{2}\lambda_2$$

$$\lambda_2 = \frac{2}{3}\lambda_1 = 440\text{nm}$$

28. Ans. (1)

Sol. $\Delta\theta \geq 1.22\dfrac{\lambda}{d}$

$$R\Delta\theta = \frac{\overline{1.22}\times\overline{550}\times 10^{-9}}{\overline{32}\times 10^{-3}} \times \overline{24}\times 10^{-2}\text{m}$$

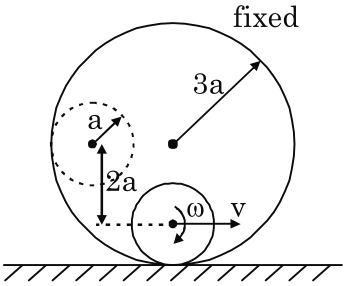
$= 503.25 \times 10^{-8}$

$= 5 \times 10^{-6}$ m $= 5\mu$m.

29. Ans. (1)

Sol. Conservation of energy

$$Mg(2a) = \frac{1}{2}Mv^2 + \frac{1}{2}(Ma^2)(\omega)^2$$



For pure rolling $v = \omega a$

$$Mg(2a) = \frac{1}{2}Mv^2 + \frac{1}{2}(Ma^2)\frac{V^2}{a^2}$$

$$Mv^2 = Mg(2a)$$

$$v = \sqrt{2ga}$$

30 Ans. (2)

Sol. $V = \dfrac{1000}{x} + \dfrac{1500}{x^2} + \dfrac{500}{x^3}$

$$E = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$$

$$\frac{\partial V}{\partial x} = -\frac{1000}{x^2} - \frac{3000}{x^3} - \frac{1500}{x^4}$$

$$\frac{\partial V}{\partial y} = 0 \;\; \& \;\; \frac{\partial V}{\partial z} = 0$$

$$\therefore \; E(x,0,0) = \left(\frac{1000}{x^2} + \frac{3000}{x^3} + \frac{1500}{x^4}\right)\hat{i} \text{ V/m}$$

$$E(1,0,0) = (1000 + 3000 + 1500)\hat{i}\text{ V/m}$$

$$= 5500\hat{i}\text{ V/m}$$

31. Ans. (3)

Sol. Spherical node $= \psi^2 = 0$

$\sigma^2 - 4\sigma + 3 = 0$; $\sigma^2 - 3\sigma - \sigma + 3 = 0$

$\sigma(\sigma - 3) - 1(\sigma - 3) = 0$; $(\sigma - 1)(\sigma - 3) = 0$

$\sigma = 1,3$; $\sigma = \dfrac{2Zr}{a_0} = 1,3$; $r = \dfrac{3}{2}\dfrac{a_0}{Z}, \dfrac{1}{2}\dfrac{a_0}{Z}$

32. Ans. (2)

Sol. $\dfrac{P_C V_C}{RT_C} = \dfrac{3}{8}$

$$\frac{73.89 \times V_C}{0.0821 \times 300} = \frac{3}{8}$$

$$V_C = \frac{1}{8} = 3b$$

$$b = \frac{1}{24}$$

$V_{actual} \times 4 = b$

$$V_{actual} = \frac{b}{4} = \frac{1}{24 \times 4}\text{for 1 mole}$$

For 24 mole

$$V_{actual} = \frac{24}{24 \times 4} \times 1000 = 250\text{ ml}$$

33. Ans. (1)

Sol. Hit & Trial Method.

34. Ans. (4)

Sol. $A(s) \rightleftharpoons B(g) + C(g)$

$K = 900 = P_1 (P_1 + P_2)$ $\qquad P_1 + P_2 \; P_1$

$D(s) \rightleftharpoons B(g) + E(g)$

$K = 1600 = P_2(P_1 + P_2)$

$P_2 + P_1 \; P_2$ $\qquad 2500 = (P_1 + P_2)^2$

$P_1 + P_2 = 50$

Ans. 100 mm of Hg

35. Ans. (3)

Sol. $q = 0$

$\Delta U = W = -P_{ext}(V_2 - V_1)$

$= -(40 - 50) = +40$ bar L

$\Delta H = \Delta U + \Delta(PV)$

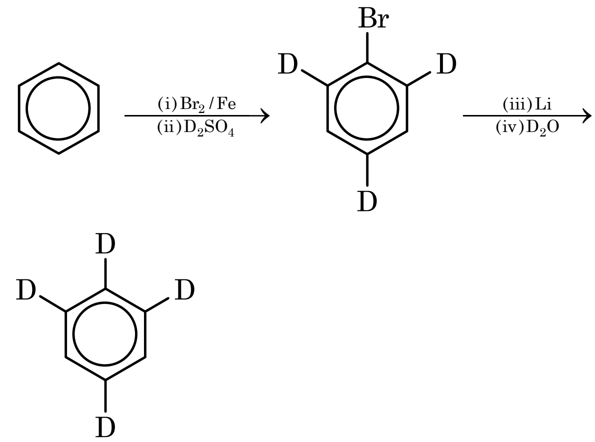
$= +40 + (P_2V_2 - P_1V_1)$bar–L

$= +40 + (200 - 150)$ bar L

$= 90$ bar–L $= 9000$ J

36. Ans. (3)

Sol.



37. Ans. (3)

Sol. Aldol condensation.

38. Ans. (2)

Sol. $2^2 = 4$.

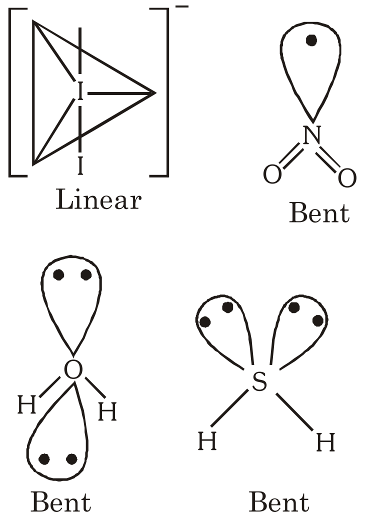
39. Ans. (2)

40. Ans. (3)

Sol. CH_3CH_2OH shows H–bonding.

41. Ans. (1)

Sol. I_3^- is correct because in a hybridisation of centre atom is sp^3d with 3 lone pair of electrons which are present at equatorial positions. The structure of these comppounds are as



42. Ans. (1)

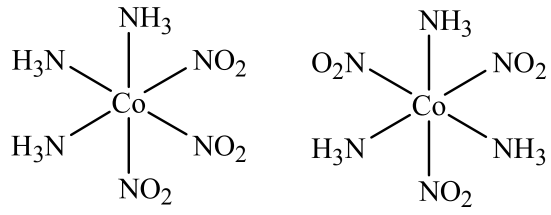
Sol. Fact based.

43. Ans. (3)

Sol. Fact based.

44. Ans. (2)

Sol.



Facial Meriodonal

(1), (3) and (4) can show cis–trans isomerism but not Fac–Mer isomerism.

45. Ans. (3)

Sol. $2AgNO_3 \xrightarrow[\text{Strongly}]{\Delta} 2Ag + 2NO_2 + O_2$

(1)is possible on heating at low temperature.

(2) is not correct because Ag_2O is unstable.

(4) is not formed.

46. Ans. (3)

Sol. $\Delta_r G^\circ = \Delta_f G^\circ(\text{Product}) - \Delta_f G^\circ(\text{reaction})$

$= -805 + 2(-228.6) - (52.3 + 0)$

$= -805 - 457.2 - 52.3 = -1314.5 \text{ kJmol}^{-1}$

47. Ans. (1)

Sol. $K_{sp} = [Ag^+][Cl^-] = 1.7 \times 10^{-10}$

Precipitation will start when I.P. exceeds K_{sp}

I.P. $= [Ag^+][Cl^-] = 1.7 \times 10^{-10}$

$$[Ag^+] = \frac{1.7 \times 10^{-10}}{0.001} = \frac{1.7 \times 10^{-10}}{10^{-4}} = 1.7 \times 10^{-6} \text{ mol L}^{-1}$$

48. Ans. (4)

Sol. ccp has 8 tetrahdral voids within the unit cells.

49. Ans. (1)

Sol. $\pi = \dfrac{n_B RT}{V}$; $n_B = \dfrac{w_B}{M_B}$

$$\pi = \frac{w_B}{M_B} \times \frac{RT}{V}$$

$$M_B = \frac{w_B}{V} \times \frac{RT}{\pi} = \frac{2 \times 0.08 \times 300 \times 760}{0.3 \times 20}$$

$= 6080$ gm mol^{-1}

50. Ans. (2)

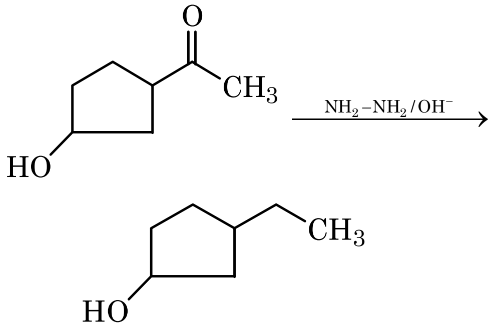
Sol. For a consecutive reaction

$$A \xrightarrow[1^{st}\text{ order}]{k_1} B \xrightarrow[1^{st}\text{ order}]{k_2} C$$

$$T_{[B]_{max}} = \frac{\ln \frac{k_1}{k_2}}{k_1 - k_2}$$

51. Ans. (2)

Sol.

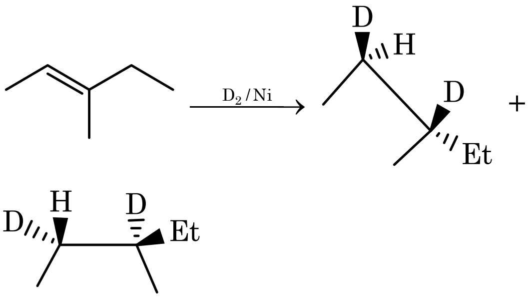


52. Ans. (3)

Sol. Fact based.

53. Ans. (4)

Sol.



54. Ans. (3)

Sol. DNA is responsible for transmission of heredity characteristics.

55. Ans. (4)

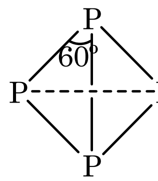
Sol. Glucose is used for silvering of mirrors.

56. Ans. (2)

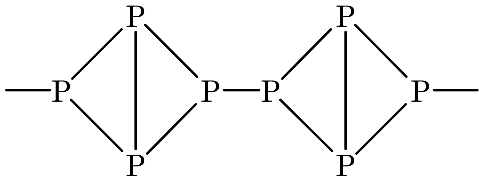
Sol. (1) is not correct because it is a correct statement. Nitrogen forms multiple bonds because two N atoms can come sufficiently close together while P atom being large in size cannot come close together and hence does not form P ≡ P.

(2) is correct because it is an incorrect statement as the P–P single bond is more stronger than N–N because in N–N, there is more repulsive force between the non–bonding electron pair.

(3) is not correct it is a correct statement as in white phosphorus, the structure

is highly strained  whereas

in red phosphorus, the structure is less stained



(4) is not correct because it is a correct statement as S—S single bond is a strong bond.

57. Ans. (4)

Sol. $l = 2$

Fe (26) = $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^6\ 4s^2$

Fe^{2+} (24) = $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^6$

$\ell = 2 \Rightarrow d$

Electron present in (ℓ = 2) d–orbitals = 6

58. Ans. (4)

Sol. (4) is correct because calcination is used to remove moisture, to remove organic matter and to convert carbonate ore to oxide.

59. Ans. (3)

Sol. N is smaller in size, therefore most electronegative, Si is largest in size, therefore less electronegative than phosphorus. P is less electronegative than carbon due to bigger size.

60. Ans. (2)

Sol. Photochemical smog is oxidising

61. Ans. (3)

Sol. $\sum_{r=1}^{9} \sin^2 \frac{r\pi}{10} = \frac{1}{2}\sum_{r=1}^{9}\left(1 - \cos\frac{2r\pi}{10}\right) = 5$

62. Ans. (1)

Sol. $\frac{\pi}{2} - \cos^{-1} x - \cos^{-1} x = \frac{\pi}{2} - \cos^{-1}(3x - 2)$

$2\cos^{-1}x = \cos^{-1}(3x - 2)$

$\Rightarrow 2x^2 - 1 = 3x - 2 \Rightarrow 2x^2 - 3x + 1 = 0$

$\Rightarrow x = 1, \frac{1}{2}$

63. Ans. (4)

Sol. Locus of mid point is $x^2 + y^2 - \frac{3}{5}x - \frac{4}{5}y = 0$

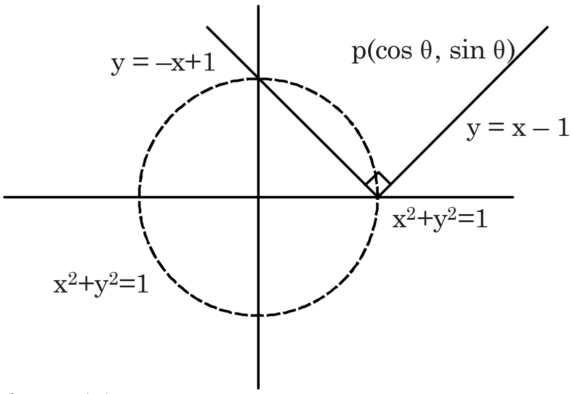
64. Ans. (3)

Sol. $a_1 = 2\cos\theta$, $b_1 = -2\cot\theta$
$a_2 = 4\sec\theta$, $b_2 = 4\tan\theta$

65. Ans. (1)

Sol. $P(\cos\theta, \sin\theta)$ lies on $x^2 + y^2 = 1$.

Clearly θ lies from $\left(0, \frac{\pi}{2}\right)$.



66. Ans. (4)

Sol. Two parabolas intersect at 4 points. Hence number of common chords are ${}^4C_2$.

67. Ans. (3)

Sol. $x + \frac{1}{x} = 2\cos\frac{\pi}{5}$

Let $x = e^{i.\frac{\pi}{5}}$

$\Rightarrow \quad x^{90} + \frac{1}{x^{90}} = 2\cos 90\frac{\pi}{5} = 2.$

68. Ans. (4)

Sol. $\sum_{k=1}^{\infty}\frac{k}{2^k}\sum_{n=1}^{\infty}\frac{1}{2^n} = \sum_{k=1}^{\infty}\frac{k}{2^k}$

69. Ans. (1)

Sol. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 5 & \lambda \end{vmatrix} \neq 0 \Rightarrow \lambda \neq 8$

70. Ans. (3)

Sol. Put x = 1 and – 1 and then add.

71. Ans. (3)

Sol. $|A| = 1$ and $A^{-1} = \frac{\text{adj}(A)}{|A|}$

72. Ans. (2)

Sol. Replace $x \to \frac{x+1}{3}$.

73. Ans. (2)

Sol. HHHH_ _ _ + THHHH _ _ + _ THHHH _ + _ _ THHHH

$\frac{1}{16} + \frac{1}{32} + \frac{1}{32} + \frac{1}{32} = \frac{5}{32}$

74. Ans. (1)

Sol. $g(x) = f(f(x) - 1) + f(5 - f(x))$
$= f((x-1)^2 - 2) + f(6 - (x-1)^2)$
$= 2(x-1)^4 - 16(x-1)^2 + 32$
$= 2((x-1)^2 - 4)^2 \geq 0, \ \forall x \in R$

75. Ans. (4)

Sol. $f'(x) = 3x^2 + 2bx + a$

Roots of $f'(x) = 0$ should be – 3 and 2.

76. Ans. (4)

Sol. limit = $e^{\lim_{x\to 0}\frac{1}{x}\log\left(\tan\left(\frac{\pi}{4}-x\right)\right)} = e^{-2}$.

77. Ans. (2)

Sol. $\Delta = 2\int_0^1\left(\sqrt{x} - x^2\right)dx$

78. Ans. (2)

Sol. $a^n + b^n = (a+b)^n$
$\therefore x^2 - 3 = 1 \Rightarrow x = \pm 2.$

79. Ans. (4)

Sol. $I = \int \frac{x^{-3}}{(4 + x^{-2})^6}dx$ put $4 + x^{-2} = t$

80. Ans. (1)

Sol.

$f(x) = x^3$	$x > 0$
$= 0$	$x = 0$
$= -x^3$	$x < 0$

81. Ans. (2)

Sol. $I = \int_{-3}^{3} x^8\{x^{11}\}dx$

$I = \int_{-3}^{3} x^8\{-x^{11}\}dx$

$2I = \int_{-3}^{3} x^8 dx = 2 \cdot 3^7$

82. Ans. (2)

Sol. $\frac{dx}{dy}=\frac{x+2y^3}{y}\Rightarrow\frac{dx}{dy}-\frac{1}{y}x=2y^2$

I.F. = $e^{\int\frac{-1}{y}dy}=\frac{1}{y}$

$\therefore\ \frac{1}{y}.x=\int\frac{1}{y}.2y^2dy=y^2+C$

83. Ans. (4)

Sol. ln f(x) = ln (x – 1) + ln (x – 2) ______ ln (x – n)

$f'(x)=f(x)\cdot\left[\frac{1}{x-1}+\frac{1}{x-2}+......\frac{1}{x-n}\right]$

f′(n) = (n – 1)(n – 2) 3.2.1

5040 = (n – 1)!

n = 8

84. Ans. (3)

Sol. $\overline{X}=\frac{\sum_{r=0}^{n}r.{}^nC_r}{\sum_{r=0}^{n}{}^nC_r}=\frac{n}{2}$

$\frac{1}{N}\sum f_ix_i^{\ 2}=\frac{1}{2^n}\sum_{r=0}^{n}r^2\,{}^nC_r=\frac{n(n+1)}{4}$

Var (x) = $\frac{1}{N}\sum f_ix_i^{\ 2}-\overline{X}^2=\frac{n}{4}$

85. Ans. (1)

86. Ans. (4)

Sol. $\frac{1}{a}+\frac{1}{d}=\frac{1}{b}+\frac{1}{c}$ Now by definition, R is Reflexive, Symmetric and Transitive.

87. Ans. (4)

Sol. $(p\to q)\equiv\sim p\vee q$

$\sim(\sim p\vee(q\wedge r))\ \equiv\ p\wedge(\sim q\vee\sim r))$

88. Ans. (1)

Sol. Let P.V. of P, A, B and C be $\vec{p},\vec{a},\vec{b}$ and $\vec{c}$.

and O is circumcentre of equilateral ΔABC.

$|\vec{b}|=|\vec{p}|=|\vec{a}|=|\vec{c}|=\frac{\ell}{\sqrt{3}}$

$\sum|\overline{PA}|^2=6\cdot\frac{\ell^2}{3}-2\vec{p}\cdot(\vec{a}+\vec{b}+\vec{c})=2\ell^2$

89. Ans. (2)

Sol. Point of intersection of lines is (4, 3, 5)

Equation of plane ⊥ to OP is

4x + 3y + 5z = 50

90. Ans. (1)

Sol. $\lim_{x\to0}\frac{(1-f(x))((f(x))^2+f(x)+1)}{5x^2}$

$=\frac{3}{5}\left(\frac{1-\cos2x.\cos4x.\cos6x.\cos8x}{x^2}\right)$

Now use expansion of

cos x = $1-\frac{x^2}{2!}+\frac{x^4}{4!}.....$