Paper Code : 1001CJA102118083

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

CLASSROOM CONTACT PROGRAMME

(Academic Session : 2018 - 2019)

JEE (Main + Advanced)

LEADER COURSE (SCORE-I) & ENTHUSIAST COURSE (SCORE-II)

ANSWER KEY **TEST DATE : 04-04-2019**

Test Type : FULL SYLLABUS Test Pattern : JEE-Main

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	**4**	**1**	**2**	**1**	**3**	**4**	**3**	**4**	**2**	**1**	**2**	**4**	**1**	**3**	**2**	**1**	**1**	**1**	**2**	**3**
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	**3**	**4**	**1**	**1**	**3**	**1**	**4**	**4**	**2**	**4**	**3**	**3**	**3**	**3**	**3**	**1**	**2**	**2**	**4**	**2**
Que.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	**4**	**4**	**2**	**2**	**3**	**4**	**4**	**3**	**2**	**3**	**2**	**3**	**4**	**4**	**2**	**4**	**3**	**4**	**4**	**1**
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	**4**	**1**	**4**	**1**	**4**	**3**	**2**	**1**	**4**	**1**	**4**	**2**	**3**	**1**	**3**	**1**	**2**	**3**	**3**	**2**
Que.	81	82	83	84	85	86	87	88	89	90										
Ans.	**3**	**2**	**2**	**4**	**2**	**1**	**3**	**3**	**4**	**3**										

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

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SOLUTION

1. Ans. (4)

Sol. p.d. across 3 ohm or 1 microfarad

$$= 9\left(\frac{3}{3+6}\right) = 3V$$

its charge = 3 micro coulomb

p.d. across 7 ohm or 3 microfarad

$$= 9\left(\frac{7}{7+2}\right) = 7V$$

Its charge = 21 μC

2. Ans. (1)

Sol. Around Ampere's loop of radius 3a

$$\oint \vec{B}.d\vec{l} = \mu_0 I_{enclosed}$$

$$B(2\pi(3a)) = \mu_0 I - \mu_0 I\left[\frac{\pi(3a)^2 - \pi(2a)^2}{\pi(4a)^2 - \pi(2a)^2}\right]$$

$$B = \frac{7\mu_0 I}{72\pi a}$$

3. Ans. (2)

Sol. $E = M\frac{di}{dt}$, M = mutual inductance

$$\frac{E}{50\times10^{-3}} = \frac{6/0.02}{8/0.5}$$

$E = 0.94V$

4. Ans. (1)

Sol. $\lambda = \frac{\ln 2}{12.7h} = 5.46\times10^{-2}h^{-1}$

$$\lambda = \lambda_{EC} + \lambda_{\beta^-} = \lambda_{EC} + \frac{39}{61}\lambda_{EC}$$

$$\lambda_{EC} = 3.329\times10^{-2}h^{-1}$$

$$t_{half\ EC} = \frac{\ln 2}{\lambda_{EC}} = 20.8h$$

$$\lambda_{\beta^-} = \frac{39}{61}\lambda_{EC} = 2.128\times10^{-2}h^{-1}$$

$$t_{half\ EC} = \frac{\ln 2}{\lambda_{\beta}} = 32.6h$$

5. Ans. (3)

Sol. $f = 25$ cm

$$m = \frac{-v}{u} = \frac{f}{f-u}$$

$$m = \frac{25}{25-(-25)} = 0.5$$

6. Ans. (4)

Sol. violet has $\mu = 1.8$

$$\frac{1}{f} = (\mu-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{v} - \frac{1}{u}$$

$$(1.8-1)\left(\frac{1}{40} - \frac{1}{-40}\right) = \frac{1}{v} - \frac{1}{-50}$$

v = 50 cm

7. Ans. (3)

Sol. $t = \sum \frac{l_i}{\sqrt{\frac{F}{\mu_i}}}$

$$t = \sum \frac{l_i}{\sqrt{F}}\sqrt{m_i/l_i}$$

$$t = \sum \frac{\sqrt{m_i l_i}}{\sqrt{F}}$$

$$t = \sqrt{\frac{ml}{F}}\left[1 + \frac{1}{2} + \frac{1}{4} + ...\right] = 2\sqrt{\frac{ml}{F}}$$

8. Ans. (4)

Sol. frequencies are f, f + 2, f + 4,......, f + 100(2)

f + 200 = 3f

f = 100 Hz

Tenth fork has frequency = f + 2(9) = 118Hz

9. Ans. (2)

Sol. optical path difference

$= (\mu_{glass} - 1)t_{glass} + (\mu_{water} - 1)t_{water} + (\mu_{glass} - 1)t_{glass}$

$= (1.5 - 1)(1) + (4/3 - 1)(21) + (1.5 - 1)(1)$

$= 8$cm

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10. Ans. (1)

Sol. force balance for liquid column above upper half of sphere

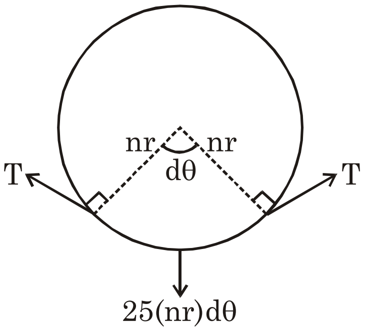
$P_0\pi r^2 + \rho V g = F$

$$P_0\pi r^2 + \rho\left[\pi r^2(3r) - \frac{2\pi r^3}{3}\right]g = F$$

$$F = \frac{\pi r^2}{3}\left[3P_0 + 7r\rho g\right]$$

11. Ans. (2)

Sol.



force balance of are element

$$2T\sin\left(\frac{d\theta}{2}\right) = 2Snrd\theta$$

T = 2Snr (tension in band)

T = k(elongation)

$2Snr = k[2\pi nr - 2\pi r]$

$$k = \frac{Sn}{\pi(n-1)}$$ (elastic force constant of band)

force balance of mass m

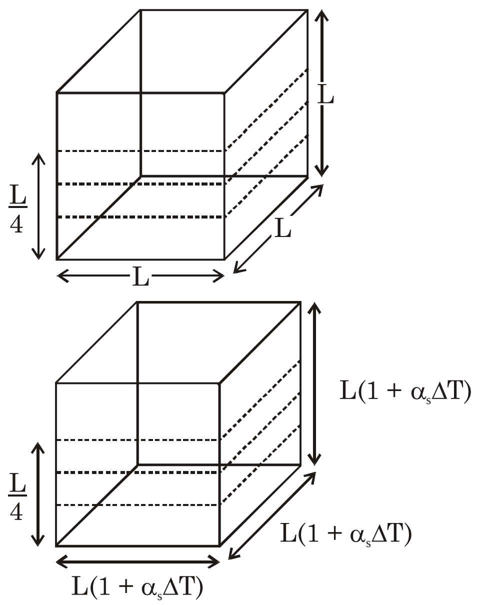
kx = mg

$$\frac{Sn}{\pi(n-1)}(2\pi r) = mg$$

$$S = \frac{mg(n-1)}{2nr}$$

12. Ans. (4)

Sol.



$$\frac{L^3}{4}(1+\gamma_l\Delta T) = \left[L(1+\alpha_s\Delta T)\right]^2\left[\frac{L}{4}\right]$$

$$(1+\gamma_l\Delta T) = \left[(1+2\alpha_s\Delta T)\right]$$

$\alpha_s = \gamma_l/2$

13. Ans. (1)

Sol. $$\text{least count} = \frac{\text{pitch}}{\text{number of CSD}} = \frac{0.01\text{m}}{2000}$$

$= 5\mu m$

14. Ans. (3)

Sol. torque by wire = magnetic torque

$k\delta = mB_h\sin\alpha$

k = torsion constant, δ = net twist

m = magnetic moment,

B_h = horizontal component of Earth's magnetic field

α = angle between magnetic moment and magnetic meridian

$$\frac{m_1}{m_2} = \frac{\delta_1}{\delta_2} = \frac{180-30}{270-30} = 5/8$$

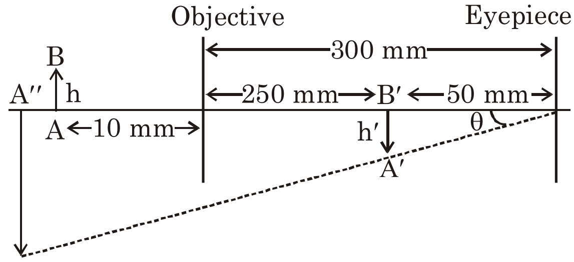
15. Ans. (2)

Sol. $$I = \frac{P}{4\pi r^2} = \frac{E_{rms}^2}{c\mu_0}$$

$$E_{rms} = \sqrt{\frac{250\times3\times10^8\times4\pi\times10^{-7}}{4\pi(0.9)^2}} = 96V/m$$

16. Ans. (1)

Sol.



$$\frac{h'}{h} = \frac{250}{10}$$

$h' = 25h$

$$\theta = \frac{h'}{50} = \frac{25h}{50} = \frac{h}{2}$$

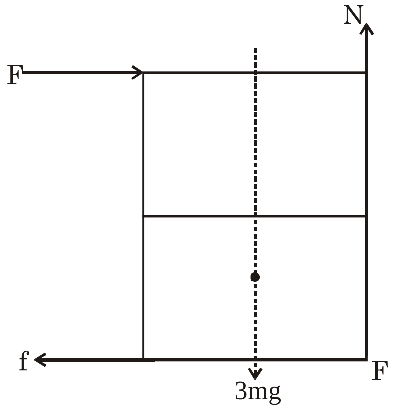
$$M = \frac{\theta}{\theta_0} = \frac{h/2}{h/250} = 125X$$

17. Ans. (1)

18. Ans. (1)

Sol. Equilibrium equations for toppling before sliding.

Normal reaction passes through F



$F = f$

$N = 3\ mg$

$\tau_F = 0$

$F(2h) = 3mg\left(\frac{h}{2}\right)$

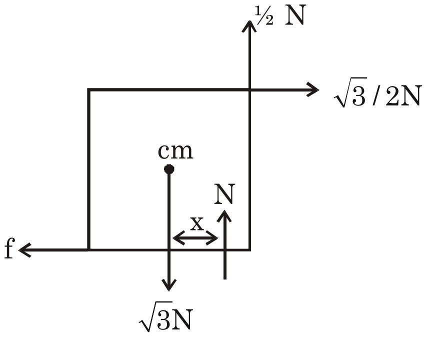
but $f \le \mu N$

$\frac{3mg}{4} \le \mu(3mg)$

$\mu \ge 0.25$

19. Ans. (2)

Sol.



$N = \sqrt{3} - 1/2$ ....(1)

$f = \sqrt{3}/2$ ....(2)

$\tau_{cm} = 0$

$f(10) + \frac{\sqrt{3}}{2} \times 10 = \frac{1}{2} \times 10 + Nx$

x = 10 cm

20. Ans. (3)

Sol. Flux through bottom circular end

$\frac{q}{4\pi\varepsilon_0}\left(2\pi(1-\cos 37^0)\right) = \frac{q}{10\varepsilon_0}$

Flux through curved surface is

$\frac{q}{2\varepsilon_0} - \frac{q}{10\varepsilon_0} = \frac{2q}{5\varepsilon_0}$

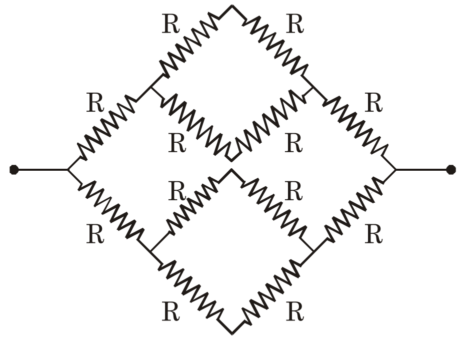
21. Ans. (3)

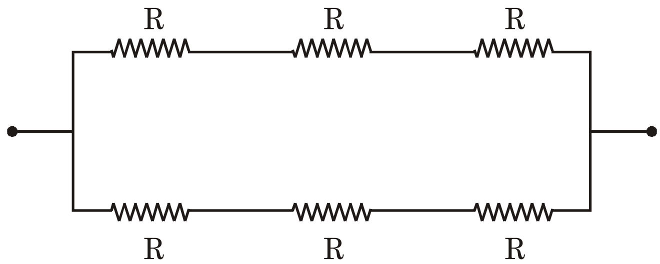
Sol. Net electric field should be normal to conductor's surface

Net electric field due to all the outside charges is zero inside a conducting shell

22. Ans. (4)

Sol.





For maximum power

$R_{eq} = r_{int}$

$\frac{3R}{2} = 3$

$R = 2\Omega$

23. Ans. (1)

Sol. $\text{energy} = \frac{kA^2}{2}$

$\frac{(k/4)A^2}{2} = \frac{k_{eq}A^2}{2}$

$\frac{1}{k_{eq}} = \frac{4}{k}$

$\frac{1}{k'} + \frac{1}{k} = \frac{4}{k}$

$k' = k/3$

24. Ans. (1)

Sol. $y_{cm} = \frac{\int_0^\pi \lambda_0 \sin\theta R d\theta R \sin\theta}{\int_0^\pi \lambda_0 \sin\theta R d\theta} = \frac{R\pi}{4}$

$x_{cm} = 0$ by symmetry.

25. Ans. (3)

Sol. $mg = B + F_V$

$mg = \frac{mg}{2} + F_V$

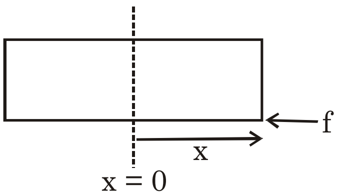
W.d. by mg = mgx

W.d. by $F_V = -\frac{mg}{2}$

ratio = $\frac{-1}{2}$

26. Ans. (1)

Sol.



At any instant frictional force is

$f = \frac{\mu mgx}{L}$

Work energy theorem

$FL - \int_0^L \frac{\mu mgx}{L}dx = \frac{1}{2}mv^2$

$v = \sqrt{\mu gL}$

27. Ans. (4)

Sol. kinetic energy of electron

$K = \frac{Qe}{4\pi\varepsilon_0 r}$

$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$

$\lambda = \frac{h}{\sqrt{2m\left(\frac{Qe}{4\pi\varepsilon_0 r}\right)}}$

$Q = 100\ \mu C$

28. Ans. (4)

Sol. upper lens

$\frac{1}{f_1} = (1.5-1)\left(\frac{2}{R}\right)$ [Lens makers formula]

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} - \frac{1}{-4R} = \frac{1}{R}$ => $v = 4R/3$

$\frac{1}{f_2} = (1.8-1)\left(\frac{2}{R}\right)$ [Lens makers formula]

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} - \frac{1}{-4R} = \frac{1.6}{R}$ => $v = 20R/27$

distance between images is

$\frac{4R}{3} - \frac{20}{27}R = \frac{16R}{27}$

29. Ans. (2)

Sol. initially

$n_h = n_c = 7 \times 10^{15}\ m^{-3}$

total charge carries = $14 \times 10^{15}\ m^{-3}$

after doping, donor impurity = $\frac{5\times10^{28}}{10^7} m^{-3}$

number of electrons = $\frac{5\times10^{28}}{2\times10^7} m^{-3}$

$n_c >> n_h$

total charge carries = n_c

$= 2.5 \times 10^{21}\ m^{-3}$

fractional change

$= \frac{2.5\times10^{21} - 14\times10^{15}}{14\times10^{15}} = 1.8\times10^5$

30. Ans. (4)

Sol. attenuation per km = $\frac{10\log_{10}\left(\frac{P_{in}}{P_{out}}\right)}{l}$

$= \frac{10\log_{10}\left(\frac{200}{10}\right)}{5} = 2.6 dB/km$

31. Ans. (3)

32. Ans. (3)

33. Ans. (3)

34. Ans. (3)

35. Ans. (3)

36. Ans. (1)

37. Ans. (2)

Sol. In $\dot{N}O_2$ & $\dot{C}lO_3$, valence e^- also form π-bond.

38. Ans. (2)

39. Ans. (4)

40. Ans. (2)

41. Ans. (4)

42. Ans. (4)

43. Ans. (2)

44. Ans. (2)

45. Ans. (3)

46. Ans. (4)

47. Ans. (4)

48. Ans. (3)

49. Ans. (2)

50. Ans. (3)

51. Ans. (2)

52. Ans. (3)

53. Ans. (4)

Sol : $H_2O_2 + O_3 \rightarrow H_2O + 2O_2$

54. Ans. (4)

55. Ans. (2)

56. Ans. (4)

57. Ans. (3)

58. Ans. (4)

59. Ans. (4)

Sol. $Ce^{+3} > La^{+3} \Rightarrow La^{+3} > Ce^{+4}$

$Ce^{+3} < La^{+3}$

$Ce^{+3} > Lu^{+3} \Rightarrow La^{+3} > Lu^{+3}$

$Eu^{+2} > Eu^{+3} > Yb^{+3}$

60. Ans. (1)

61. Ans. (4)

Sol. $-[x]^2 + 5[x] - 6 \geq 0$

$\Rightarrow [x]^2 - 5[x] + 6 \leq 0 \Rightarrow 2 \leq [x] \leq 3$

$\Rightarrow 2 \leq x < 4$

and $\sin x \geq 0$ when $x \in [2n\pi, (2n+1)\pi]$

$\therefore x \in [2, \pi]$

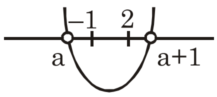
62. Ans. (1)

Sol. $(x - a)(x - (a + 1)) < 0 \quad \forall x \in (-1, 2)$

$\Rightarrow a \leq -1$ and $a + 1 \geq 2$

$\Rightarrow a \leq -1$ and $a \geq 1$

$\Rightarrow a \in \phi$



63. Ans. (4)

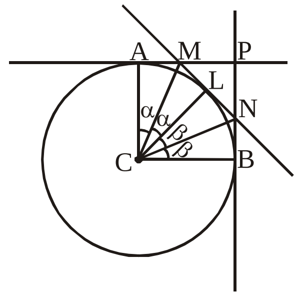
Sol. $P = 1 - \left[\frac{2}{10} \times \frac{8}{9} \times \frac{1}{8} + \frac{4}{10} \times \left[\frac{3}{9} \times \frac{2}{8} + \frac{6}{9} \times \frac{3}{8}\right] \times 2\right]$

$= 1 - \left(\frac{16 + 48 + 144}{10.9.8}\right) = 1 - \frac{208}{10.9.8}$

$= 1 - \frac{26}{10.9} = 1 - \frac{13}{45} = \frac{32}{45}$

64. Ans. (1)

Sol.



Since, $2\alpha + 2\beta = 90°$

$\Rightarrow \alpha + \beta = 45°$

$\Rightarrow \angle MCN = 45°$

65. Ans. (4)

Sol. $\vec{a} = \hat{i} + 2\hat{i} + (3 - \pi)\hat{i}$

and, $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$

So, $\vec{a} \cdot \vec{b} = 1 + 4 + 9 - 3\pi = 14 - 3\pi$

66. Ans. (3)

Sol. $\lim_{x \to 0} \frac{\frac{(2p-3)}{x} + \frac{\sin x}{x}}{2 - \frac{\tan x}{x}} = q$

$\therefore p = \frac{3}{2}$ and $q = \frac{1}{2-1} = 1$

So, $p + q = \frac{5}{2}$

67. Ans. (2)

Sol. $(1 + {}^7C_1x^4 + ...)(1 + x^2)^3(1 - x)^3$

$(1 + {}^7C_1x^4)(1 + x^6 + 3x^4 + 3x^2)(1 - x^3 + 3x^2 - 3x)$

$\therefore$ coefficient of x^5

$= -9 - 3 - 21 = -33$

68. Ans. (1)

Sol. Differentiating w.r.t x

$f(x) \cdot [\sin^2 x + \cos x] = 2 f(x) \cdot f'(x)$

$\therefore f'(x) = \frac{\sin^2 x + \cos x}{2}, f(x) \neq 0$

$\lim_{x \to 0} f'(x) = \frac{1}{2} = f'(0)$ (as $f'(x)$ is continuous)

Also, note that $f(0) = 0$

69. Ans. (4)

Sol. $\sim(\sim p \vee (\sim q \vee r))$

$= p \wedge (\sim(\sim q \vee r)) = p \wedge (q \wedge \sim r)$

70. Ans. (1)

Sol. $e^2 = 1+\sqrt{\sin^4\alpha+\cos^4\alpha}$

$= 1+\sqrt{1-2\sin^2\alpha\cdot\cos^2\alpha}$

$= 1+\sqrt{1-\frac{1}{2}\sin^2 2\alpha}$,

is maximum when $\sin 2\alpha = 0$

$\therefore b^2 = 1 \quad \therefore 2b = 2$

71. Ans. (4)

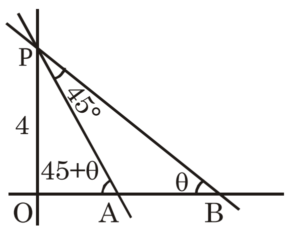
Sol. $\{1, 3\} \times \{2, 4\} \subseteq A \times B$

$\Rightarrow \{(1, 2), (1, 4), (3, 2), (3, 4)\} \subseteq \{(1, 3), (1, 4), (1, y), (2, 3), (2, 4), (2, y), (x, 3), (x, 4), (x, y)\}$

$\Rightarrow y = 2, x = 3$

72. Ans. (2)

Sol. $AB = 4\cot\theta - 4\cot(45^\circ + \theta)$



$= 4\left[\cot\theta-\frac{\cot\theta-1}{\cot\theta+1}\right]$

$= 4\left[(\cot\theta+1)-1-\left(\frac{(\cot\theta+1)-2}{\cot\theta+1}\right)\right]$

$\Rightarrow AB = 4\left[(\cot\theta+1)+\frac{2}{(\cot\theta+1)}-2\right]$

Now, $0<\theta<\frac{\pi}{2}$ and $0<\theta+\frac{\pi}{4}<\frac{\pi}{2}$

$\Rightarrow 0<\theta<\frac{\pi}{4}$

So, $AB = f(x) = 4\left[x+\frac{2}{x}-2\right]$,

$x = \cot\theta + 1 \in (2, \infty)$

$f'(x) = 4\left[1-\frac{2}{x^2}\right] > 0$

$\therefore$ f(x) is increasing function

$f(x) > f(2) = 4$

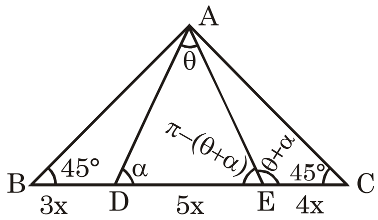
73. Ans. (3)

Sol. $\frac{x^2+2x^3+\frac{1}{x}+\frac{1}{x^4}}{4} \ge \sqrt[4]{x^2\cdot 2x^3\cdot\frac{1}{x}\cdot\frac{1}{x^4}} = \sqrt[4]{2}$

$\Rightarrow x^2+2x^3+\frac{1}{x}+\frac{1}{x^4} \ge 4\sqrt[4]{2} > 4$

74. Ans. (1)

Sol.



In ΔABE :

$(3 + 5)\cot\alpha = 5 + 3\cot(\theta + \alpha)$......(i)

In ΔADC :

$9\cot(\theta + \alpha) = 4\cot\alpha - 5$

$\Rightarrow 3[8\cot\alpha - 5] = 4\cot\alpha - 5$ (from (i))

$\Rightarrow \cot\alpha = \frac{1}{2}$

$\therefore \tan(\theta + \alpha) = -3$

$\frac{\tan\theta+2}{1-2\tan\theta} = -3$

$\Rightarrow \tan\theta + 2 = -3 + 6\tan\theta \Rightarrow \tan\theta = 1 \Rightarrow \theta = 45^\circ$

75. Ans. (3)

Sol. $\int_0^1 (1-2x)^{10}\,dx$

$= \int_0^1 [{}^{10}C_0 - {}^{10}C_1(2x) + {}^{10}C_2(2x)^2 - {}^{10}C_3(2x)^3$

$+\ldots\ldots + {}^{10}C_{10}(2x)^{10}]\,dx$

$\Rightarrow \left[\frac{(1-2x)^{11}}{-22}\right]_0^1 = {}^{10}C_0 - {}^{10}C_1 + \frac{{}^{10}C_2\cdot 2^2}{3} - \frac{{}^{10}C_3\cdot 2^3}{4}$

$+\ldots\ldots + \frac{{}^{10}C_{10}2^{10}}{11}$

76. Ans. (1)

Sol. $\begin{vmatrix} 2 & 1 & 0 \\ k & 2k & k-1 \\ k+1 & k & 1 \end{vmatrix} = 0$

$\Rightarrow 2(2k - k^2 + k) - 1\,(k - k^2 + 1) = 0$

$\Rightarrow 4k - 2k^2 + 2k - k + k^2 - 1 = 0$

$\Rightarrow -k^2 + 5k - 1 = 0$

$\Rightarrow k^2 - 5k + 1 = 0$

$\Rightarrow k = \frac{5\pm\sqrt{25-4}}{2} = \frac{5\pm\sqrt{21}}{2}$

77. Ans. (2)

Sol. Let x_i be the actual (correct) measurement

So, given that $\frac{\sum(x_i+3)}{10} = 20$

and $\frac{\sum(x_i+3)^2}{10} - (20)^2 = 15$

Hence, $\frac{\sum x_i}{10} = 17$

and, $\frac{\sum x_i^2}{10} - (17)^2 = 15$

(note that variance is independent of shifting of origin)

78. Ans. (3)

Sol. $\frac{a+1}{-2a-3} = \frac{2}{a} \neq \frac{a+2}{a^2-1}$

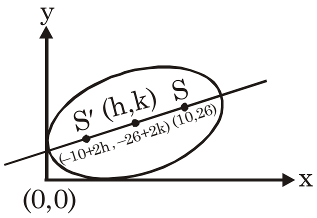
$\Rightarrow a^2 + a = -4a - 6$

$\Rightarrow a^2 + 5a + 6 = 0 \Rightarrow (a+2)(a+3) = 0$

$\Rightarrow a = -2$ or -3

79. Ans. (3)

Sol.



$(-26 + 2k)(26) = (-10 + 2h)(10)$

$\Rightarrow (k - 13)13 = (h - 5)5$

$\Rightarrow 5h - 13k + 169 - 25 = 0$

$\Rightarrow 5x - 13y + 144 = 0$

80. Ans. (2)

Sol. $\frac{dy}{dx} = xy^2 + x - y^2 \sin x - \sin x$

$\Rightarrow \frac{dy}{dx} = (x - \sin x)(y^2 + 1)$

Hence, $\int \frac{dy}{y^2+1} = \int (x - \sin x)dx$

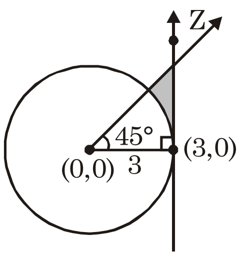
$\Rightarrow \tan^{-1} y = \frac{x^2}{2} + \cos x + C$

Since, $y(0) = 0 \Rightarrow c = -1$

So, $y = \tan\left(\frac{x^2}{2} + \cos x - 1\right)$

81. Ans. (3)

Sol.



shaded area $= \frac{1}{2} \times 3 \times 3 - \frac{\pi \times 3^2}{8} = \frac{36 - 9\pi}{8}$

82. Ans. (2)

Sol. $2\sin^{-1}\sqrt{x} = \cos^{-1} x^2$

$\Rightarrow \cos(2\sin^{-1}\sqrt{x}) = \cos(\cos^{-1} x^2)$

$\Rightarrow 1 - 2(\sqrt{x})^2 = x^2 \quad (0 \le x \le 1)$

$\Rightarrow 1 - 2x = x^2$

$\Rightarrow x^2 + 2x - 1 = 0$

$\Rightarrow x = \frac{-2 \pm \sqrt{8}}{2} = -1 \pm \sqrt{2}$

$\therefore x = \sqrt{2} - 1$

83. Ans. (2)

Sol. $f(0) = \lim_{x \to 0} f(x) = 0$

Also, $f'(0) = \lim_{h \to 0} \frac{f(h) - f(0)}{h} = 0$

84. Ans. (4)

Sol. $|M| = -2ab - 1(-ab - 2ab)$

$= -2ab + ab + 2ab = ab$

So, $\det(\text{Adj.}M) = |M|^2 = (ab)^2$

$\therefore$ Its maximum value is $(2 \cdot 2)^2 = 16$

85. Ans. (2)

Sol. $x_1 + x_2 + x_3 + x_4 + x_5 = 7$

So, required number of ways

$= {}^{7+5-1}C_{5-1} = {}^{11}C_4 = {}^{11}C_7$

86. Ans. (1)

Sol. $\cos x \cdot \sin x \cdot \cos 2x = \frac{\sqrt{2}}{8} = \frac{1}{4\sqrt{2}}$

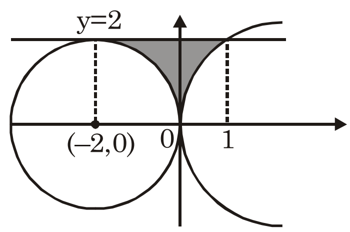
$\sin 2x \cdot \cos 2x = \frac{1}{2\sqrt{2}} \Rightarrow \sin 4x = \frac{1}{\sqrt{2}}$

$\therefore 4x = n\pi + (-1)^n \frac{\pi}{4}$

$\Rightarrow x = \frac{n\pi}{4} + (-1)^n \frac{\pi}{16}, n \in I$

87. Ans. (3)

Sol.



$y = 2$ and $y^2 = 4x$ meet at $(1, 2)$

$A = \left(4 - \frac{\pi(2)^2}{4}\right) + \int_0^2 \frac{y^2}{4} dy$

$= 4 - \pi + \frac{1}{4.3} \cdot 2^3 = 4 - \pi + \frac{2}{3} = \frac{14}{3} - \pi$

88. Ans. (3)

Sol. Let the common tangent be $y = mx + \frac{12}{m}$

So, $\left(\frac{12}{m}\right)^2 = 72(1 + m^2)$

$\Rightarrow m^4 + m^2 - 2 = 0$

$\Rightarrow m^2 = 1 \Rightarrow \frac{12}{m} = \pm 12$

89. Ans. (4)

Sol. $\sin A = \frac{3}{5} \Rightarrow \cos A = \pm\frac{4}{5}$

Also, $\cos B = \frac{5}{13} \Rightarrow \sin B = \frac{12}{13}$

Now, $\sin C = \sin(A + B)$

$= \sin A\cos B + \cos A\sin B$

$= \frac{3}{5}\cdot\frac{5}{13} \pm \frac{4}{5}\cdot\frac{12}{13}$

$= \frac{63}{65}$ $\left(-\frac{23}{65}\right.$ is not possible)

$\Rightarrow \cos A = \frac{4}{5}$

$\therefore$ $\cos C = -[\cos A\cos B - \sin A\sin B]$

$= -\frac{4}{5}\cdot\frac{5}{13} + \frac{3}{5}\cdot\frac{12}{13} = \frac{16}{65}$

90. Ans. (3)

Sol. $f'(x) = 4\cos x - 4\cos x\sin x$

$= 4\cos x\,(1 - \sin x)$

$\therefore$ It is non-monotonic in $\left(\frac{\pi}{2}, \frac{5\pi}{3}\right)$