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ALLEN CAREER INSTITUTE
KOTA (RAJASTHAN)
path to success

CLASSROOM CONTACT PROGRAMME

(Academic Session : 2018 - 2019)

JEE (Main + Advanced) : ENTHUSIAST COURSE : SCORE-II

ANSWER KEY **TEST DATE : 06-03-2019**

Test Type : FULL SYLLABUS Test Pattern : JEE-Main

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	1	3	3	2	2	3	2	2	4	2	1	1	3	1	2	2	1	4	4	2
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	3	2	3	2	3	2	4	3	1	1	2	2	3	3	2	1	1	3	1	1
Que.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	3	3	4	3	4	3	1	3	1	1	3	1	4	3	2	2	4	3	2	3
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	4	3	3	1	3	2	1	2	3	2	3	2	1	3	3	3	4	3	4	2
Que.	81	82	83	84	85	86	87	88	89	90										
Ans.	4	2	4	2	4	3	4	4	2	4										

JEE (Main + Advanced) : ENTHUSIAST COURSE
SCORE : II

Test Type : FULL SYLLABUS   Test Pattern : JEE-Main

TEST DATE : 06 - 03 - 2019

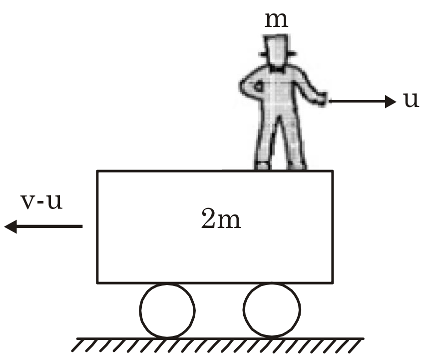
SOLUTION

1. Ans. (1)

Sol. Let the velocity of man after jumping be 'u' towards right. Then speed of cart is (v-u) towards left. From conservation of momentum mu = 2m(v – u)

$$\therefore u = \frac{2v}{3}$$

hence workdone by man = change in K.E. of system



$$= \frac{1}{2}mu^2 + \frac{1}{2}2m(v-u)^2$$

$$= \frac{1}{2}m\left(\frac{2v}{3}\right)^2 + \frac{1}{2}2m\left(\frac{v}{3}\right)^2$$

$$= \frac{mv^2}{3} = \frac{60 \times 2^2}{3} = 80J$$

2. Ans. (3)

Sol. $I = Mv_{cm}$

Angular impulse equation about point of contact

$$I(h+r) = \frac{7}{5}Mr^2\omega$$

$v_{cm} = \omega r$

$$\frac{h}{r} = \frac{2}{5}$$

3. Ans. (3)

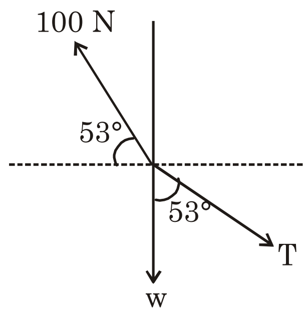
Sol. $$< \vec{V} > = \frac{\vec{V}_1 + \vec{V}_2}{2}$$

$$= \frac{(10\cos 37^\circ \hat{i} + 10\sin 37^\circ \hat{j} + 10\cos 37^\circ \hat{i})}{2}$$

$\Rightarrow \; < \vec{V} > = 8\hat{i} + 3\hat{j}$

4. Ans. (2)

Sol.



$T \sin 53° – 100 \cos 53° = 0 \;\; \Rightarrow T = 75$ N

Also $100 \sin 53° – T \cos 53° – W = 0$

$\Rightarrow$ W = 35 N

5. Ans. (2)

Sol. $Fx - \frac{1}{2}kx^2 - \mu_1 m_1 gx = 0$

$Kx = \mu_2 \, m_2 g$

6. Ans. (3)

Sol. For C, $\frac{d^2U}{dx^2} < 0$

7. Ans. (2)

Sol. For equilibrium $T_1 . \frac{\ell}{2} = T_2 . \frac{\ell}{4}$

$$\frac{T_1}{T_2} = \frac{1}{2}$$

8. Ans. (2)

Sol. $v = \sqrt{Rg\tan\theta} \Rightarrow \tan\theta = \frac{v^2}{Rg} = \frac{(20)^2}{10.10}$

$\tan\theta = 4$

$\theta = \tan^{-1} 4$

9. Ans. (4)

Sol. We can write the given force as

$$F = -k\left(y - \frac{C}{k}\right)$$

Let us define $s = y - \frac{C}{k}$ (1)

where s is the displacement from mean position because at $y = C/k$, F is zero.

Thus, the mean position is at y = C/k.

Using Eq. 1, force can be written as

$F = -ks$ (2)

which is the equation of SHM about y = C/k.

Differentiating Eq. 1 with respect to time twice,

we get

$$\frac{d^2y}{dt^2} = \frac{d^2s}{dt^2}$$

Thus, force $F = m\left(d^2y/dt^2\right)$ can also be written as $F = m\left(d^2s/dt^2\right)$.

Substituting in Eq. 2

$$m\frac{d^2s}{dt^2} = -ks$$

Comparing with standard equation we know that the solution of the above SHM equation is

$$s = y_m \sin\left(\omega t + \phi\right)$$

where $\omega = \sqrt{\frac{k}{m}}$

Replacing with value of s from Eq. 1, we get

$$y = y_m \sin\left(\omega t + \phi\right) + \frac{C}{k}$$

therefore : this particle is undergoing SHM about y = C/k. Since no information is given about initial position phase ϕ and amplitude y_m as unknown.

(b) If in part (a) F = –10y + 20, and mass m of the particle is 2.5 kg and it is released from rest from y = +3, at t = 0, write the equation of SHM.

10. Ans. (2)

Sol. In same direction and of same frequency

$x_1 = A_1 \sin \omega t$

$x_2 = A_2 \sin (\omega t + \theta)$, then resultant displacement

$x = x_1 + x_2 = A_1 \sin \omega t + A_2 \sin (\omega t + \theta) = A \sin (\omega t + \phi)$

where $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\theta}$ &

$$\phi = \tan^{-1}\left[\frac{A_2\sin\theta}{A_1 + A_2\cos\theta}\right]$$

If $\theta = 0$, both SHMs are in phase and $A = A_1 + A_2$

If $\theta = \pi$, both SHMs are out of phase and $A = |A_1 - A_2|$

The resultant amplitude due to superposition of two or more than two SHMs of this case can also be found by phasor diagram also.

$x_1 = A_1 \sin \omega_1 t$

$x_2 = A_2 \sin \omega_2 t$

then resultant displacement

$x = x_1 + x_2 = A_1 \sin \omega_1 t + A_2 \sin \omega_2 t$

This resultant motion is not SHM.

$x = A \sin \omega t$

$y = B \sin (\omega t + \theta)$

Case (i) : If $\theta = 0$ or π then $y = \pm (B/A)\, x$. So path will be straight line & resultant displacement will be $r = (x^2 + y^2)^{½} = (A^2 + B^2)^{½} \sin\omega t$

which is equation of SHM having amplitude $\sqrt{A^2 + B^2}$

Case (ii) : If $\theta = \frac{\pi}{2}$ then $x = A \sin \omega t$

$y = B \sin (\omega t + \pi/2) = B \cos \omega t$

so, resultant will be $\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$. i.e. equation of an ellipse and if A = B, then superposition will be an equation of circle.

11. Ans. (1)

Sol. Since soap film on A-side is broken

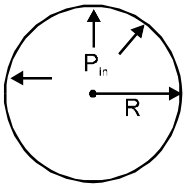
$\Rightarrow$ Soap film from B-side will pull the thread. Thus thread will be concave towards A.

12. Ans. (1)

Sol. we have initially $P_{in} = \frac{4T}{R}$ ....(1)

Now when extra charge is given, for equilibrium we have

$$P'_{in} + \frac{\sigma^2}{2\varepsilon_0} = \left(\frac{4T}{2R}\right) \quad ....(1)$$



Now we have $P'_{in} = \frac{(P_{in})v_i}{v_f}$

[isothermal change]

where v_i - initial volume of bubble
v_f - final volume of bubble

so, $P'_{in} = \frac{(P_{in})}{8}$

so, (1) equation will give

$$\frac{4T}{8R} + \frac{\sigma^2}{2\varepsilon_0} = \frac{4T}{2R} \Rightarrow \frac{\sigma^2}{2\varepsilon_0} = \frac{4T}{2R}\left(1 - \frac{1}{4}\right)$$

$$= \frac{12T}{8R}$$

so, $\sigma^2 = \frac{12T\varepsilon_0}{4R} = \left(\frac{3T\varepsilon_0}{R}\right)$ so, $\sigma = \sqrt{\frac{3T\varepsilon_0}{R}}$

now charge $Q = \sigma \, . \, 4\pi\,(2R)^2 = 4\pi.(4R^2)$

$$\sqrt{\frac{3T\varepsilon_0}{R}} = \sqrt{256 \times 3T\varepsilon_0 R^2 \pi^2}$$

$$Q = \sqrt{768\,\pi^2 R^3 T\varepsilon_0}$$

13. Ans. (3)

Sol. $(x-1)^2 + (y-2)^2 + (z-0)^2 = r^2$

$$\Rightarrow x^2 + y^2 + z^2 - 3x - 4y + 5 = r^2$$

14. Ans. (1)

Sol. $\frac{E_{out}}{E_{in}} = \frac{kq/r_{out}^2}{kq_{rin}/R^3} = \frac{R^3}{r_{in} \times r_{out}^2} = \frac{R^3}{\frac{R}{2} \times \frac{9R^2}{4}}$

$$\frac{E_{out}}{90} = \frac{8}{9}$$

$$\therefore E_{out} = \frac{8}{9} \times 90 = 80\,N/c$$

15. Ans. (2)

Sol. Number for yellow is 4, Number of violet is 7

Brown colour gives multiplier 10^1, Gold gives a tolerance of ± 5%

So resistance of resistor is $47 \times 10^1\ \Omega \pm 5\%$ or $470 \pm 5\%\ \Omega$.

16. Ans. (2)

Sol. $B_m = \mu_0 M$

$M = \frac{B_m}{\mu_0}$ = magnetization per unit volume

and volume of one atom

$$= \frac{\text{molecular weight}}{\rho \times N_A}$$

so, magnetization per atom = M × volume of one atom $= \frac{B_m}{\mu_0} \times \frac{M.W.}{\rho N_A}$

$$= \frac{1}{4\pi \times 10^{-7}} \times \frac{56 \times 10^{-6}}{7.8 \times 6.023 \times 10^{23}}$$

$= 9.5 \times 10^{-24}\ Am^2$

17. Ans. (1)

Sol. $B_{coil} = B_H \tan\theta$

$$\frac{4\pi \times 10^{-7} \times 20 \times i}{2 \times 20 \times 10^{-2}} = 0.34 \times 10^{-4}(\tan 45°)$$

$$i = \frac{3.4}{2\pi} = 0.6A$$

18. Ans. (4)

Sol. Initial energy stored in capacitor 2 μF

$$U_i = \frac{1}{2}2(V)^2 = V^2$$

Final voltage after switch 2 is ON

$$V_f = \frac{C_1V_1}{C_1 + C_2} = \frac{2V}{10} = 0.2V$$

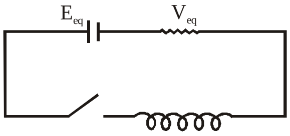
Final energy in both the capacitors

$$U_f = \frac{1}{2}(C_1 + C_2)V_f^2 = \frac{1}{2}10\left(\frac{2V}{10}\right)^2 = 0.2V^2$$

So energy dissipated

$$= \frac{V^2 - 0.2V^2}{V^2} \times 100 = 80\%$$

19. Ans. (4)

Sol.


$$E_{eq}.\ \frac{\frac{4}{3}+\frac{-8}{6}}{\frac{1}{3}+\frac{1}{6}}=\frac{\frac{8}{3}}{\frac{1}{2}}=\frac{8}{3}\times\frac{2}{1}=\frac{16}{3}volt$$

$$R_{eq}.=\frac{3\times6}{9}=2\Omega$$

$$I_{max}=\frac{V}{R}=\frac{16}{3\times2}=\frac{8}{3}A$$

$$I=\frac{8}{3}\left(1-e^{-1000t}\right)$$

20. Ans. (2)

Sol. $z=\sqrt{R^2+x_L^2}$

$$z=\sqrt{(100)^2+(100)^2}$$

$$=\sqrt{2}(100)$$

$$\cos\phi=\frac{R}{2}=\frac{100}{100\sqrt{2}}=\frac{1}{\sqrt{2}}$$

21. Ans. (3)

Sol. $f=mg$

$$=20\times10^{-3}\times10$$

$$=10^{-1}\times2$$

$$=2\times10^{-1}\,N$$

$$F_{photon}=fg$$

$$\frac{P}{C}=2\times10^{-1}$$

$$P=2\times10^{-1}\times3\times10^{8}$$

$$P=6\times10^{7}W$$

22. Ans. (2)

Sol. Equation of demped oscillation

$$x=\left[Ae^{-b/2mt}\right]\sin\omega t$$

$$I=\left[I_0e^{-\frac{R}{2L}t}\right]\sin\omega t$$

$$I_0e^{-(R/2L)t}=\frac{I_0}{e}\qquad\text{For maximum}$$

$$\frac{R}{2L}t=1\qquad\phi=\omega t$$

$$t=\frac{2L}{R}\qquad\phi=\frac{2L\omega}{R}\left(X_L=\omega L\right)$$

$$(2\pi)n=\frac{2\times200}{20}=n=\frac{10}{\pi}$$

23. Ans. (3)

24. Ans. (2)

Sol. $m=\frac{D}{f}$

25. Ans. (3)

Sol. $2\mu t\cos r=\frac{\lambda}{2}$ (For I_{max} for first time)

$$\Rightarrow\quad t\propto\lambda$$

$$\Rightarrow\quad\frac{t}{t+0.05}=\frac{4000}{6000}=\frac{2}{3}$$

$$\Rightarrow\quad t=0.1\,\mu m$$

26. Ans. (2)

Sol. $A=A_0\left(\frac{\sin\beta}{\beta}\right)$

or $\frac{A}{A_0}=\frac{\sin\beta}{\beta}$

Now $\beta=\frac{\pi a\sin\theta}{\lambda}=\frac{\pi}{2}$

$$\therefore\quad\frac{A}{A_0}=\frac{\sin\left(\pi/2\right)}{\left(\pi/2\right)}=\frac{2}{\pi}$$

27. Ans. (4)

Sol. $\frac{\Delta d}{d}\times100=\frac{0.003}{0.3}\times100+2\times\frac{0.005}{0.5}\times100+\frac{0.06}{6}\times100$

= 4%

28. Ans. (3)

Sol. Power of side band $=\frac{\mu^2}{2}\times P_c$

$P_c\rightarrow$ power of carrier wave.

29. Ans. (1)

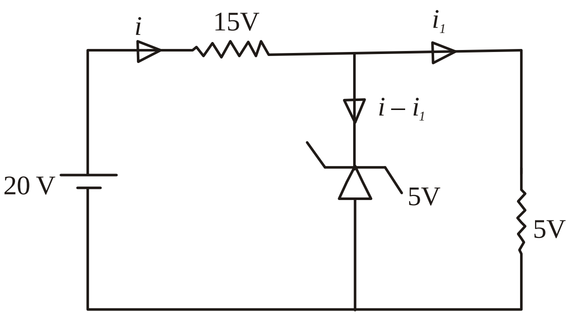
Sol. $\frac{1}{\alpha}=\frac{1}{\beta}+1$ = β=99

$\beta=\frac{I_c}{I_b}=99 \Rightarrow I_c = 990\mu A$

$I_E = I_c + I_b$ = 1000 μA = 1 mA

30. Ans. (1)

Sol.



$i-i_1=\frac{15}{1000}-\frac{5}{2000}=12.5mA$

31. Ans. (2)

meq of $K_2C_2O_4.3H_2C_2O_4.4H_2O=$ meq of $KMnO_4$

$0.1\times V_1 = 20\times 0.05\times 5$

$V_1 = 50mL$

meq of $K_2C_2O_4.3H_2C_2O_4.4H_2O=$ = meq of NaOH

$6\times\frac{0.1}{8}\times 50=\frac{1}{8}\times 1\times V_2$

$V_2 = 30\,mL$

[n-factor of $K_2C_2O_4.3H_2C_2O_4.4H_2O$ with $KMnO_4$ is 8 and with NaOH is 6]

32. Ans. (2)

∵ N_2 remain inert in proces, final volume must be of N_2 only

$V_{N_2} = 360\,mL$

$V_{O_2(initial)} = 450-360$

$V_{O_2(initial)} = 90\,mL$

$V_{O_2\,consumed} = 54\,mL$

$V_{CO_2} = 36\,mL \quad V_{H_2o} = 36$

$C_xHy + O_2 \rightarrow CO_2 + H_2O$
12 36 36

formula = C_3H_6

33. Ans. (3)

$Fe_2(C_2O_4)_3 \longrightarrow 2Fe^{+++} + 6CO_2\,[n.factor = 6]$

$Br_2 \longrightarrow Br^- + BrO_3^-\,[n-factor = 5/3]$

$SnCl_2 + HCl_2 + O_3 \longrightarrow \underset{+SnCl_4}{H_2O}\left[\underset{of\ O_3}{n-factors} = 6\right]$

$S_2O_3^{--} \longrightarrow SO_4^{--}\,(n-factors = 8)$

34. Ans. (3)

for ideal gas $d=\frac{PM}{RT}$

$\frac{d}{p}=\frac{M}{RT}$

as per question at T = 300 K $\frac{d}{p}=1$

$1=\frac{M}{0.08\times 300}$

M = 24 gm/mole

35. Ans. (2)

$pK_a = \frac{3.0+4.4}{2}=3.7$

∴ $K_{In} = 2\times 10^{-4}$

36. Ans. (1)

Balmer series $n_1 = 1$ α line $n_2 = 3$
β line $n_2 = 4$

$\frac{1}{\lambda_a}=R\left[\frac{1}{2^2}-\frac{1}{3^2}\right]=\frac{5R}{36}$

$\lambda_\alpha = \frac{36}{5R}$

$\frac{1}{\lambda_\beta} = R\times 2^2\left[\frac{1}{2^2}-\frac{1}{4^2}\right]=\frac{3R}{16}$

$\lambda_\beta = \frac{16}{3R}$

$\frac{\lambda_\alpha}{\lambda_\beta}=\frac{36/5R}{16/3R}=\frac{27}{20}$

37. Ans. (1)

$$C_3H_8^{(g)} + 5O_2^{(g)} \rightarrow 3CO_2^{(g)} + 4H_2O\ (l)$$

$\Delta ng = 3 - (1 + 5) = -3$

$$\Delta H = \Delta U + \Delta n_g RT$$

$$\Delta H = -2201.1 + \frac{-3 \times 8.314 \times 298}{1000}$$

$$\Delta H = -2201.1 - 7.432$$

$$\Delta H = -2208.53\ KJ$$

38. Ans. (3)

CH_3COOH is a weak acid hence it will absorb some amount of energy as conisation enthalpy hence heat energy will decome.

39. Ans. (1)

A →	B + C	
100 – x	x	x

100 + x = 140

x = 40

A = 60 mm

$$K = \frac{2.303}{30} \log_{10} \frac{100}{60}$$

$$K = \frac{2.303}{30} \times (1 - 0.8)$$

$$K = \frac{2.303 \times 0.2}{30}$$

$$T_{1/2} = \frac{2.303 \times 0.3}{2.303 \times 0.2} \times 30 = 45 \min$$

40. Ans. (1)

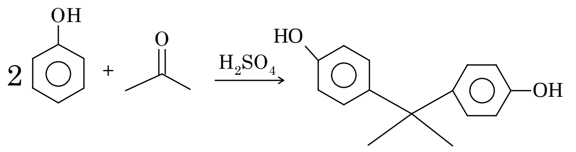
$$E = E^o - \frac{0.059}{1} \log_{10} \frac{[Fe^{+3}]}{[Fe^{+2}][Ag^+]}$$

$$0 = (0.8 - 0.77) - \frac{0.059}{1} \log \frac{1}{[Ag^+]}$$

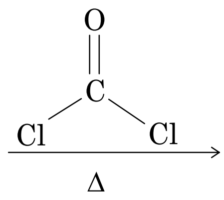
$$\log[Ag^+] = \frac{0.03}{0.059} \approx -\frac{1}{2}$$

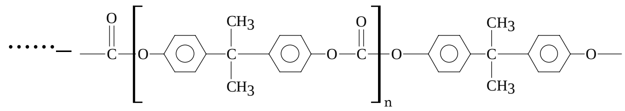
$[Ag^+]$ = antilog (– 0.5)

41. Ans. (3)



Bisphenol A

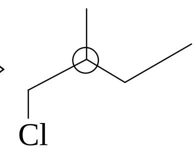




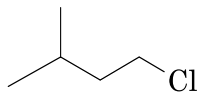
Lexan

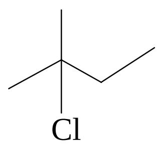
42. Ans. (3)

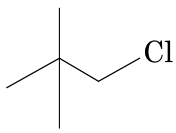
R–Cl ⇒

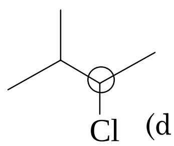


(d + l)



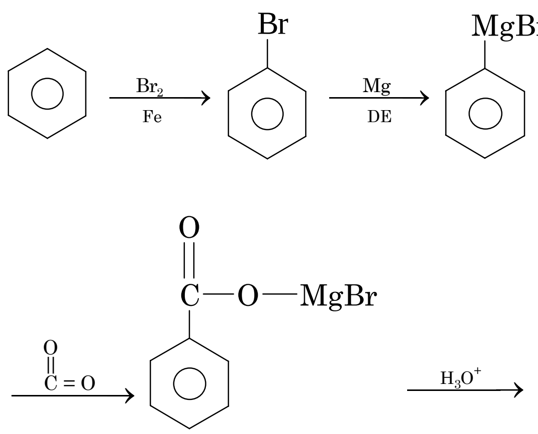


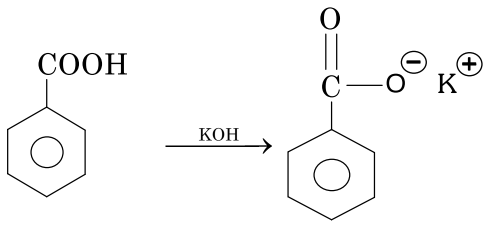


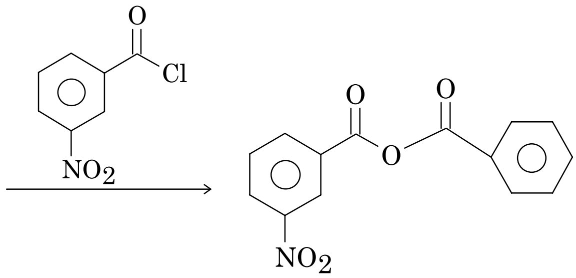


(d + l)

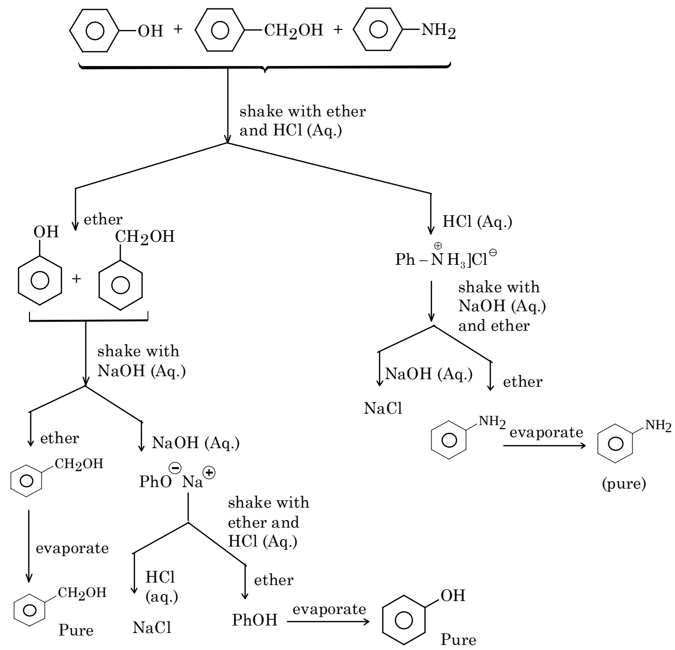
43. Ans. (4)







44. Ans. (3)



45. Ans. (4)
46. Ans. (3)
47. Ans. (1)
48. Ans. (3)
49. Ans. (1)
50. Ans. (1)
51. Ans.(3)
52. Ans.(1)
53. Ans. (4)
54. Ans. (3)
55. Ans.(2)
56. Ans.(2)
57. Ans.(4)
58. Ans.(3)
59. Ans. (2)

60. Ans. (3)

Calgon process involves the formation of a soluble complex.

61. Ans. (4)

Sol. S.D. = $\sqrt{\frac{\sum(x_j-8)^2}{18}-\left(\frac{\sum(x_j-8)}{18}\right)^2}$

$=\sqrt{\frac{45}{18}-\frac{1}{4}}=\sqrt{\frac{90-9}{36}}=\sqrt{\frac{81}{36}}=\frac{9}{6}=\frac{3}{2}$

62. Ans. (3)

Sol. $\vec a.\vec b=\vec a.\vec c=0,\vec b.\vec c=|\vec b||\vec c|\cos\frac{2\pi}{3}=3\times4\times\frac{-1}{2}=-6$

So, $\left[\vec a\,\vec b\,\vec c\right]^2=\begin{vmatrix}\vec a.\vec a. & \vec a.\vec b & \vec a.\vec c\\ \vec b.\vec a & \vec b.\vec b & \vec b.\vec c\\ \vec c.\vec a & \vec c.\vec b & \vec c.\vec c\end{vmatrix}=\begin{vmatrix}4 & 0 & 0\\ 0 & 9 & -6\\ 0 & -6 & 16\end{vmatrix}$

$=4(144-36)=4\times108=432$

$\Rightarrow\left[\vec a\,\vec b\,\vec c\right]=2\times2\times3\sqrt{3}=12\sqrt{3}$

63. Ans. (3)

Sol.

p	q	$\sim p\vee q$	$\sim p \wedge \sim q$	$(\sim p\vee q)\wedge(\sim p\wedge\sim q)$
T	T	T	F	F
T	F	F	F	F
F	T	T	F	F
F	F	T	T	T

$\because$ neither tautology nor contradiction.

64. Ans. (1)

Sol. Any point on the line is $(3\lambda+2, 4\lambda-1, 12\lambda+2)$

Given, $x-y+z=5$

$\Rightarrow 3\lambda+2-4\lambda+1+12\lambda+2=5$

$\Rightarrow 11\lambda=0 \Rightarrow \lambda=0$

So, $P(2,-1,2)$ *and* $Q(-1,\ -5,-10)$

Hence, Distance

$=\sqrt{(2+1)^2+(-1+5)^2+(2+10)^2}$

$=\sqrt{9+16+144}=\sqrt{169}=13$

65. Ans. (3)

Sol. $2\sin x(1+2(1-2\sin^2 x))(\cos 3x-\sin 3x)+1$

$=2(3\sin x-4\sin^3 x)(\cos 3x-\sin 3x)+1$

$=2\sin 3x\cos 3x-2\sin^2 3x+1$

$=\sin 6x+\cos 6x$

66. Ans.(2)

Sol. $x=\frac{1}{2}\left(\sqrt{5}+\frac{1}{\sqrt{5}}\right)\Rightarrow x=\frac{3}{\sqrt{5}}$

Now, $\log_8\left(\frac{\sqrt{x^2-1}}{x-\sqrt{x^2-1}}\right)=\log_8\left(\frac{\sqrt{\frac{9}{5}-1}}{\frac{3}{\sqrt{5}}-\sqrt{\frac{9}{5}-1}}\right)$

$=\log_8\left(\frac{2/\sqrt{5}}{1/\sqrt{5}}\right)=\log_{2^3}2^1=\frac{1}{3}$

67. Ans.(1)

Sol. $\because x=1$ (domain)

$\Rightarrow \tan^{-1}\sqrt{\frac{1-y}{1+y}}+\cos^{-1}|y|=\frac{\pi}{4}$

If $y>0:\tan^{-1}\sqrt{\frac{1-y}{1+y}}+\cos^{-1}y=\frac{\pi}{4}$

Let $\cos^{-1}y=\theta;\ \theta\in\left[0,\frac{\pi}{2}\right]$

$\Rightarrow \tan^{-1}\left|\tan\frac{\theta}{2}\right|+\theta=\frac{\pi}{4}$

$\Rightarrow \theta=\frac{\pi}{6}\quad\Rightarrow y=\frac{\sqrt{3}}{2}$

If $y<0:\ \tan^{-1}\sqrt{\frac{1-y}{1+y}}+\pi-\cos^{-1}y=\frac{\pi}{4}$

Let $\cos^{-1}y=\theta;\theta\in\left[\frac{\pi}{2},\pi\right]$

$\Rightarrow \tan^{-1}\left|\tan\frac{\theta}{2}\right|+\pi-\theta=\frac{\pi}{4}$

$\Rightarrow \theta=\frac{3\pi}{2}$ (not possible)

68. Ans. (2)

Sol. Given $\frac{f(x).f'(x)}{\sqrt{1-(f(x))^4}}\ge x$

$\Rightarrow \frac{2f(x).f'(x)}{\sqrt{1-(f(x))^4}}\ge 2x$

Now if $g(x)=\sin^{-1}\{(f(x)^2\}-x^2$ then $g'(x)\ge 0$

$\Rightarrow$ g(x) is a non-decreasing function

$\Rightarrow \lim\limits_{x\to\alpha^+}g(x)\le\lim\limits_{x\to\beta^-}g(x)$

$\Rightarrow \lim\limits_{x\to\alpha^+}\left(\{\sin^{-1}(f(x))^2\}-x^2\right)\le\lim\limits_{x\to\beta^-}\left(\{\sin^{-1}(f(x))^2\}-x^2\right)$

$\Rightarrow \frac{\pi}{2}-\alpha^2\le\frac{\pi}{6}-\beta^2$

$\Rightarrow \alpha^2-\beta^2\ge\frac{\pi}{3}\Rightarrow[\alpha^2-\beta^2]\ge 1$

69. Ans. (3)

Sol. $\frac{dy}{dx}=e^{x-y}\left(e^x-e^y\right)$

$e^y\frac{dy}{dx}+e^xe^y=e^{2x}$ **... (1)**

Putting $e^y=v\Rightarrow e^y\frac{dy}{dx}=\frac{dv}{dx}$

$\therefore \frac{dv}{dx}+ve^x=e^{2x}$

$I.F.=e^{\int e^xdx}=e^{e^x}$

So, $ve^{e^x}=\int e^{e^x}.e^{2x}\,dx$

$\Rightarrow ve^{e^x}=\left(e^x-1\right).e^{e^x}+c$

$\Rightarrow e^y=e^x-1+ce^{-e^x}$

Since x = 0, y = 1

So, $c=e^2\Rightarrow e^y=e^x-1+e^{2-e^x}$

70. Ans. (2)

Sol. $a^2+b^2=2+2\cos(\theta_1+\theta_2)$

71. Ans. (3)

Sol. $\because$ circumcenter is (0, 0) and centroid is $\left(\frac{3+\sqrt{5}}{3},\frac{9+2\sqrt{5}}{3}\right)$

i.e. orthocenter is $\left(3+\sqrt{5},9+2\sqrt{5}\right)$

72. Ans. (2)

Sol. ${}^nC_2={}^nC_3\Rightarrow n=5$

73. Ans. (1)

Sol. Let $P(at_1^2, 2at_1), Q(at_2^2, 2at_2)$

Slope of $\overline{PQ}$ is $1 \Rightarrow t_1 + t_2 = 2$

Any normal to the parabola is

$y + xt = 2at + at^3$

$\Rightarrow at^3 + t(2a - h) - k = 0$ (locus is (h,k))

So, $t_1 + t_2 + t_3 = 0 \Rightarrow t_3 = -2$

$\Rightarrow 8a + 2(2a - h) + k = 0$

74. Ans. (3)

Sol. $\begin{vmatrix} x^2 & x & 1 \\ x & 1 & x^2 \\ 1 & x^2 & x \end{vmatrix} = 0$

$$\Rightarrow \left(x^2 + x + 1\right)\begin{vmatrix} 1 & 1 & 1 \\ x & 1 & x^2 \\ 1 & x^2 & x \end{vmatrix} = 0 \left(R_1 \to R_1 + R_2 + R_3\right)$$

$$\Rightarrow \left(x^2 + x + 1\right)\begin{vmatrix} 0 & 0 & 1 \\ x-1 & 1-x^2 & x^2 \\ -\left(x^2-1\right) & -\left(x-x^2\right) & x \end{vmatrix} = 0$$

$$\begin{pmatrix} c_1 \to c_1 - c_2 \\ c_2 \to c_2 - c_3 \end{pmatrix}$$

$$\Rightarrow \left(x^2+x+1\right)\left(x-1\right)\left(1-x\right)\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1+x & x^2 \\ -\left(x+1\right) & -x & x \end{vmatrix} = 0$$

$$\Rightarrow \left(x^2 + x + 1\right)\left(x-1\right)^2\left\{x - \left(x+1\right)^2\right\} = 0$$

$$\Rightarrow \left(x^2 + x + 1\right)^2\left(x-1\right)^2 = 0$$

75. Ans. (3)

Sol. Equation of tangent is $y = 2x \pm \sqrt{4a^2 + b^2}$

It is normal to the circle $x^2 + y^2 + 4x + 1 = 0$

Hence, this tangent passes through $(-2, 0)$

$\Rightarrow 0 = -4 \pm \sqrt{4a^2 + b^2} \Rightarrow 4a^2 + b^2 = 16$

Using A.M. $\geq$ G.M, we get

$$\frac{4a^2 + b^2}{2} \geq \sqrt{4a^2b^2} \Rightarrow ab \leq 4$$

76. Ans. (3)

Sol. There are 45 two digit even numbers. So, reduced sample space is S and n(S) = 45

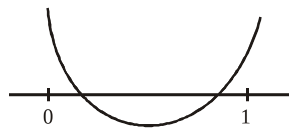
Now, if an even number is divisible by three then, it must be divisible by six.

So, E = {12, 18, ..., 96} $\Rightarrow$ n(E) = 15

Hence, $P(E) = \frac{15}{45} = \frac{1}{3}$

77. Ans. (4)

Sol. Given equation $f(x) = x^2 + ax + 1 = 0$



$\because$ since roots are real, distinct and lying between 0 and 1.

So,

i) $a^2 - 4 > 0$

ii) $0 < -a/2 < 1$

iii) f(0) > 0 and f(1) > 0

Hence, $a \in \phi$

78. Ans. (3)

Sol. Equation of director circles of ellipse and hyperbola are respectively.

$x^2 + y^2 = a^2 + b^2$ and $x^2 + y^2 = a^2 - b^2$

$a^2 + b^2 = 4r^2$ ...(1)

$a^2 - b^2 = r^2$ ...(2)

So $a^2 = \frac{5r^2}{2}, \quad b^2 = \frac{3r^2}{2}$

Now $e_e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{3r^2}{2} \times \frac{2}{5r^2} = 1 - \frac{3}{5} = \frac{2}{5}$;

Also, $e_h^2 = 1 + \frac{b^2}{a^2} = 1 + \frac{3}{5} = \frac{8}{5}$

So, $4e_h^2 - e_e^2 = 4 \times \frac{8}{5} - \frac{2}{5} = \frac{30}{5} = 6$

79. Ans. (4)

Sol. $P = {}^nC_6\left(3^{\frac{1}{3}}\right)^{n-6} \cdot \left(4^{\frac{-1}{3}}\right)^6$

$Q = {}^nC_{n-6}\left(3^{\frac{1}{3}}\right)^6 \cdot \left(4^{\frac{-1}{3}}\right)^{n-6}$

$\therefore \frac{Q}{P} = 12 \Rightarrow (12)^{\frac{n-6}{3}} = (12)^1$

$\Rightarrow \frac{n-6}{3} = 1 \Rightarrow n = 9$ **Ans.**]

80. Ans. (2)

Sol. | E | | | | | E |

First place can be filled in only 1 way similarly last place can be filled in 1 way remaining 4 places can be filled in 4! ways

$\Rightarrow$ required permulation = 1 × 4! × 1

= 24

81. Ans. (4)

Sol. $\frac{3}{1^2.2^2}+\frac{7}{1^3.2^3}+\frac{5}{2^2.3^2}+\frac{19}{2^3.3^3}+\frac{7}{3^2.4^2}+\frac{37}{3^3.4^4}+.....$ 20 terms

$$=\left(\frac{3}{1^2.2^2}+\frac{5}{2^2.3^2}+\frac{7}{3^2.4^2}.....10\,terms\right)$$

$$+\left(\frac{7}{1^3.2^3}+\frac{19}{2^3.3^3}+\frac{37}{3^3.4^3}.....10\,terms\right)$$

$$=\left(\frac{2^2-1^2}{1^2.2^2}+\frac{3^2-2^2}{2^2.3^2}+\frac{4^2-3^2}{3^2.4^2}...10\ terms\right)$$

$$+\left(\frac{2^3-1^3}{1^3.2^3}+\frac{3^3-2^3}{2^3.3^3}+\frac{4^3-3^3}{3^3.4^3}...10\ terms\right)$$

$$=\left(\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{2^2}-\frac{1}{3^2}......+\frac{1}{10^2}+\frac{1}{11^2}\right)$$

$$+\left(\frac{1}{1^3}-\frac{1}{2^3}+\frac{1}{2^2}-\frac{1}{3^3}.....+\frac{1}{10^3}-\frac{1}{11^3}\right)$$

$$=1-\frac{1}{11^2}+1-\frac{1}{11^3}=2-\frac{1}{121}-\frac{1}{1331}$$

$$=\frac{2650}{1331}$$

82. Ans. (2)

Sol. $2\sum_{1\le i<j\le 100}\sum\left(\alpha_i\alpha_j\right)^5$

$$=\left(\alpha_1^5+\alpha_2^5+\alpha_3^5+\alpha_4^5+....\right)^2$$

$$-\left(\alpha_1^{10}+\alpha_2^{10}+\alpha_3^{10}+\alpha_4^{10}+....\right)=0-0=0$$

Since, $\left(\alpha_1^r+\alpha_2^r+...\alpha_{100}^r\right)=\begin{cases}100, \text{ if } r=100k\\ 0, \text{ if } r\neq 100k\end{cases}$

83. Ans. (4)

Sol. Let h(x) = $f(f(x))$

So, h(–x) = $f(f(-x)) = f(-f(x))$

$= -f(f(x)) = -h(x)$

84. Ans. (2)

Sol. $f'(x) = 6x^2 - 6(2+\lambda)x + 12\lambda$

$\because$ For exactly one local max and min

$f'(x) = 0$ will have two real and unequal roots.

$\Rightarrow$ D > 0 $\Rightarrow$ $(2+\lambda)^2 - 8\lambda > 0$

$\Rightarrow$ $(\lambda-2)^2 > 0$ $\Rightarrow$ $\lambda \in R - \{2\}$

85. Ans. (4)

Sol. $k = \lim\limits_{x\to 0}\frac{\sqrt{1+\sin^2(x^2)}-\cos^3(x^2)}{x^4}$

$$=\frac{1}{2}\lim_{x\to 0}\frac{1+\sin^2(x^2)-\cos^6(x^2)}{x^4}$$

$$=\frac{1}{2}\lim_{x\to 0}\frac{\left(1-\cos^3(x^2)\right)\left(1+\cos^3(x^2)\right)}{x^4}+\underbrace{\frac{\sin^2(x^2)}{x^4}}_{=1}$$

$$=\frac{1}{2}\lim_{x\to 0}\left[\frac{(1-\cos x^2)\left(1+\cos x^2+\cos^2(x^2)\right)\left(1+\cos^3(x^2)\right)}{x^4}+1\right]$$

$$=\frac{1}{2}\left[\frac{1}{2}\times 3\times 2+1\right] = 2 \textbf{ Ans.}]$$

86. Ans. (3)

Sol. $f'(x)=\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$$=\lim_{h\to 0}\frac{f(x+h)-f(x+0)}{h}$$

$$=\lim_{h\to 0}\frac{f(x)+f(h)+h\sqrt{f(x)}-\{f(x)+f(0)+0\sqrt{f(x)}\}}{h}$$

$$=\lim_{h\to 0}\left\{\frac{f(h)-f(0)}{h}\right\}+\sqrt{f(x)}$$

$$= f'(0)+\sqrt{f(x)}$$

$$= 0+\sqrt{f(x)} = \sqrt{f(x)}$$

So, $\int\frac{f'(x)}{\sqrt{f(x)}}dx = \int dx \Rightarrow 2\sqrt{f(x)} = x + c$

(c = 0, as f(0) = 0)

$$\Rightarrow f(x)=\frac{x^2}{4}$$

Note that when α = 0 area is minimum.

$\Rightarrow$ Required minimum area = $2\int_0^9 2\sqrt{y}\,dy$ = 72

87. Ans. (4)

Sol. $f(t) = a^{t\ell nb}$

$g(t) = b^{-t\ell na}$

$= (b^{\ell na})^{-t} = (a^{\ell nb})^{-t}$

$= a^{-t\ell nb}$

Now $x = a^{t\ell nb}$ and $y = a^{-t\ell nb}$

$\Rightarrow$ xy = 1 or f(t).g(t) = 1

$\Rightarrow \frac{dy}{dx} = \frac{-1}{x^2}$

88. Ans. (4)

$$f(-x) = \begin{vmatrix} -x & \cos x & e^{x^6} \\ -\sin^5 x & x^4 & \sec x \\ -\tan^3 x & 10 & 20 \end{vmatrix}$$

$= -f(x)$

$\Rightarrow$ f(x) is an odd function.

89. Ans. (2)

Sol. This can happen, if three lines are real and distinct and acute angle between any pair is $\frac{\pi}{3}$.

$\Rightarrow$ $f(m) = bm^3 + dm^2 + cm + a$ has 3 distinct real roots and

$$\frac{m_1 - m_2}{1 + m_1 m_2} = \frac{m_2 - m_3}{1 + m_2 m_3} = \frac{m_3 - m_1}{1 + m_3 m_2} = \pm\sqrt{3}$$

$\Rightarrow 3 + m_1 m_2 + m_2 m_3 + m_3 m_1 = 0$

$\Rightarrow$ $3b + c = 0$ and $d^2 > 3bc$

(as $f'(m) = 0$ has two real and distinct roots)

90. Ans. (4)

Sol. Let $N = M - \frac{M^2}{3} + \frac{M^3}{9} - \frac{M^4}{27} + \ldots\ldots \infty$

$\therefore -\frac{MN}{3} = 0 - \frac{M^2}{3} + \frac{M^3}{9} \ldots\ldots \infty$

$\Rightarrow \left(I + \frac{M}{3}\right)N = M$

$\Rightarrow N = \begin{bmatrix} 3 & -9 \\ -3 & 3 \end{bmatrix} \begin{bmatrix} 4 & -3 \\ -1 & 4 \end{bmatrix}^{-1} =$

$\frac{1}{13}\begin{bmatrix} 3 & -27 \\ -9 & 3 \end{bmatrix} = \frac{3}{13}\begin{bmatrix} 1 & -9 \\ -3 & 1 \end{bmatrix}$

$\therefore \alpha = -9,\ \beta = -3$

$\Rightarrow \left|\frac{\alpha}{\beta}\right| = \left|\frac{-9}{-3}\right| = 3$ **Ans.**]