SOLVED EXAMPLES

Ex.1 The lines $2x - 3y = 5$ and $3x - 4y = 7$ are diameters of a circle of area 154 sq. units. The equation of this circle is -

(A) $x^2 + y^2 - 2x - 2y = 47$

(B) $x^2 + y^2 - 2x - 2y = 62$

(C) $x^2 + y^2 - 2x + 2y = 47$

(D) $x^2 + y^2 - 2x + 2y = 62$

Sol. The point of intersection of the given lines is (1,–1) which is the centre of the required circle. Also if its radius be r, then as given

$$\pi r^2 = 154$$

$$\Rightarrow r^2 = \frac{154 \times 7}{22} = 49 \Rightarrow r = 7$$

$\therefore$ reqd. equation is $(x-1)^2 + (y + 1)^2 = 49$

$\Rightarrow x^2 + y^2 - 2x + 2y = 47$ **Ans. [C]**

Ex.2 The equation of a circle which passes through the point (1,–2) and (4,–3) and whose centre lies on the line $3x + 4y = 7$ is-

(A) $15 (x^2 + y^2) - 94 x + 18 y - 55 = 0$

(B) $15(x^2 + y^2) - 94 x + 18 y + 55 = 0$

(C) $15 (x^2 + y^2) + 94 x - 18 y + 55 = 0$

(D) None of these

Sol. Let the circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(1)

Hence, substituting the points, (1,–2) and (4,–3) in equation (1)

$$\left.\begin{array}{l} 5 + 2g - 4f + c = 0 \ \ldots\ldots(2) \\ 25 + 89 - 6f + c = 0 \ldots\ldots(3) \end{array}\right\}$$

= centre (–g,–f) lies on line $3x + 4y = 7$

solving for g,f,c

Hence $-3g - 4f = 7$ ...(4)

Here $g = \frac{-47}{15}$, $f = \frac{9}{15}$, $c = \frac{55}{15}$

Hence the equation is

$15 (x^2 + y^2) - 94 x + 18y + 55 = 0$

Ans. [B]

Note: Trial method : In such cases, substitute the given points in the answer (A),(B),(C) and hence locate the correct answer. This may save time and energy.

Ex.3 The equation of a circle passing through (–4, 3) and touching the lines $x+ y = 2$, $x- y = 2$ is -

(A) $x^2 + y^2 - 20 x - 55 = 0$

(B) $x^2 + y^2 + 20 x + 55 = 0$

(C) $x^2 + y^2 - 20 x - 55 = 0$

(D) None of these

Sol. Let the circle be $x^2 + y^2 + 2gx + 2fy + c = 0$.

Passes through (–4, 3)

$25 - 8g + 6f + c = 0$ ...(1)

Touches both lines $\Rightarrow \frac{-g-f-2}{\sqrt{2}}$

$$= \sqrt{g^2 + f^2 - c} = \frac{-g+f-2}{\sqrt{2}}$$

$\therefore f = 0 \quad \therefore g^2 - 4g - 4 - 2c = 0$

Also $c = 8g - 25 \quad \therefore g = 10 \pm 3\sqrt{6}$, $f = 0$, $c = 55 \pm 24\sqrt{6}$

It is easy to see that the answers given are not near to the values of g,f,c. Hence none of these is the correct option. **Ans. [D]**

Note : Correct Answer :

$x^2 + y^2 + 2 (10 \pm 3\sqrt{6}) x + (55 \pm 24\sqrt{6}) = 0$

Ex.4 The equation of the circle which touches the axis of y at the origin and passes through (3,4) is -

(A) $4 (x^2 + y^2) - 25 x = 0$

(B) $3 (x^2 + y^2) - 25 x = 0$

(C) $2 (x^2 + y^2) - 3x = 0$

(D) $4 (x^2 + y^2) - 25 x + 10 = 0$

Sol. The centre of the circle lies on x– axis. Let a be the radius of the circle. Then, coordinates of the centre are (a,0). The circle passes through (3,4). Therefore,

$$\sqrt{(a-3)^3 + (0-4)^2} = a$$

$$\Rightarrow -6a + 25 = 0 \qquad \Rightarrow a = \frac{25}{6}$$

So, equation of the circle is

$(x- a)^2 + (y - 0)^2 = a^2$

or, $x^2 + y^2 - 2ax = 0$

or $3(x^2 + y^2) - 25 x = 0$ **Ans.[B]**

Ex.5 The equation of a circle which touches x-axis and the line $4x – 3y + 4 = 0$, its centre lying in the third quadrant and lies on the line $x – y – 1 = 0$, is -

(A) $9 (x^2 + y^2) + 6x + 24 y + 1 = 0$

(B) $9 (x^2 + y^2) – 6x – 24 y + 1 = 0$

(C) $9 (x^2 + y^2) –6x + 2y + 1 = 0$

(D) None of these

Sol. Let centre be (–h,–k) equation

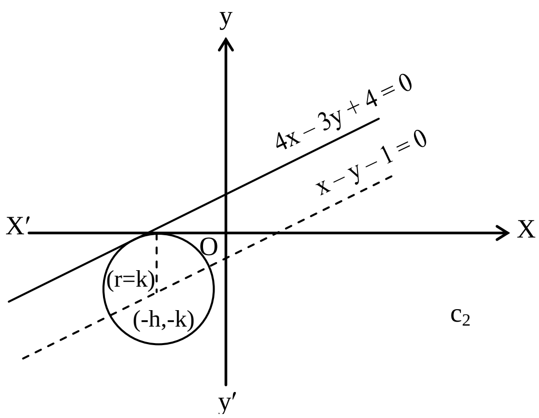
$(x+ h)^2 + (y+k)^2 = k^2$ ...(1)

Also $– h + k = 1$ ...(2)

$\therefore$ h = k–1radius = k (touches x- axis)

Touches the line $4x–3y + 4 = 0$

$$\left|\frac{-4h-3(-k)+4}{5}\right| = k \quad ...(3)$$



Solving (2) and (3), $h = \frac{1}{3}$, $k = \frac{4}{3}$

Hence the circle is

$$\left(x-\frac{4}{5}\right)^2 + \left(y+\frac{4}{3}\right)^2 = \left(\frac{4}{3}\right)^2$$

$\Rightarrow$ $9 (x^2 + y^2) + 6x + 24 y + 1 = 0$ **Ans.[A]**

Ex.6 The equation to a circle passing through the origin and cutting of intercepts each equal to + 5 of the axes is -

(A) $x^2 + y^2 + 5x – 5y = 0$

(B) $x^2 + y^2 – 5x + 5y = 0$

(C) $x^2 + y^2 – 5x – 5y = 0$

(D) $x^2 + y^2 + 5x + 5y = 0$

Sol. Let the circle cuts the x – axis and y– axis at A and B respectively. If O is the origin, then $\angle AOB = 90^o$, and A (5,0); B (0,5) is the diameter of the circle.

Then using diameter from the equation to the circle, we get

$(x – 5) (x – 0) + (y – 0) (y – 5) = 0$

$\Rightarrow$ $x^2 + y^2 – 5x – 5y = 0$ **Ans.[C]**

Ex.7 The equation of the circle whose radius is 3 and which touches the circle $x^2 + y^2 –4x – 6y – 12 = 0$ internally at the point (–1,–1) is -

(A) $(x – 4/5) + (4 + 7/5)^2 = 3^2$

(B) $(x – 4/5) + (4 – 7/5)^2 = 3^2$

(C) $(x–8)^2 + (y–1)^2 = 3^2$

(D) None of these

Sol. Let C be the centre of the given circle and C_1 be the centre of the required circle.

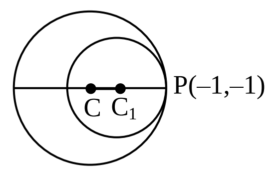
Now C = (2,3),

CP = radius = 5

$\because$ $C_1 P = 3$

$\Rightarrow CC_1 = 2$

$\therefore$ The point C_1 divides internally, the line joining C and P in the ratio 2: 3



$\therefore$ coordinates of C_1 are $\left(\frac{4}{5},\frac{7}{5}\right)$

Hence (B) is the required circle. **Ans. [B]**

Ex.8 The equation of a circle which passes through the three points (3, 0) (1, –6), (4, –1) is -

(A) $2x^2 + 2y^2 + 5x – 11 y + 3 = 0$

(B) $x^2 + y^2 – 5x + 11 y – 3 = 0$

(C) $x^2 + y^2 + 5x – 11 y + 3 = 0$

(D) $2x^2 + 2y^2 – 5x + 11 y – 3 = 0$

Sol. Let the circle be

$x^2 + y^2 + 2gx + 2fy+ c = 0$ ...(1)

$9 + 0 + 6 g + 0 + c = 0$ ...(2)

$1 + 36 + 2g – 12 f + c = 0$ ...(3)

$16 + 1 + 8g – 2f + c = 0$ ...(4)

from (2) – (3) , $–28 + 4g + 12f = 0$

$g + 3f – 7 = 0$ ...(5)

from (3) – (4), $20 – 6g– 10f = 0$

$3g + 5f – 10 = 0$ ...(6)

Solving $\frac{g}{-30+35} = \frac{f}{-21+10} = \frac{1}{5-9}$

$\therefore$ $g = -\frac{5}{4}$, $f = \frac{11}{4}$, $c = -\frac{3}{2}$

Hence the circle is

$2x^2 + 2y^2 - 5x + 11y - 3 = 0$ **Ans.[D]**

Ex.9 The equation of the circle which is touched by $y = x$, has its centre on the positive direction of the x- axis and cuts off a chord of length 2 units along the line $\sqrt{3}y - x = 0$ is -

(A) $x^2 + y^2 - 4x + 2 = 0$ (B) $x^2 + y^2 - 8x + 8 = 0$

(C) $x^2 + y^2 - 4x + 1 = 0$ (D) $x^2 + y^2 - 4y + 2 = 0$

Sol. Since the required circle has its centre on X-axis, So, let the coordinates of the centre be (a,0). The circle touches y = x. Therefore,

radius = length of the perpendicular from (a,0) on $x - y = 0$

$= \frac{a}{\sqrt{2}}$

The circle cuts off a chord of length 2 units along $x - \sqrt{3}y = 0$.

$$\left(\frac{a}{\sqrt{2}}\right)^2 = 1^2 + \left(\frac{a - \sqrt{3}\times 0}{\sqrt{1^2 + (\sqrt{3})^2}}\right)^2$$

$$\Rightarrow \frac{a^2}{2} = 1 + \frac{a^2}{4} \Rightarrow a = 2$$

Thus, centre of the circle is at (2,0) and radius $= \frac{a}{\sqrt{2}} = \sqrt{2}$.

So, its equation is $x^2 + y^2 - 4x + 2 = 0$ **Ans.[A]**

Ex.10 The greatest distance of the point P (10, 7) from the circle $x^2 + y^2 - 4x - 2y - 20 = 0$ is -

(A) 5 (B) 15

(C) 10 (D) None of these

Sol. Since $S_1 = 10^2 + 7^2 - 4 \times 10 - 2 \times 7 - 20 > 0$. So, P lies outside the circle. Join P with the centre C (2,1) of the given circle. Suppose PC cuts the circle at A and B. Then, PB is the greatest distance of P from the circle.

We have : $PC = \sqrt{(10-2)^2 + (7-1)^2} = 10$

and CB = radius $= \sqrt{4+1+20} = 5$

$\therefore$ PB = PC + CB = (10 + 5) = 15 **Ans.[B]**

Ex.11 The length of intercept on y– axis, by a circle whose diameter is the line joining the points (–4, 3) and (12, –1) is -

(A) $2\sqrt{13}$ (B) $\sqrt{13}$

(C) $4\sqrt{13}$ (D) None of these

Sol. Here equation of the circle

$(x+4)(x-12) + (y-3)(y+1) = 0$

or $x^2 + y^2 - 8x - 2y - 51 = 0$

Hence intercept on y– axis

$= 2\sqrt{f^2 - c} = 2\sqrt{1-(-51)} = 4\sqrt{13}$ **Ans.[C]**

Ex.12 For the circle $x^2 + y^2 + 4x - 7y + 12 = 0$ the following statement is true -

(A) the length of tangent from (1, 2) is 7

(B) Intercept on y- axis is 2

(C) intercept on x– axis is $2 - \sqrt{2}$

(D) None of these

Sol. Here

(A) Putting y = 0, $x^2 + 4x + 12 = 0$ imaginary roots, not true

(B) Put x = 0, $y^2 - 7y + 12 = 0$
or (y–3) (y– 4) = 0 intercept = 4–3 = 12

(C) Length of tangent $= \sqrt{1+4+4-14+12} = \sqrt{7}$

Hence" none of these" is true. **Ans.[D]**

Ex.13 The equation of tangent drawn from the origin to the circle $x^2 + y^2 - 2rx - 2hy + h^2 = 0$ is -

(A) y = 0

(B) x – y = 0

(C) $(h^2 - r^2)x - 2rhy = 0$

(D) None of these

Sol. Equation of tangent line drawn form origin can be taken as y = mx

The centre of the given circle is (r, h) and radius is = r.

Now by condition of tangency p = r, we have

$$\frac{mr - h}{\sqrt{1+m^2}} = \pm r$$

$\Rightarrow m^2r^2 + h^2 - 2mhr = r^2(1+m^2)$

$\Rightarrow m = \frac{h^2 - r^2}{2hr}$

Putting this value in y = mx, we get the required equation of tangent (C). **Ans.[C]**

Remark : Since we can write equation of circle in the following form $(x-r)^2 + (y-h)^2 = r^2$

Obviously, the other tangent through origin is y-axis i.e. x = 0.

Ex.14 If the squares of the lengths of the tangents from a point P to the circles $x^2 + y^2 = a^2$, $x^2 + y^2 = b^2$ and $x^2 + y^2 = c^2$ are in A.P., then

(A) a, b, c are in GP

(B) a, b, c are in AP

(C) a^2, b^2, c^2 are in AP

(D) a^2, b^2, c^2 are in GP

Sol. Let P (x_1, y_1) be the given point and PT_1, PT_2, PT_3 be the lengths of the tangents from P to the circles $x^2 + y^2 = a^2$, $x^2 + y^2 = b^2$ and $x^2 + y^2 = c^2$ respectively. Then,

$PT_1 = \sqrt{x_1^2 + y_1^2 - a^2}$, $PT_2 = \sqrt{x_1^2 + y_1^2 - b^2}$ and

$PT_3 = \sqrt{x_1^2 + y_1^2 - c^2}$

Now, PT_1^2, PT_2^2, PT_3^2 are in AP

$\Rightarrow 2\,PT_2^2 = PT_1^2 + PT_3^2$

$\Rightarrow 2(x_1^2 + y_1^2 - b^2) = (x_1^2 + y_1^2 - a^2) + (x_1^2 + y_1^2 - c^2)$

$\Rightarrow 2b^2 = a^2 + c^2$

$\Rightarrow a^2, b^2, c^2$ are in AP. **Ans.[C]**

Ex.15 The area of the triangle formed by the tangents from an external point (h, k) to the circle $x^2 + y^2 = a^2$ and the chord of contact, is -

(A) $\frac{1}{2} a\left(\frac{h^2+k^2-a^2}{\sqrt{h^2+k^2}}\right)$

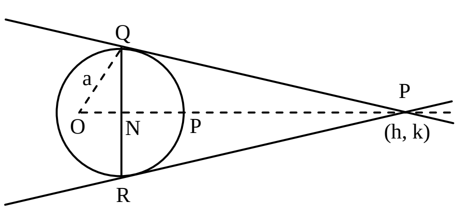
(B) $\frac{a(h^2+k^2-a^2)^{3/2}}{2(h^2+k^2)}$

(C) $\frac{a(h^2+k^2-a^2)^{3/2}}{(h^2+k^2)}$

(D) None of these

Sol. Here area of Δ PQR is required

Now chord of contact w.r. to circle $x^2 + y^2 = a^2$, and point (h, k) $hx + ky - a^2 = 0$



Perp. from (h, k), $PN = \frac{h^2+k^2-a^2}{\sqrt{h^2+k^2}}$

Also length $QR = 2\sqrt{a^2 - \frac{(a^2)^2}{h^2-k^2}}$

$= \frac{2a\sqrt{h^2+k^2-a^2}}{\sqrt{h^2+k^2}}$

$\therefore\ \Delta PQR = \frac{1}{2}(QR)(PN)$

$= \frac{1}{2}\,2a\sqrt{\frac{h^2+k^2-a^2}{h^2+k^2}}\ \frac{(h^2+k^2-a^2)}{\sqrt{h^2+k^2}}$

$= a\,\frac{(h^2+k^2-a^2)^{3/2}}{\sqrt{h^2+k^2}}$ **Ans.[C]**

Ex.16 If the line y = x + 3 meets the circle $x^2 + y^2 = a^2$ at A and B, then the equation of the circle having AB as a diameter will be -

(A) $x^2 + y^2 + 3x - 3y - a^2 + 9 = 0$

(B) $x^2 + y^2 + 3x + 3y - a^2 + 9 = 0$

(C) $x^2 + y^2 - 3x + 3y - a^2 + 9 = 0$

(D) None of these

Sol. Let the equation of the required circle be

$(x^2 + y^2 - a^2) + \lambda\,(y - x - 3) = 0$

since its centre $(\lambda/2, -\lambda/2)$ lies on the given line, so we have $-\lambda/2 = \lambda/2 + 3 = -3$

Putting this value of in (A) we get the reqd. eqn. as

$x^2 + y^2 + 3x - 3y - a^2 + 9 = 0$ **Ans. [A]**

Ex.17 The equation of the circle passing through the point of intersection of the circles $x^2 + y^2 = 6$ and $x^2 + y^2 - 6x + 8 = 0$, and also through the point (1, 1) is -

(A) $x^2 + y^2 - 4y + 2 = 0$

(B) $x^2 + y^2 - 3x + 1 = 0$

(C) $x^2 + y^2 - 6x + 4 = 0$

(D) None of these

Sol. Let the equation of the required circle be

$(x^2 + y^2 - 6x + 8) + (x^2 + y^2 - 6) = 0$

Since it passes through (1, 1), so we have

$1 + 1 - 6 + 8 + \lambda\,(1 + 1 - 6) = 0 = 1$

$\therefore$ the required equation is

$x^2 + y^2 - 3x + 1 = 0$ **Ans. [B]**

Ex.18 If y = 2x is a chord of the circle $x^2 + y^2 = 10x$, then the equation of the circle whose diameter is this chord is -

(A) $x^2 + y^2 + 2x + 4y = 0$

(B) $x^2 + y^2 + 2x - 4y = 0$

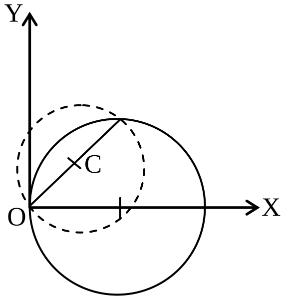
(C) $x^2 + y^2 - 2x - 4y = 0$

(D) None of these

Sol. Here equation of the circle

$(x^2 + y^2 - 10x) + \lambda(y - 2x) = 0$

Now centre C $(5 + \lambda, -\lambda/2)$ lies on the



chord again

$\therefore \frac{-\lambda}{2} = 2(5 + \lambda) \Rightarrow \frac{-5\lambda}{2} = 10$

$\therefore \lambda = -4$

Hence $x^2 + y^2 = 10x - 4y + 8x = 0$

or $x^2 + y^2 - 2x - 4y = 0$ **Ans.[C]**

Ex.19 The circle S_1 with centre C_1 (a_1, b_1) and radius r_1 touches externally the circle S_2 with centre C_2 (a_2, b_2) and radius r_2. If the tangent at their common point passes through the origin, then

(A) $(a_1^2 + a_2^2) + (b_1^2 + b_2^2) = r_1^2 + r_2^2$

(B) $(a_1^2 - a_2^2) + (b_1^2 - b_2^2) = r_2^2 - r_1^2$

(C) $(a_1^2 - b_2^2) + (a_2^2 + b_2^2) = r_1^2 + r_2^2$

(D) $(a_1^2 - b_1^2) + (a_2^2 + b_2^2) = r_1^2 + r_2^2$

Sol. The two circles are

$S_1 = (x - a_1)^2 + (y - b_1^2) = r_1^2$ ...(i)

$S_2 = (x - a_2)^2 + (y - b_2^2) = r_2^2$ ...(ii)

The equation of the common tangent of these two circles is given by $S_1 - S_2 = 0$

i.e., $2x(a_1 - a_2) + 2y(b_1 - b_2) + (a_2^2 + b_2^2)$

$- (a_1^2 + b_1^2) + r_1^2 - r_2^2 = 0$

If this passes through the origin, then

$(a_2^2 + b_2^2) - (a_1^2 + b_1^2) + r_1^2 - r_2^2 = 0$

$(a^2{}_2 - a_1^2) + (b_2^2 - b_1^2) = r^2{}_2 - r_1^2$ **Ans.[B]**

Ex.20 The length of the common chord of the circles $(x-a)^2 + y^2 = c^2$ and $x^2 + (y-b)^2 = c^2$ is -

(A) $\sqrt{c^2 + a^2 + b^2}$ (B) $\sqrt{4c^2 + a^2 + b^2}$

(C) $\sqrt{4c^2 - a^2 - b^2}$ (D) $\sqrt{c^2 - a^2 - b^2}$

Sol. The equation of the common chord is

$[(x-a)^2 + y^2 - c^2] - [x^2 + (y-b)^2 - c^2] = 0$

$\Rightarrow 2ax - 2by - a^2 + b^2 = 0$ ...(1)

Now p = length of perpendicular from (a, 0) on (1)

$= \frac{2a^2 - a^2 + b^2}{\sqrt{4a^2 + 4b^2}} = \frac{1}{2}\sqrt{a^2 + b^2}$

$\therefore$ length of common chord

$= 2\sqrt{c^2 - p^2} = 2\sqrt{c^2 - \frac{a^2 + b^2}{4}}$

$= \sqrt{4c^2 - a^2 - b^2}$ **Ans.[C]**

Ex.21 The angle of intersection of the two circles $x^2 + y^2 - 2x - 2y = 0$ and $x^2 + y^2 = 4$, is -

(A) 30º (B) 60º (C) 90º (D) 45º

Sol. Here circles are

$x^2 + y^2 - 2x - 2y = 0$ ...(1)

$x^2 + y^2 = 4$ ...(2)

Now c_1 (1, 1), $r_1 = \sqrt{1^2 + 1^2} = \sqrt{2}$

c_2 (0, 0), $r_2 = 2$

If θ is the angle of intersection then

$\cos\theta = \frac{r_1^2 + r_2^2 - (c_1c_2)^2}{2r_1r_2}$

$= \frac{2 + 4 - (\sqrt{2})^2}{2.\sqrt{2}.2.} = \frac{1}{\sqrt{2}}$

$= \theta = 45º$ **Ans.[D]**

Ex.22 If a circle passes through the point (1,2) and cuts the circle $x^2 + y^2 = 4$ orthogonally, then the locus of its centre is -

(A) $x^2 + y^2 - 2x - 6y - 7 = 0$

(B) $x^2 + y^2 - 3x - 8y + 1 = 0$

(C) $2x + 4y - 9 = 0$

(D) $2x + 4y - 1 = 0$

Sol. Let the equation of the circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$,

Since it passes through (1, 2), so

$1 + 4 + 2g + 4f + c = 0$

$\Rightarrow 2g + 4f + c + 5 = 0$ ...(1)

Also this circle cuts $x^2 + y^2 = 4$

orthogonally, so $2g(0) + 2f(0) = c - 4$

$\Rightarrow$ c = 4 ...(2)

From (1) and (2) eliminating c, we have

$2g + 4f + 9 = 0$

Hence locus of the centre $(-g, -f)$ is

$2x + 4y - 9 = 0$ **Ans.[C]**

Ex.23 Circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 2x - 4y + 3 = 0$

(A) touch each other externally

(B) touch each other internally

(C) intersect each other

(D) do not intersect

Sol. Here C_1 (0, 0) and C_2 (1, 2)

$\therefore C_1C_2 = \sqrt{1+4} = \sqrt{5} = 2.23.$

Also $r_1 = 2$, $r_2 = \sqrt{1+4-3} = \sqrt{2} = 1.41$

$\therefore r_1 - r_2 < C_1C_2 < r_1 + r_2$

$\Rightarrow$ circles intersect each other. **Ans.[C]**

Ex.24 The circles $x^2 + y^2 + 2x - 2y + 1 = 0$ and $x^2 + y^2 - 2x - 2y + 1 = 0$ touch each other -

(A) externally at (0,1) (B) internally at (0,1)

(C) externally at (1,0) (D) internally at (1,0)

Sol. The centres of the two circles are C_1 (–1, 1) and C_2 (1, 1) and both have radii equal to 1. We have:

$C_1C_2 = 2$ and sum of the radii = 2

So, the two circles touch each other externally. The equation of the common tangent is obtained by subtracting the two equations.

The equation of the common tangent is

$4x = 0 \Rightarrow x = 0.$

Putting x = 0 in the equation of the either circle, we get

$y^2 - 2y + 1 = 0 \Rightarrow (y-1)^2 = 0 \Rightarrow y = 1.$

Hence, the points where the two circles touch is (0,1). **Ans.[A]**

Ex.25 The total number of common tangents to the two circles $x^2 + y^2 - 2x - 6y + 9 = 0$ and $x^2 + y^2 + 6x - 2y + 1 = 0$, is -

(A) 1 (B) 2 (C) 3 (D) 4

Sol. Here

c_1 (1, 3), $r_1 = \sqrt{1+9-9} = 1$

c_2 (–3, 1), $r_2 = \sqrt{9+1-1} = 3$

Now $c_1c_2 = \sqrt{(1+3)^2 + (3-2)^2}$

$= \sqrt{16+1} = \sqrt{17}$

$c_1c_2 > r_1 + r_2$

Hence the circles are non- intersecting externally. Hence 4 tangents, two direct and two transverse tangents may be drawn.

Ans.[D]

Ex.26 If (4, –2) is a point on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, which is concentric to $x^2 + y^2 - 2x + 4y + 20 = 0$, then value of c is -

(A) – 4 (B) 0

(C) 4 (D) 1

Sol. Since the first circle is concentric to $x^2 + y^2 - 2x + 4y + 20 = 0$, therefore its equation can be written as

$x^2 + y^2 - 2x + 4y + c = 0$

If it passes through (4,–2), then

$16 + 4 - 8 - 8 + c = 0$

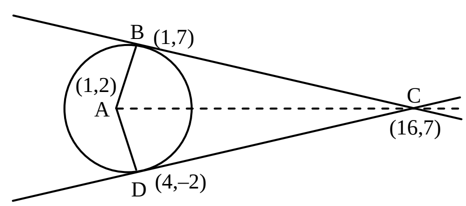
$\Rightarrow c = -4$ **Ans. [A]**

Ex.27 Let A be the centre of the circle $x^2 + y^2 - 2x - 4y - 20 = 0$, and B(1,7) and D(4,–2) are points on the circle then, if tangents be drawn at B and D, which meet at C, a then area of quadrilateral ABCD is -

(A) 150 (B) 75

(C) 75/2 (D) None of these

Sol.



Here centre A (1,2), and Tangent at (1,7) is

$x.1 + y.7 - 1(x+1) - 2(y+7) - 20 = 0$

or $y = 7$ ...(1)

Tangent at D (4,–2) is

$3x - 4y - 20 = 0$ ...(2)

Solving (1) and (2), C is (16, 7)

Area ABCD = AB x BC

$= 5 \times \sqrt{256+49-32-28-20}$

$= 5 \times 15 = 75$ units **Ans.[B]**

Ex.28 The abscissa of two points A and B are the roots of the equation $x^2 + 2ax - b^2 = 0$ and their ordinates are the roots of the equation $y^2 + 2py - q^2 = 0$. The radius of the circle with AB as a diameter will be -

(A) $\sqrt{a^2+b^2+p^2+q^2}$ (B) $\sqrt{b^2+q^2}$

(C) $\sqrt{a^2+b^2-p^2-q^2}$ (D) $\sqrt{a^2+p^2}$

Sol. Let A ≡ (α, β); B ≡ (γ, δ). Then

$\alpha + \gamma = -2a$, $\alpha\gamma = -b^2$

and $\beta + \delta = -2p$, $\beta\delta = -q^2$

Now equation of the required circle is

$(x - \alpha)(x - \gamma) + (y - \beta)(y - \delta) = 0$

$\Rightarrow x^2 + y^2 - (\alpha + \gamma)x - (\beta + \delta) + \alpha\gamma + \beta\delta = 0$

$\Rightarrow x^2 + y^2 + 2ax + 2py - b^2 - q^2 = 0$

Its radius = $\sqrt{a^2+b^2+p^2+q^2}$ **Ans.[A]**

Ex.29 Two rods of length a and b slide on the axes in such a way that their ends are always concylic. The locus of centre of the circle passing through the ends is -

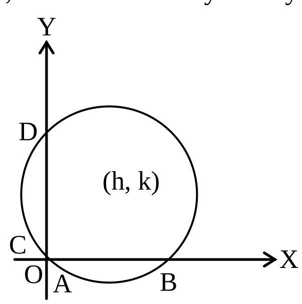
(A) $4(x^2 - y^2) = a^2 - b^2$

(B) $x^2 - y^2 = a^2 - b^2$

(C) $x^2 - y^2 = 4(a^2 - b^2)$

(D) $x^2 + y^2 = a^2 + b^2$

Sol. Let a rod AB of length 'a' slides on x-axis and rod CD of length 'b' slide on y - axis so that ends A, B, C and D are always concyclic.



Let equation of circle passing through these ends is

$x^2 + y^2 + 2gx + 2fy + c = 0$

Obviously $2\sqrt{g^2 - c} = a$ and $2\sqrt{f^2 - c} = b$

$\therefore\ 4(g^2 - f^2) = a^2 - b^2$

$\Rightarrow 4[(-g)^2 - (-f)^2] = a^2 - b^2$

therefore locus of centre (–g, –f) is

$4(x^2 - y^2) = a^2 - b^2$. **Ans.[A]**

Ex.30 The angle between the tangents from α, β to the circle $x^2 + y^2 = a^2$ is -

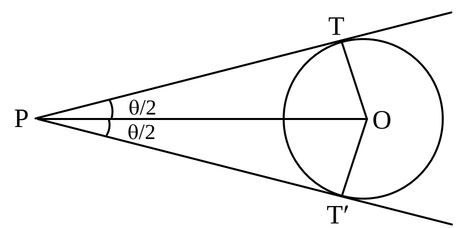
(A) $\tan^{-1}\left(\frac{a}{\sqrt{S_1}}\right)$ (B) $2\tan^{-1}\left(\frac{a}{\sqrt{S_1}}\right)$

(C) $2\tan^{-1}\left(\frac{\sqrt{S_1}}{a}\right)$ (D) None of these

Where $S_1 = \alpha^2 + \beta^2 - a^2$

Sol. Let PT and PT' be the tangents drawn from P (α, β) to the circle $x^2 + y^2 = a^2$, and let ∠TPT´ = θ. If O is the centre of the circle, then ∠TPO = ∠T´PO = θ/2.

$\therefore \tan\frac{\theta}{2} = \frac{OT}{OP} = \frac{a}{\sqrt{S_1}}$



$\Rightarrow \frac{\theta}{2} = \tan^{-1}\left(\frac{a}{\sqrt{S_1}}\right)$

$\Rightarrow \theta = 2\tan^{-1}\left(\frac{a}{\sqrt{S_1}}\right)$ **Ans. [B]**

LEVEL-1

Question based on Standard forms of Equation of a Circle

Q.1 The length of the diameter of the circle $x^2 + y^2 - 4x - 6y + 4 = 0$ is -
(A) 9 (B) 3 (C) 4 (D) 6

Q.2 Which of the following is the equation of a circle?
(A) $x^2 + 2y^2 - x + 6 = 0$
(B) $x^2 - y^2 + x + y + 1 = 0$
(C) $x^2 + y^2 + xy + 1 = 0$
(D) $3(x^2 + y^2) + 5x + 1 = 0$

Q.3 The equation of the circle passing through (3, 6) and whose centre is (2, –1) is -
(A) $x^2 + y^2 - 4x + 2y = 45$
(B) $x^2 + y^2 - 4x - 2y + 45 = 0$
(C) $x^2 + y^2 + 4x - 2y = 45$
(D) $x^2 + y^2 - 4x + 2y + 45 = 0$

Q.4 If (4, 3) and (–12, –1) are end points of a diameter of a circle, then the equation of the circle is-
(A) $x^2 + y^2 - 8x - 2y - 51 = 0$
(B) $x^2 + y^2 + 8x - 2y - 51 = 0$
(C) $x^2 + y^2 + 8x + 2y - 51 = 0$
(D) None of these

Q.5 The radius of the circle passing through the points (0, 0), (1, 0) and (0, 1) is-
(A) 2 (B) $1/\sqrt{2}$ (C) $\sqrt{2}$ (D) 1/2

Q.6 The radius of a circle with centre (a, b) and passing through the centre of the circle $x^2 + y^2 - 2gx + f^2 = 0$ is-
(A) $\sqrt{(a-g)^2 + b^2}$ (B) $\sqrt{a^2 + (b+g)^2}$
(C) $\sqrt{a^2 + (b-g)^2}$ (D) $\sqrt{(a+g)^2 + b^2}$

Q.7 If (x, 3) and (3, 5) are the extremities of a diameter of a circle with centre at (2, y). Then the value of x and y are-
(A) x = 1, y = 4 (B) x = 4, y = 1
(C) x = 8, y = 2 (D) None of these

Q.8 If (0, 1) and (1, 1) are end points of a diameter of a circle, then its equation is-
(A) $x^2 + y^2 - x - 2y + 1 = 0$
(B) $x^2 + y^2 + x - 2y + 1 = 0$
(C) $x^2 + y^2 - x - 2y - 1 = 0$
(D) None of these

Q.9 The coordinates of any point on the circle $x^2 + y^2 = 4$ are-
(A) $(\cos\alpha, \sin\alpha)$ (B) $(4\cos\alpha, 4\sin\alpha)$
(C) $(2\cos\alpha, 2\sin\alpha)$ (D) $(\sin\alpha, \cos\alpha)$

Q.10 The parametric coordinates of any point on the circle $x^2 + y^2 - 4x - 4y = 0$ are-
(A) $(-2 + 2\cos\alpha, -2 + 2\sin\alpha)$
(B) $(2 + 2\cos\alpha, 2 + 2\sin\alpha)$
(C) $(2 + 2\sqrt{2}\cos\alpha, 2 + 2\sqrt{2}\sin\alpha)$
(D) None of these

Q.11 The parametric coordinates of a point on the circle $x^2 + y^2 - 2x + 2y - 2 = 0$ are -
(A) $(1 - 2\cos\alpha, 1 - 2\sin\alpha)$
(B) $(1 + 2\cos\alpha, 1 + 2\sin\alpha)$
(C) $(1 + 2\cos\alpha, -1 + 2\sin\alpha)$
(D) $(-1 + 2\cos\alpha, 1 + 2\sin\alpha)$

Q.12 The equation $k(x^2 + y^2) - x - y + k = 0$ represents a real circle, if-
(A) $k < \sqrt{2}$ (B) $k > \sqrt{2}$
(C) $k > 1/\sqrt{2}$ (D) $0 < |k| \le \frac{1}{\sqrt{2}}$

Q.13 If the equation $px^2 + (2-q)xy + 3y^2 - 6qx + 30y + 6q = 0$ represents a circle, then the values of p and q are -
(A) 2, 2 (B) 3, 1 (C) 3, 2 (D) 3, 4

Q.14 The circle represented by the equation $x^2 + y^2 + 2gx + 2fy + c = 0$ will be a point circle, if-
(A) $g^2 + f^2 = c$ (B) $g^2 + f^2 + c = 0$
(C) $g^2 + f^2 > c$ (D) None of these

Q.15 The equation of the circum-circle of the triangle formed by the lines x = 0, y = 0, $\frac{x}{a} - \frac{y}{b} = 1$, is -

(A) $x^2 + y^2 + ax - by = 0$
(B) $x^2 + y^2 - ax + by = 0$
(C) $x^2 + y^2 - ax - by = 0$
(D) $x^2 + y^2 + ax + by = 0$

Q.16 The circum-circle of the quadrilateral formed by the lines x = a, x = 2a, y = – a, y = a is-

(A) $x^2 + y^2 - 3ax - a^2 = 0$
(B) $x^2 + y^2 + 3ax + a^2 = 0$
(C) $x^2 + y^2 - 3ax + a^2 = 0$
(D) $x^2 + y^2 + 3ax - a^2 = 0$

Q.17 The x coordinates of two points A and B are roots of equation $x^2 + 2x - a^2 = 0$ and y coordinate are roots of equation $y^2 + 4y - b^2 = 0$ then equation of the circle which has diameter AB is-

(A) $(x - 1)^2 + (y - 2)^2 = 5 + a^2 + b^2$
(B) $(x + 1)^2 + (y + 2)^2 = \sqrt{(5 + a^2 + b^2)}$
(C) $(x + 1)^2 + (y + 2)^2 = (a^2 + b^2)$
(D) $(x + 1)^2 + (y + 2)^2 = 5 + a^2 + b^2$

Question based on Equation of Circle in special cases

Q.18 A circle touches both the axes and its centre lies in the fourth quadrant. If its radius is 1 then its equation will be -

(A) $x^2 + y^2 - 2x + 2y + 1 = 0$
(B) $x^2 + y^2 + 2x - 2y - 1 = 0$
(C) $x^2 + y^2 - 2x - 2y + 1 = 0$
(D) $x^2 + y^2 + 2x - 2y + 1 = 0$

Q.19 The equation to a circle with centre (2, 1) and touching x axis is -

(A) $x^2 + y^2 + 4x + 2y + 4 = 0$
(B) $x^2 + y^2 - 4x - 2y + 4 = 0$
(C) $x^2 + y^2 - 4x - 2y + 1 = 0$
(D) None of these

Q.20 The equation to the circle whose radius is 4 and which touches the x–axis at a distance –3 from the origin is-

(A) $x^2 + y^2 - 6x + 8y - 9 = 0$
(B) $x^2 + y^2 \pm 6x - 8y + 9 = 0$
(C) $x^2 + y^2 + 6x \pm 8y + 9 = 0$
(D) $x^2 + y^2 \pm 6x - 8y - 9 = 0$

Q.21 The equation of the circle touching the lines x = 0, y = 0 and x = 2c is-

(A) $x^2 + y^2 + 2cx + 2cy + c^2 = 0$
(B) $x^2 + y^2 - 2cx + 2cy + c^2 = 0$
(C) $x^2 + y^2 \pm 2cx - 2cy + c^2 = 0$
(D) $x^2 + y^2 - 2cx \pm 2cy + c^2 = 0$

Q.22 The circle $x^2 + y^2 - 4x - 4y + 4 = 0$ is-

(A) touches x–axes only
(B) touches both axes
(C) passes through the origin
(D) touches y–axes only

Q.23 If a be the radius of a circle which touches x-axis at the origin, then its equation is-

(A) $x^2 + y^2 + ax = 0$ (B) $x^2 + y^2 \pm 2ya = 0$
(C) $x^2 + y^2 \pm 2xa = 0$ (D) $x^2 + y^2 + ya = 0$

Q.24 The point where the line x = 0 touches the circle $x^2 + y^2 - 2x - 6y + 9 = 0$ is-

(A) (0, 1) (B) (0, 2)
(C) (0, 3) (D) No where

Q.25 Circle $x^2 + y^2 + 6y = 0$ touches -

(A) x– axis at the point (3, 0)
(B) x– axis at the origin
(C) y – axis at the origin
(D) The line y + 3 = 0

Question based on Position of Point w.r.t. Circle

Q.26 Position of the point (1, 1) with respect to the circle $x^2 + y^2 - x + y - 1 = 0$ is -

(A) Outside the circle (B) Inside the circle
(C) Upon the circle (D) None of these

Q.27 The point (0.1, 3.1) with respect to the circle $x^2 + y^2 - 2x - 4y + 3 = 0$, is -

(A) Inside the circle but not at the centre
(B) At the centre of the circle
(C) On the circle
(D) Outside the circle

Question based on Line & Circle

Q.28 The straight line (x – 2) + (y + 3) = 0 cuts the circle $(x - 2)^2 + (y - 3)^2 = 11$ at-

(A) no points (B) two points
(C) one point (D) None of these

Q.29 If the line 3x + 4y = m touches the circle $x^2 + y^2 = 10x$, then m is equal to-

(A) 40, 10 (B) 40, –10

(C) –40, 10 (D) –40, –10

Q.30 Circle $x^2 + y^2 - 4x - 8y - 5 = 0$ will intersect the line $3x - 4y = m$ in two distinct points, if -

(A) $-10 < m < 5$ (B) $9 < m < 20$

(C) $-35 < m < 15$ (D) None of these

Q.31 The length of the intercept made by the circle $x^2 + y^2 = 1$ on the line x + y = 1 is-

(A) $1/\sqrt{2}$ (B) $\sqrt{2}$

(C) 2 (D) $2\sqrt{2}$

Q.32 If a circle with centre (0, 0) touches the line 5x + 12y = 1 then its equation will be-

(A) $13(x^2 + y^2) = 1$ (B) $x^2 + y^2 = 169$

(C) $169(x^2 + y^2) = 1$ (D) $x^2 + y^2 = 13$

Q.33 The equation of circle which touches the axes of coordinates and the line $\frac{x}{3} + \frac{y}{4} = 1$ and whose centre lies in the first quadrant is $x^2 + y^2 - 2cx - 2cy + c^2 = 0$, where c is-

(A) 2 (B) 0

(C) 3 (D) 6

Q.34 For the circle $x^2 + y^2 - 2x + 4y - 4 = 0$, the line $2x - y + 1 = 0$ is a-

(A) chord (B) diameter

(C) tangent line (D) None of these

Q.35 The line y = x + c will intersect the circle $x^2 + y^2 = 1$ in two coincident points, if-

(A) $c = -\sqrt{2}$ (B) $c = \sqrt{2}$

(C) $c = \pm\sqrt{2}$ (D) None of these

Q.36 Centre of a circle is (2, 3). If the line x + y = 1 touches it. Find the equation of circle-

(A) $x^2 + y^2 - 4x - 6y + 5 = 0$

(B) $x^2 + y^2 - 4x - 6y - 4 = 0$

(C) $x^2 + y^2 - 4x - 6y - 5 = 0$

(D) None of these

Q.37 The lines $12x - 5y - 17 = 0$ and $24x - 10y + 44 = 0$ are tangents to the same circle. Then the radius of the circle is-

(A) 1 (B) $1\frac{1}{2}$

(C) 2 (D) None of these

Q.38 If the circle $x^2 + y^2 = a^2$ cuts off a chord of length 2b from the line y = mx + c, then-

(A) $(1-m^2)(a^2 - b^2) = c^2$

(B) $(1+m^2)(a^2 - b^2) = c^2$

(C) $(1-m^2)(a^2 + b^2) = c^2$

(D) None of these

Question based on Equation of Tangent & Normal

Q.39 $\ell x + my + n = 0$ is a tangent line to the circle $x^2 + y^2 = r^2$, if-

(A) $\ell^2 + m^2 = n^2 r^2$ (B) $\ell^2 + m^2 = n^2 + r^2$

(C) $n^2 = r^2(\ell^2 + m^2)$ (D) None of these

Q.40 The equation of the tangent to the circle $x^2 + y^2 = 25$ which is inclined at 60º angle with x-axis, will be-

(A) $y = \sqrt{3}x \pm 10$ (B) $y = \sqrt{3}x \pm 2$

(C) $\sqrt{3}y = x \pm 10$ (D) None of these

Q.41 The gradient of the tangent line at the point $(a\cos\alpha, a\sin\alpha)$ to the circle $x^2 + y^2 = a^2$, is-

(A) $\tan(\pi - \alpha)$ (B) $\tan\alpha$

(C) $\cot\alpha$ (D) $-\cot\alpha$

Q.42 If y = c is a tangent to the circle $x^2 + y^2 - 2x + 2y - 2 = 0$ at (1, 1), then the value of c is-

(A) 1 (B) 2

(C) –1 (D) – 2

Q.43 Line 3x + 4y = 25 touches the circle $x^2 + y^2 = 25$ at the point-

(A) (4, 3) (B) (3, 4)

(C) (–3,–4) (D) None of these

Q.44 The equations of the tangents drawn from the point (0, 1) to the circle $x^2 + y^2 - 2x + 4y = 0$ are-
(A) $2x - y + 1 = 0,\ x + 2y - 2 = 0$
(B) $2x - y - 1 = 0,\ x + 2y - 2 = 0$
(C) $2x - y + 1 = 0,\ x + 2y + 2 = 0$
(D) $2x - y - 1 = 0,\ x + 2y + 2 = 0$

Q.45 The tangent lines to the circle $x^2 + y^2 - 6x + 4y = 12$ which are parallel to the line $4x + 3y + 5 = 0$ are given by-
(A) $4x + 3y - 7 = 0,\ 4x + 3y + 15 = 0$
(B) $4x + 3y - 31 = 0,\ 4x + 3y + 19 = 0$
(C) $4x + 3y - 17 = 0,\ 4x + 3y + 13 = 0$
(D) None of these

Q.46 The equations of tangents to the circle $x^2 + y^2 - 22x - 4y + 25 = 0$ which are perpendicular to the line $5x + 12y + 8 = 0$ are-
(A) $12x - 5y + 8 = 0,\ 12x - 5y = 252$
(B) $12x - 5y - 8 = 0,\ 12x - 5y + 252 = 0$
(C) $12x - 5y = 0,\ 12x - 5y = 252$
(D) None of these

Q.47 The equation of the normal to the circle $x^2 + y^2 = 9$ at the point $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ is-
(A) $x - y = \frac{\sqrt{2}}{3}$ (B) $x + y = 0$
(C) $x - y = 0$ (D) None of these

Q.48 The equation of the normal at the point (4, –1) of the circle $x^2 + y^2 - 40x + 10y = 153$ is-
(A) $x + 4y = 0$ (B) $4x + y = 3$
(C) $x - 4y = 0$ (D) $4x - y = 0$

Q.49 The equation of the normal to the circle $x^2 + y^2 - 8x - 2y + 12 = 0$ at the points whose ordinate is – 1, will be-
(A) $2x - y - 7 = 0,\ 2x + y - 9 = 0$
(B) $2x + y - 7 = 0,\ 2x + y + 9 = 0$
(C) $2x + y + 7 = 0,\ 2x + y + 9 = 0$
(D) $2x - y + 7 = 0,\ 2x - y + 9 = 0$

Q.50 The line $ax + by + c = 0$ is a normal to the circle $x^2 + y^2 = r^2$. The portion of the line $ax + by + c = 0$ intercepted by this circle is of length-
(A) r^2 (B) r
(C) $2r$ (D) $\sqrt{r}$

Question based on Length of Tangent & Pair of Tangents

Q.51 If the length of tangent drawn from the point (5,3) to the circle $x^2 + y^2 + 2x + ky + 17 = 0$ is 7, then k =
(A) – 6 (B) – 4 (C) 4 (D) 13/2

Q.52 The length of tangent from the point (5, 1) to the circle $x^2 + y^2 + 6x - 4y - 3 = 0$, is-
(A) 29 (B) 81
(C) 7 (D) 21

Q.53 The length of the tangent drawn from the point (2, 3) to the circle $2(x^2 + y^2) - 7x + 9y - 11 = 0$
(A) 18 (B) 14 (C) $\sqrt{14}$ (D) $\sqrt{28}$

Q.54 If the lengths of the tangents drawn from the point (1, 2) to the circles $x^2 + y^2 + x + y - 4 = 0$ and $3x^2 + 3y^2 - x - y + k = 0$ be in the ratio 4 : 3, then k =
(A) 21/2 (B) 7/2 (C) –21/4 (D) 7/4

Q.55 A pair of tangents are drawn from the origin to the circle $x^2 + y^2 + 20(x + y) + 20 = 0$. The equation of the pair of tangents is-
(A) $x^2 + y^2 + 5xy = 0$ (B) $x^2 + y^2 + 10xy = 0$
(C) $2x^2 + 2y^2 + 5xy = 0$ (D) $2x^2 + 2y^2 - 5xy = 0$

Q.56 If the equation of one tangent to the circle with centre at (2, –1) from the origin is $3x + y = 0$, then the equation of the other tangent through the origin is-
(A) $x + 3y = 0$ (B) $3x - y = 0$
(C) $x - 3y = 0$ (D) $x + 2y = 0$

Q.57 The equation of the pair of tangents drawn to the circle $x^2 + y^2 - 2x + 4y + 3 = 0$ from (6, –5) is-
(A) $7x^2 + 23y^2 + 30xy + 66x + 50y - 73 = 0$
(B) $7x^2 + 23y^2 - 30xy - 66x - 50y + 73 = 0$
(C) $7x^2 + 23y^2 + 30xy - 66x - 50y - 73 = 0$
(D) None of these

Q.58 The angle between the tangents drawn from the origin to the circle $(x-7)^2 + (y + 1)^2 = 25$ is-
(A) $\pi/3$ (B) $\pi/6$
(C) $\pi/2$ (D) $\pi/8$

Question based on Chord of Contact

Q.59 The equation of the chord of contact of the circle $x^2 + y^2 + 4x + 6y - 12 = 0$ with respect to the point (2, –3) is-
(A) $4x = 17$ (B) $4x + y = 17$
(C) $4y = 17$ (D) None of these

Q.60 The equation of the chord of contact, if the tangents are drawn from the point (5, –3) to the circle $x^2 + y^2 = 10$, is-
(A) $5x - 3y = 10$ (B) $3x + 5y = 10$
(C) $5x + 3y = 10$ (D) $3x - 5y = 10$

Question based on Director Circle

Q.61 The equation of director circle to the circle $x^2 + y^2 = 8$ is-
(A) $x^2 + y^2 = 8$ (B) $x^2 + y^2 = 16$
(C) $x^2 + y^2 = 4$ (D) $x^2 + y^2 = 12$

Q.62 Two perpendicular tangents to the circle $x^2 + y^2 = a^2$ meet at P. Then the locus of P has the equation-
(A) $x^2 + y^2 = 2a^2$ (B) $x^2 + y^2 = 3a^2$
(C) $x^2 + y^2 = 4a^2$ (D) None of these

Question based on Position of Two Circle

Q.63 Consider the circle $x^2 + (y - 1)^2 = 9$, $(x - 1)^2 + y^2 = 25$. They are such that-
(A) each of these circles lies outside the other
(B) one of these circles lies entirely inside the other
(C) these circles touch each other
(D) they intersect in two points

Q.64 Circles $x^2 + y^2 - 2x - 4y = 0$ and $x^2 + y^2 - 8y - 4 = 0$
(A) touch each other internally
(B) cuts each other at two points
(C) touch each other externally
(D) None of these

Q.65 The number of common tangents of the circle $x^2 + y^2 - 2x - 1 = 0$ and $x^2 + y^2 - 2y - 7 = 0$ is-
(A) 1 (B) 3
(C) 2 (D) 4

Q.66 If the circles $x^2 + y^2 + 2x - 8y + 8 = 0$ and $x^2 + y^2 + 10x - 2y + 22 = 0$ touch each other, their point of contact is-
(A) $\left(-\frac{17}{5}, \frac{11}{5}\right)$ (B) $\left(\frac{11}{3}, 2\right)$
(C) $\left(\frac{17}{5}, \frac{11}{5}\right)$ (D) $\left(-\frac{11}{3}, 2\right)$

Q.67 For the given circles $x^2 + y^2 - 6x - 2y + 1 = 0$ and $x^2 + y^2 + 2x - 8y + 13 = 0$, which of the following is true-
(A) one circle lies completely outside the other
(B) one circle lies inside the other
(C) two circle intersect in two points
(D) they touch each other

Q.68 If circles $x^2 + y^2 = r^2$ and $x^2 + y^2 - 20x + 36 = 0$ intersect at real and different points, then-
(A) $r < 2$ and $r > 18$ (B) $2 < r < 18$
(C) $r = 2$ and $r = 18$ (D) None of these

Q.69 The number of common tangents that can be drawn to the circles $x^2 + y^2 - 4x - 6y - 3 = 0$ and $x^2 + y^2 + 2x + 2y + 1 = 0$ is-
(A) 1 (B) 2 (C) 3 (D) 4

Question based on Equation of a chord whose middle point is given

Q.70 Find the locus of mid point of chords of circle $x^2 + y^2 = 25$ which subtends right angle at origin-
(A) $x^2 + y^2 = 25/4$ (B) $x^2 + y^2 = 5$
(C) $x^2 + y^2 = 25/2$ (D) $x^2 + y^2 = 5/2$

Q.71 The equation to the chord of the circle $x^2 + y^2 = 16$ which is bisected at (2, – 1) is-
(A) $2x + y = 16$ (B) $2x - y = 16$
(C) $x + 2y = 5$ (D) $2x - y = 5$

Q.72 The equation of the chord of the circle $x^2 + y^2 - 6x + 8y = 0$ which is bisected at the point (5, –3) is-
(A) $2x - y + 7 = 0$ (B) $2x + y - 7 = 0$
(C) $2x + y + 7 = 0$ (D) $2x - y - 7 = 0$

Question based on Circle through the Point of Intersection

Q.73 The equation of the circle passing through the point (1, 1) and through the point of intersection of circles $x^2 + y^2 + 13x - 3y = 0$ and $2x^2 + 2y^2 + 4x - 7y - 25 = 0$ is-
(A) $4x^2 + 4y^2 - 17x - 10y + 25 = 0$
(B) $4x^2 + 4y^2 + 30x - 13y - 25 = 0$
(C) $4x^2 + 4y^2 - 30x - 10y - 25 = 0$
(D) None of these

Q.74 The equation of circle passing through the points of intersection of circles $x^2 + y^2 = 6$ and $x^2 + y^2 - 6x + 8 = 0$ and the point (1, 1) is-
(A) $x^2 + y^2 - 4y + 2 = 0$
(B) $x^2 + y^2 - 3x + 1 = 0$
(C) $x^2 + y^2 - 6x + 4 = 0$
(D) None of these

Q.75 The equation of the circle whose diameter is the common chord of the circles $x^2 + y^2 + 3x + 2y + 1 = 0$ and $x^2 + y^2 + 3x + 4y + 2 = 0$ is-
(A) $x^2 + y^2 + 3x + y + 5 = 0$
(B) $x^2 + y^2 + x + 3y + 7 = 0$
(C) $x^2 + y^2 + 2x + 3y + 1 = 0$
(D) $2(x^2 + y^2) + 6x + 2y + 1 = 0$

Question based on Common chord of two Circles

Q.76 The common chord of $x^2 + y^2 - 4x - 4y = 0$ and $x^2 + y^2 = 16$ subtends at the origin an angle equal to-
(A) $\pi/6$ (B) $\pi/4$
(C) $\pi/3$ (D) $\pi/2$

Q.77 The distance from the centre of the circle $x^2 + y^2 = 2x$ to the straight line passing through the points of intersection of the two circles $x^2 + y^2 + 5x - 8y + 1 = 0$, $x^2 + y^2 - 3x + 7y - 25 = 0$ is-
(A) 1 (B) 2
(C) 3 (D) None of these

Q.78 The length of the common chord of the circle $x^2 + y^2 + 4x + 6y + 4 = 0$ and $x^2 + y^2 + 6x + 4y + 4 = 0$ is-
(A) $\sqrt{10}$ (B) $\sqrt{22}$
(C) $\sqrt{34}$ (D) $\sqrt{38}$

Q.79 The length of the common chord of circle $x^2 + y^2 - 6x - 16 = 0$ and $x^2 + y^2 - 8y - 9 = 0$ is-
(A) $10\sqrt{3}$ (B) $5\sqrt{3}$
(C) $5\sqrt{3}/2$ (D) None of these

Q.80 Length of the common chord of the circles $x^2 + y^2 + 5x + 7y + 9 = 0$ and $x^2 + y^2 + 7x + 5y + 9 = 0$ is-
(A) 8 (B) 9 (C) 7 (D) 6

Question based on Angle of intersection of two Circles

Q.81 Two given circles $x^2 + y^2 + ax + by + c = 0$ and $x^2 + y^2 + dx + ey + f = 0$ will intersect each other orthogonally, only when-
(A) ad + be = c + f
(B) a + b + c = d + e + f
(C) ad + be = 2c + 2f
(D) 2ad + 2be = c + f

Q.82 If the circles of same radius a and centres at (2, 3) and (5, 6) cut orthogonally, then a is equal to-
(A) 6 (B) 4 (C) 3 (D) 10

Q.83 The angle of intersection of circles $x^2 + y^2 + 8x - 2y - 9 = 0$ and $x^2 + y^2 - 2x + 8y - 7 = 0$ is -
(A) 60º (B) 90º (C) 45º (D) 30º

Q.84 The angle of intersection of two circles is 0º if -
(A) they are separate
(B) they intersect at two points
(C) they intersect only at a single point
(D) it is not possible

Q.85 If a circle passes through the point (1, 2) and cuts the circle $x^2 + y^2 = 4$ orthogonally, then the equation of the locus of its centre is -
(A) $x^2 + y^2 - 2x - 6y - 7 = 0$
(B) $x^2 + y^2 - 3x - 8y + 1 = 0$
(C) $2x + 4y - 9 = 0$
(D) $2x + 4y - 1 = 0$

Q.86 The equation of the circle which passes through the origin has its centre on the line x + y = 4 and cuts the circle $x^2 + y^2 - 4x + 2y + 4 = 0$ orthogonally, is -
(A) $x^2 + y^2 - 2x - 6y = 0$
(B) $x^2 + y^2 - 6x - 3y = 0$
(C) $x^2 + y^2 - 4x - 4y = 0$
(D) None of these

LEVEL-2

Q.1 If θ is the angle subtended at $P(x_1, y_1)$ by the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ then -

(A) $\tan\theta = \frac{2\sqrt{g^2+f^2-c}}{\sqrt{S_1}}$

(B) $\cot\frac{\theta}{2} = \frac{\sqrt{S_1}}{\sqrt{g^2+f^2-c}}$

(C) $\cot\theta = \frac{\sqrt{S_1}}{\sqrt{g^2+f^2-c}}$

(D) None of these

Q.2 The circle $(x-2)^2 + (y-5)^2 = a^2$ will be inside the circle $(x-3)^2 + (y-6)^2 = b^2$ if -

(A) $b > a + \sqrt{2}$ (B) $a < \sqrt{2} - b$

(C) $a - b < \sqrt{2}$ (D) $a + b > \sqrt{2}$

Q.3 If the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ cut the coordinate axes in concyclic points, then -

(A) $a_1 a_2 = b_1 b_2$ (B) $a_1b_1 = a_2b_2$

(C) $a_1 b_2 = a_2 b_1$ (D) None of these

Q.4 Four distinct points (2k, 3k), (1, 0), (0, 1) and (0,0) lie on a circle for -

(A) All integral values of k

(B) $0 < k < 1$

(C) $k < 0$

(D) 5/13

Q.5 The circle $x^2 + y^2 + 2gx + 2fy + c = 0$ bisects the circumference of the circle

$x^2 + y^2 + 2ax + 2by + d = 0$, then -

(A) $2a(g - a) + 2b(f - b) = c - d$

(B) $2a(g + a) + 2b(f + b) = c + d$

(C) $2g(g - a) + 2f(f - b) = d - c$

(D) $2g(g + a) + 2f(f + b) = c + d$

Q.6 Three equal circles each of radius r touch one another. The radius of the circle which touching by all the three given circles internally is -

(A) $(2 + \sqrt{3})\, r$ (B) $\frac{(2+\sqrt{3})}{\sqrt{3}}\, r$

(C) $\frac{(2-\sqrt{3})}{\sqrt{3}}\, r$ (D) $(2 - \sqrt{3})\, r$

Q.7 The equation of the in-circle of the triangle formed by the axes and the line $4x + 3y = 6$ is -

(A) $x^2 + y^2 - 6x - 6y + 9 = 0$

(B) $4(x^2 + y^2 - x - y) + 1 = 0$

(C) $4(x^2 + y^2 + x + y) + 1 = 0$

(D) None of these

Q.8 The equation of circle passing through the points of intersection of circle $x^2 + y^2 = 6$ and $x^2 + y^2 - 6x + 8 = 0$ and the point (1, 1) is -

(A) $x^2 + y^2 - 3x + 1 = 0$

(B) $x^2 + y^2 - 6x + 4 = 0$

(C) $x^2 + y^2 - 4y + 2 = 0$

(D) none of these

Q.9 If the two circles $(x-1)^2 + (y-3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points then -

(A) $2 < r < 8$ (B) $r < 2$

(C) $r = 2, r = 8$ (D) $r > 2$

Q.10 If from any point P on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, tangents are drawn to the circle $x^2 + y^2 + 2gx + 2fy + c\sin^2\alpha + (g^2 + f^2)\cos^2\alpha = 0$, then the angle between the tangents is -

(A) α (B) 2α

(C) $\alpha / 2$ (D) None of these

Q.11 The circles whose equations are $x^2 + y^2 + c^2 = 2ax$ and $x^2 + y^2 + c^2 - 2by = 0$ will touch one another externally if -

(A) $\frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{a^2}$ (B) $\frac{1}{c^2} + \frac{1}{a^2} = \frac{1}{b^2}$

(C) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$ (D) None of these

Q.12 The possible values of p for which the line $x\cos\alpha + y\sin\alpha = p$ is a tangent to the circle $x^2 + y^2 - 2qx\cos\alpha - 2qy\sin\alpha = 0$ is/are -

(A) q and 2q (B) 0 and q

(C) 0 and 2q (D) q

Q.13 The length of the tangent drawn from any point on the circle $x^2 + y^2 + 2gx + 2fy + \alpha = 0$ to the circle $x^2 + y^2 + 2gx + 2fy + \beta = 0$ is -

(A) $\sqrt{\beta-\alpha}$ (B) $\sqrt{\alpha\beta}$

(C) $\sqrt{\alpha-\beta}$ (D) $\sqrt{(\alpha/\beta)}$

Q.14 The locus of centre of the circle which cuts the circle $x^2 + y^2 = k^2$ orthogonally and passes through the point (p,q) is -

(A) $2px + 2qy - (p^2 + q^2 + k^2) = 0$

(B) $x^2 + y^2 - 3px - 4qy - (p^2 + q^2 - k^2) = 0$

(C) $2px + 2qy - (p^2 - q^2 + k^2) = 0$

(D) $x^2 + y^2 - 2px - 3qy - (p^2 - q^2 - k^2) = 0$

Q.15 If the line $(x + g) \cos \theta + (y + f) \sin \theta = k$ touches the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, then -

(A) $g^2 + f^2 = k^2 + c^2$ (B) $g^2 + f^2 = k + c$

(C) $g^2 + f^2 = k^2 + c$ (D) None of these

Q.16 The locus of the point which moves so that the lengths of the tangents from it to two given concentric circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2$ are inversely as their radii has equation -

(A) $x^2 + y^2 = (a + b)^2$

(B) $x^2 + y^2 = a^2 + b^2$

(C) $(a^2 + b^2)(x^2 + y^2) = 1$

(D) $x^2 + y^2 = a^2 - b^2$

Q.17 The equation of the circle which passes through (1, 0) and (0, 1) and has its radius as small as possible, is -

(A) $2x^2 + 2y^2 - 3x - 3y + 1 = 0$

(B) $x^2 + y^2 - x - y = 0$

(C) $x^2 + y^2 - 2x - 2y + 1 = 0$

(D) $x^2 + y^2 - 3x - 3y + 2 = 0$

Q.18 The distance between the chords of contact of the tangents to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ from the origin and from the point (g, f) is -

(A) $g^2 + f^2$ (B) $\frac{1}{2}(g^2 + f^2 + c)$

(C) $\frac{1}{2}\frac{g^2+f^2+c}{\sqrt{g^2+f^2}}$ (D) $\frac{1}{2}\frac{g^2+f^2-c}{\sqrt{g^2+f^2}}$

Q.19 The area of the triangle formed by the tangents from the points (h, k) to the circle $x^2 + y^2 = a^2$ and the line joining their points of contact is -

(A) $a\frac{(h^2+k^2-a^2)^{3/2}}{h^2+k^2}$ (B) $a\frac{(h^2+k^2-a^2)^{1/2}}{h^2+k^2}$

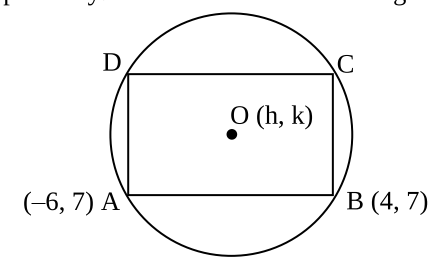
(C) $\frac{(h^2+k^2-a^2)^{3/2}}{h^2+k^2}$ (D) $\frac{(h^2+k^2-a^2)^{1/2}}{h^2+k^2}$

Q.20 Tangents drawn from origin to the circle $x^2 + y^2 - 2ax - 2by + b^2 = 0$ are perpendicular to each other, if -

(A) $a - b = 1$ (B) $a + b = 1$

(C) $a^2 = b^2$ (D) $a^2 + b^2 = 1$

Q.21 A rectangle ABCD is inscribed in a circle with a diameter lying along the line 3y = x + 10. If A and B are the points (–6, 7) and (4, 7) respectively. Find the area of the rectangle -



(A) 40 (B) 80 (C) 20 (D) 160

Q.22 If P is a point such that the ratio of the squares of the lengths of the tangents from P to the circles $x^2 + y^2 + 2x - 4y - 20 = 0$ and $x^2 + y^2 - 4x + 2y - 44 = 0$ is 2 : 3 then the locus of P is a circle with centre

(A)(7, – 8) (B) (–7, 8)

(C) (7, 8) (D) (–7, –8)

Q.23 Consider four circles $(x \pm 1)^2 + (y \pm 1)^2 = 1$, then the equation of smaller circle touching these four circle is

(A) $x^2 + y^2 = 3 - \sqrt{2}$ (B) $x^2 + y^2 = 6 - 3\sqrt{2}$

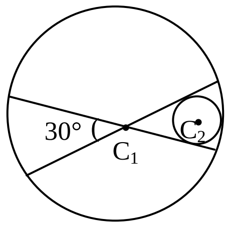
(C) $x^2 + y^2 = 5 - 2\sqrt{2}$ (D) $x^2 + y^2 = 3 - 2\sqrt{2}$

Q.24 In a system of three curves C_1, C_2 and C_3. C_1 is a circle whose equation is $x^2 + y^2 = 4$. C_2 is the locus of the point of intersection of orthogonal tangents drawn on C_1. C_3 is the locus of the point of intersection of perpendicular tangents drawn on C_2. Area enclosed between the curve C_2 and C_3 is-

(A) 8π sq. units (B) 16π sq. units

(C) 32π sq. units (D) None of these

Q.25 Consider the figure and find radius of bigger circle. C_1 is centre of bigger circle and radius of smaller circle is unity-



(A) $1 + \sqrt{2} - \sqrt{6}$ (B) $\sqrt{2} + \sqrt{3}$

(C) $-1 + \sqrt{2} + \sqrt{6}$ (D) $1 + \sqrt{2} + \sqrt{6}$

Q.26 Locus of centre of circle touching the straight lines $3x + 4y = 5$ and $3x + 4y = 20$ is -

(A) $3x + 4y = 15$ (B) $6x + 8y = 15$

(C) $3x + 4y = 25$ (D) $6x + 8y = 25$

Q.27 If $(-3, 2)$ lies on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ which is concentric with the circle $x^2 + y^2 + 6x + 8y - 5 = 0$, then c is -

(A) 11 (B) – 11

(C) 24 (D) None of these

Q.28 The locus of the centre of a circle of radius 2 which rolls on the outside of the circle $x^2 + y^2 + 3x - 6y - 9 = 0$ is

(A) $x^2 + y^2 + 3x - 6y + 5 = 0$

(B) $x^2 + y^2 + 3x - 6y - 31 = 0$

(C) $x^2 + y^2 + 3x - 6y + \frac{29}{4} = 0$

(D) $x^2 + y^2 + 3x - 6y - 5 = 0$

Q.29 Equation of a circle whose centre is origin and radius is equal to the distance between the lines $x = 1$ and $x = -1$ is

(A) $x^2 + y^2 = 1$ (B) $x^2 + y^2 = \sqrt{2}$

(C) $x^2 + y^2 = 4$ (D) $x^2 + y^2 = -4$

LEVEL-3

Q.1 If the circle $x^2 + y^2 + 2x - 4y - k = 0$ is midway between two circles
$x^2 + y^2 + 2x - 4y - 4 = 0$ and
$x^2 + y^2 + 2x - 4y - 20 = 0$, then K =
(A) 8 (B) 9 (C) 11 (D) 12

Q.2 Equation of circle touching the lines $|x| + |y| = 4$ is -
(A) $x^2 + y^2 = 12$ (B) $x^2 + y^2 = 16$
(C) $x^2 + y^2 = 4$ (D) $x^2 + y^2 = 8$

Q.3 One possible equation of the chord of $x^2 + y^2 = 100$ that passes through (1, 7) and subtends an angle $\frac{2\pi}{3}$ at origin is -
(A) $3y + 4x - 25 = 0$ (B) $x + y - 8 = 0$
(C) $3x + 4y - 31 = 0$ (D) None of these

Q.4 A circle C_1 of unit radius lies in the first quadrant and touches both the co-ordinate axes. The radius of the circle which touches both the co-ordinate axes and cuts C_1 so that common chord is longest -
(A) 1 (B) 2 (C) 3 (D) 4

Q.5 From a point P tangent is drawn to the circle $x^2 + y^2 = a^2$ and a tangent is drawn to $x^2 + y^2 = b^2$. If these tangent are perpendicular, then locus of P is -
(A) $x^2 + y^2 = a^2 + b^2$ (B) $x^2 + y^2 = a^2 - b^2$
(C) $x^2 + y^2 = (ab)^2$ (D) $x^2 + y^2 = a + b$

Q.6 A circle is inscribed in an equilateral triangle of side 6. Find the area of any square inscribed in the circle -
(A) 36 (B) 12 (C) 6 (D) 9

Q.7 The tangent at any point to the circle $x^2 + y^2 = r^2$ meets the coordinate axes at A and B. If lines drawn parallel to the coordinate axes through A and B intersect at P, the locus of P is
(A) $x^2 + y^2 = r^{-2}$ (B) $x^{-2} + y^{-2} = r^2$
(C) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{r^2}$ (D) $\frac{1}{x^2} - \frac{1}{y^2} = \frac{1}{r^2}$

Q.8 If $(a \cos \theta_i, a \sin \theta_i) i = 1, 2, 3$ represent the vertices of an equilateral triangle inscribed in a circle, then -
(A) $\cos \theta_1 + \cos \theta_2 + \cos \theta_3 = 0$
(B) $\sin \theta_1 + \sin \theta_2 + \sin \theta_3 \neq 0$
(C) $\tan \theta_1 + \tan \theta_2 + \tan \theta_3 = 0$
(D) $\cot \theta_1 + \cot \theta_2 + \cot \theta_3 = 0$

Q.9 Of the two concentric circles the smaller one has the equation $x^2 + y^2 = 4$. If each of the two intercepts on the line $x + y = 2$ made between the two circles is 1, the equation of the larger circle is -
(A) $x^2 + y^2 = 5$ (B) $x^2 + y^2 = 5 + 2\sqrt{2}$
(C) $x^2 + y^2 = 7 + 2\sqrt{2}$ (D) $x^2 + y^2 = 11$

Q.10 A point on the line x = 3 from which tangent drawn to the circle $x^2 + y^2 = 8$ are at right angles -
(A) $(3, \sqrt{7})$ (B) $(3, \sqrt{23})$
(C) $(3, -\sqrt{23})$ (D) None of these

Q.11 If the equation of the in-circle of an equilateral triangle is $x^2 + y^2 + 4x - 6y + 4 = 0$, then equation of circum-circle of the triangle is-

(A) $x^2 + y^2 + 4x + 6y - 23 = 0$

(B) $x^2 + y^2 + 4x - 6y - 23 = 0$

(C) $x^2 + y^2 - 4x - 6y - 23 = 0$

(D) None of these

Q.12 The angle between tangents drawn from a point P to the circle $x^2 + y^2 + 4x - 2y - 4 = 0$ is 60°. Then locus of P is -

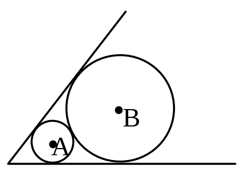
(A) $x^2 + y^2 + 4x - 2y - 31 = 0$

(B) $x^2 + y^2 + 4x - 2y - 21 = 0$

(C) $x^2 + y^2 + 4x - 2y - 11 = 0$

(D) $x^2 + y^2 + 4x - 2y = 0$

Q.13 A circle with centre A and radius 7 is tangent to the sides of an angle of 60°. A larger circle with centre B is tangent to the sides of the angle and to the first circle. The radius of the larger circle is



(A) $30\sqrt{3}$ (B) 21

(C) $20\sqrt{3}$ (D) 30

Assertion-Reason Type Question

The following questions (Q. 14 to 23) given below consist of an "Assertion" Statement-(1) and "Reason " Statement- (2) Type questions. Use the following key to choose the appropriate answer.

(A) Both Statement- (1) and Statement- (2) are true and Statement- (2) is the correct explanation of Statement- (1)

(B) Both Statement- (1) and Statement- (2) are true but Statement- (2) is not the correct explanation of Statement- (1)

(C) Statement- (1) is true but Statement- (2) is false

(D) Statement- (1) is false but Statement- (2) is true

Q. 14 **Statement (1):** Two points A(10, 0) and O(0, 0) are given and a circle $x^2 + y^2 - 6x + 8y - 11 = 0$. The circle always cuts the line segments OA.

Statement (2) : The centre of the circle, point A and the point O are not collinear.

Q.15 **Statement (1) :** If a line L = 0 is a tangents to the circle S = 0 then it will also be a tangent to the circle $S + \lambda L = 0$.

Statement (2) : If a line touches a circles then perpendicular distance from centre of the circle on the line must be equal to the radius.

Q.16 Consider the following statements:-

Statement (1): The circle $x^2 + y^2 = 1$ has exactly two tangents parallel to the x-axis

Statement (2): $\frac{dy}{dx} = 0$ on the circle exactly at the point (0, ±1).

Q.17 **Statement (1):** The equation of chord of the circle $x^2 + y^2 - 6x + 10y - 9 = 0$, which is bisected at (–2, 4) must be $x + y - 2 = 0$.

Statement (2) : In notations the equation of the chord of the circle S = 0 bisected at (x_1, y_1) must be $T = S_1$.

Q.18 **Statement (1):** If two circles $x^2 + y^2 + 2gx + 2fy = 0$ and $x^2 + y^2 + 2g'x + 2f'y = 0$ touch each other then $f'g = fg'$.

Statement (2) : Two circle touch each other, if line joining their centres is perpendicular to all possible common tangents.

Q.19 **Statement (1):** If a circle passes through points of intersection of co-ordinate axes with the lines $\lambda x - y + 1 = 0$ and $x - 2y + 3 = 0$ then value of λ is 2.

Statement (2): If lines $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$ intersects. Coordinate axes at concyclic points then $\frac{a_1}{a_2} = \frac{b_1}{b_2}$.

Q.20 **Statement (1):** Equation of circle passing through two points (2, 0) and (0, 2) and having least area is $x^2 + y^2 - 2x - 2y = 0$.

Statement (2): The circle of smallest radius passing through two given points A and B must be of radius $\frac{AB}{2}$.

Q.21 Tangents are drawn from the point (2, 3) to the circle $x^2 + y^2 = 9$, then

Statement (1): Tangents are mutually perpendicular.

Statement (2): Locus of point of intersection of perpendicular tangents is $x^2 + y^2 = 18$.

Q.22 Let 'θ' is the angle of intersection of two circles with centres C_1 and C_2 and radius r_1 and r_2 respectively then.

Statement (1): If $\cos\theta = \pm 1$ then, the circles touch each other.

Statement (2): Two circles touch each other if $|C_1C_2| = |r_1 \pm r_2|$

Q.23 **Statement (1):** The locus of mid point of chords of circle $x^2 + y^2 = a^2$ which are making right angle at centre is $x^2 + y^2 = \frac{a^2}{2}$.

Statement (2): The locus of mid point of chords of circle $x^2 + y^2 - 2x = 0$ which passes through origin is $x^2 + y^2 - x = 0$.

Passage I (Question 24 to 26)

Let C_1, C_2 are two circles each of radius 1 touching internally the sides of triangles POA_1, PA_1A_2 respectively where $P \equiv (0, 4)$ O is origin, A_1, A_2 are the points on positive x-axis.

On the basis of above passage, answer the following questions:

Q.24 Angle subtended by circle C_1 at P is-

(A) $\tan^{-1}\frac{2}{3}$ (B) $2\tan^{-1}\frac{2}{3}$

(C) $\tan^{-1}\frac{3}{4}$ (D) $2\tan^{-1}\frac{3}{4}$

Q.25 Centre of circle C_2 is-

(A) (3, 1) (B) $(3\frac{1}{2}, 1)$

(C) $(3\frac{3}{4}, 1)$ (D) None of these

Q.26 Length of tangent from P to circle C_2-

(A) 4 (B) $\frac{9}{2}$

(C) 5 (D) $\frac{19}{4}$

Passage II (Question 27 to 29)

Two circles $S_1 : x^2 + y^2 - 2x - 2y - 7 = 0$ and $S_2 : x^2 + y^2 - 4x - 4y - 1 = 0$ intersects each other at A and B.

On the basis of above passage, answer the following questions:

Q.27 Length of AB is-

(A) 6 (B) $\sqrt{33}$

(C) $\sqrt{34}$ (D) $\sqrt{35}$

Q.28 Equation of circle passing through A and B whose AB is diameter-

(A) $x^2 + y^2 - 3x - 3y - 5 = 0$

(B) $x^2 + y^2 - 3x - 3y - 4 = 0$

(C) $x^2 + y^2 + 3x + 3y - 4 = 0$

(D) $x^2 + y^2 + 3x + 3y - 5 = 0$

Q.29 Mid point of AB is-

(A) $\left(\frac{5}{2}, \frac{1}{2}\right)$ (B) $\left(\frac{3}{2}, \frac{3}{2}\right)$

(C) (2, 1) (D) (1, 2)

Passage-III (Question 30 to 32)

To the circle $x^2 + y^2 = 4$ two tangents are drawn from P(–4, 0), which touches the circle at A and B and a rhombus PA P′B is completed.

On the basis of above passage, answer the following questions :

Q. 30 Circumcentre of the triangle PAB is at

(A) (–2, 0) (B) (2, 0)

(C) $\left(\frac{\sqrt{3}}{2}, 0\right)$ (D) None of these

Q.31 Ratio of the area of triangle PAP′ to that of P′AB is

(A) 2 : 1 (B) 1 : 2

(C) $\sqrt{3} : 2$ (D) None of these

Q.32 If P is taken to be at (h, 0) such that P′ lies on the circle, the area of the rhombus, is

(A) $6\sqrt{3}$ (B) $2\sqrt{3}$

(C) $3\sqrt{3}$ (D) None of these

LEVEL- 4

(Question asked in previous AIEEE and IIT-JEE)

Section –A

Q.1 The square of the length of tangent from (3, – 4) on the circle $x^2 + y^2 - 4x - 6y + 3 = 0$- **[AIEEE-2002]**

(A) 20 (B) 30 (C) 40 (D) 50

Q.2 If the two circles $(x-1)^2 + (y - 3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then **[AIEEE-2003]**

(A) $r > 2$ (B) $2 < r < 8$

(C) $r < 2$ (D) $r = 2$

Q.3 The lines $2x - 3y = 5$ and $3x - 4y = 7$ are diameters of a circle having area as 154 sq. units. Then the equation of the circle is - **[AIEEE-2003]**

(A) $x^2 + y^2 - 2x + 2y = 62$

(B) $x^2 + y^2 + 2x - 2y = 62$

(C) $x^2 + y^2 + 2x - 2y = 47$

(D) $x^2 + y^2 - 2x + 2y = 47$

Q.4 If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = 4$ orthogonally, then the locus of its centre is- **[AIEEE-2004]**

(A) $2ax + 2by + (a^2 + b^2 + 4) = 0$

(B) $2ax + 2by - (a^2 + b^2 + 4) = 0$

(C) $2ax - 2by + (a^2 + b^2 + 4) = 0$

(D) $2ax - 2by - (a^2 + b^2 + 4) = 0$

Q.5 A variable circle passes through the fixed point A(p, q) and touches x- axis. The locus of the other end of the diameter through A is- **[AIEEE-2004]**

(A) $(x - p)^2 = 4qy$ (B) $(x - q)^2 = 4py$

(C) $(y - p)^2 = 4qx$ (D) $(y - q)^2 = 4px$

Q.6 If the lines $2x + 3y + 1 = 0$ and $3x - y - 4 = 0$ lie along diameters of a circle of circumference 10π, then the equation of the circle is- **[AIEEE-2004]**

(A) $x^2 + y^2 - 2x + 2y - 23 = 0$

(B) $x^2 + y^2 - 2x - 2y - 23 = 0$

(C) $x^2 + y^2 + 2x + 2y - 23 = 0$

(D) $x^2 + y^2 + 2x - 2y - 23 = 0$

Q.7 If the circles $x^2 + y^2 + 2ax + cy + a = 0$ and $x^2 + y^2 - 3ax + dy - 1 = 0$ intersect in two distinct point P and Q then the line $5x + by - a = 0$ passes through P and Q for - **[AIEEE-2005]**

(A) exactly one value of a

(B) no value of a

(C) infinitely many values of a

(D) exactly two values of a

Q.8 A circle touches the x-axis and also touches the circle with centre at (0, 3) and radius 2. The locus of the centre of the circle is- **[AIEEE-2005]**

(A) an ellipse (B) a circle

(C) a hyperbola (D) a parabola

Q.9 If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = p^2$ orthogonally, then the equation of the locus of its centre is - **[AIEEE-2005]**

(A) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - p^2) = 0$

(B) $2ax + 2by - (a^2 - b^2 + p^2) = 0$

(C) $x^2 + y^2 - 2ax - 3by + (a^2 - b^2 - p^2) = 0$

(D) $2ax + 2by - (a^2 + b^2 + p^2) = 0$

Q.10 If the pair of lines $ax^2 + 2(a + b)xy + by^2 = 0$ lie along diameters of a circle and divide the circle into four sectors such that the area of one of the sectors is thrice the area of another sector then – **[AIEEE-2005]**

(A) $3a^2 – 10ab + 3b^2 = 0$ (B) $3a^2 – 2ab + 3b^2 = 0$
(C) $3a^2 + 10ab + 3b^2 = 0$ (D) $3a^2 + 2ab + 3b^2 = 0$

Q.11 If the lines $3x – 4y – 7 = 0$ and $2x – 3y – 5 = 0$ are two diameters of a circle of area 49π square units, the equation of the circle is– **[AIEEE-2006]**

(A) $x^2 + y^2 + 2x – 2y – 62 = 0$
(B) $x^2 + y^2 – 2x + 2y – 62 = 0$
(C) $x^2 + y^2 – 2x + 2y – 47 = 0$
(D) $x^2 + y^2 + 2x – 2y – 47 = 0$

Q.12 Let C be the circle with centre (0, 0) and radius 3 units. The equation of the locus of the mid points of the chords of the circle C that subtend an angle of $\frac{2\pi}{3}$ at its centre is – **[AIEEE-2006]**

(A) $x^2 + y^2 = 1$ (B) $x^2 + y^2 = \frac{27}{4}$
(C) $x^2 + y^2 = \frac{9}{4}$ (D) $x^2 + y^2 = \frac{3}{2}$

Q.13 Consider a family of circles which are passing through the point (-1, 1) and are tangent to x-axis. If (h, k) are the co-ordinates of the centre of the circles, then the set of values of k is given by the interval- **[AIEEE-2007]**

(A) $0 < k < 1/2$ (B) $k \ge 1/2$
(C) $– 1/2 \le k \le 1/2$ (D) $k \le 1/2$

Q.14 The point diametrically opposite to the point P(1, 0) on the circle $x^2 + y^2 + 2x + 4y –3 = 0$ is - **[AIEEE-2008]**

(A) (–3, 4) (B) (–3, –4)
(C) (3, 4) (D) (3, – 4)

Q.15 If P and Q are the points of intersection of the circles $x^2 + y^2 + 3x + 7y + 2p – 5 = 0$ and $x^2 + y^2 + 2x + 2y – p^2 = 0$, then there is a circle passing through P, Q, and (1, 1) for- **[AIEEE- 2009]**

(A) All except one value of p
(B) All except two values of p
(C) Exactly one value of p
(D) All values of p

Q.16 The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x – 4y = m$ at two distinct points if - **[AIEEE- 2010]**

(A) $– 85 < m < – 35$ (B) $– 35 < m < 15$
(C) $15 < m < 65$ (D) $35 < m < 85$

Q.17 The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2 (c > 0)$ touch each other if - **[AIEEE- 2011]**

(A) $2|a| = c$ (B) $|a| = c$
(C) $a = 2c$ (D) $|a| = 2c$

Q.18 The equation of the circle passing through the point (1, 0) and (0, 1) and having the smallest radius is - **[AIEEE- 2011]**

(A) $x^2 + y^2 – 2x – 2y + 1 = 0$
(B) $x^2 + y^2 – x – y = 0$
(C) $x^2 + y^2 + 2x + 2y – 7 = 0$
(D) $x^2 + y^2 + x + y – 2 = 0$

Section –B

Q.1 The centre of the circle passing through points (0, 0), (1, 0) and touching the circle $x^2 + y^2 = 9$ is **[IIT-1992]**

(A) (3/2, 1/2) (B) (1/2, 3/2)
(C) (1/2, 1/2) (D) $(1/2, –2^{1/2})$

Q.2 The equation of the circle which touches both the axes and the straight line $4x + 3y = 6$ in the first quadrant and lies below it is- **[IIT-1992]**

(A) $4x^2 + 4y^2 – 4x – 4y + 1 = 0$
(B) $x^2 + y^2 – 6x – 6y + 9 = 0$
(C) $x^2 + y^2 – 6x – y + 9 = 0$
(D) $4(x^2 + y^2 – x – 6y) + 1 = 0$

Q.3 The slope of the tangent at the point (h, h) of the circle $x^2 + y^2 = a^2$ is - **[IIT-1993]**

(A) 0 (B) 1
(C) –1 (D) depends on h

Q.4 The intercept on the line y = x by the circle $x^2 + y^2 – 2x = 0$ is AB. Equation of the circle with AB as a diameter is- **[IIT-96/AIEEE -04]**

(A) $x^2 + y^2 + x + y = 0$ (B) $– x^2 + y^2 + x – y = 0$

(C) $x^2 + y^2 - x - y = 0$ (D) None of these

Q.5 If a circle passes thro' the points of intersection of the co - ordinate axes with the lines $\lambda x - y + 1 = 0$ and $x - 2y + 3 = 0$, then the value of λ is- **[IIT-1997]**

(A) 2 (B) 4 (C) 6 (D) 3

Q.6 The number of common tangents to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 6x - 8y = 24$ is **[IIT-1998]**

(A) 0 (B) 1 (C) 3 (D) 4

Q.7 Let L_1 be a straight line passing through the origin and L_2 be the straight line $x + y = 1$. If the intercepts made by the circle $x^2 + y^2 - x + 3y = 0$ on L_1 and L_2 are equal, then which of the following equations can represent L_1 – **[IIT-1999]**

(A) $x + y = 0$ (B) $x - y = 0$

(C) $x + 7y = 0$ (D) None of these

Q.8 If the circles $x^2 + y^2 + 2x + 2ky + 6 = 0$ and $x^2 + y^2 + 2ky + k = 0$ intersect orthogonally, then k is - **[IIT-2000]**

(A) 2 or – 3/2 (B) –2 or –3/2

(C) 2 or 3/2 (D) –2 or 3/2

Q.9 The triangle PQR is inscribed in the circle $x^2 + y^2 = 25$. If Q and R have co-ordinates (3, 4) and (–4, 3) respectively, then angle QPR is equal to – **[IIT-2000]**

(A) $\pi/2$ (B) $\pi/3$

(C) $\pi/4$ (D) $\pi/6$

Q.10 Let PQ and RS be tangents at the extremities of the diameter PR of a circle of radius r. If PS and RQ intersect at a point X on the circumference of the circle, then 2r equals **[IIT-2001]**

(A) $\sqrt{PQ.RS}$ (B) $\frac{PQ+RS}{2}$

(C) $\frac{2PQ.RS}{PQ+RS}$ (D) $\sqrt{\frac{PQ^2+RS^2}{2}}$

Q.11 If the tangent at the point P on the circle $x^2 + y^2 + 6x + 6y = 2$ meets the straight line $5x - 2y + 6 = 0$ at a point Q on the y-axis, then the length of PQ is - **[IIT-2002]**

(A) 4 (B) 2

(C) 5 (D) 3

Q.12 If $a > 2b > 0$ then the positive value of m for which $y = mx - b\sqrt{1+m^2}$ is a common tangent to $x^2 + y^2 = b^2$ and $(x - a)^2 + y^2 = b^2$ is – **[IIT-2002]**

(A) $\frac{2b}{\sqrt{a^2-4b^2}}$ (B) $\frac{\sqrt{a^2-4b^2}}{2b}$

(C) $\frac{2b}{a-2b}$ (D) $\frac{b}{a-2b}$

Q.13 Diameter of the given circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is the chord of another circle C having centre (2, 1), the radius of the circle C is- **[IIT 2004]**

(A) $\sqrt{3}$ (B) 2

(C) 3 (D) 1

Q.14 Locus of the centre of circle touching to the x-axis & the circle $x^2 + (y - 1)^2 = 1$ externally is – **[IIT-2005]**

(A) $\{(0, y) ; y \le 0\} \cup (x^2 = 4y)$

(B) $\{(0, y) ; y \le 0\} \cup (x^2 = y)$

(C) $\{(x, y) ; y \le y\} \cup (x^2 = 4y)$

(D) $\{(0, y) ; y \ge 0\} \cup (x^2 + (y - 1)^2 = 4$

Q.15 Tangents are drawn from the point (17, 7) to the circle $x^2 + y^2 = 169$. **[IIT 2007]**

STATEMENT-1: The tangents are mutually perpendicular.

Because

STATEMENT-2: The locus of the points from which mutually perpendicular tangents can be drawn to given circle is $x^2 + y^2 = 338$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement-1, is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

Q.16 Tangents drawn from the point P(1, 8) to the circle $x^2 + y^2 - 6x - 4y - 11 = 0$ touch the circle at the points A and B. The equation of the circumcircle of the triangle PAB is - **[IIT-2009]**

(A) $x^2 + y^2 + 4x - 6y + 19 = 0$

(B) $x^2 + y^2 - 4x - 10y + 19 = 0$

(C) $x^2 + y^2 - 2x + 6y - 29 = 0$

(D) $x^2 + y^2 - 6x - 4y + 19 = 0$

Q.17 The centres of two circles C_1 and C_2 each of unit radius are at a distance of 6 units from each other. Let P be the mid point of the line segment joining the centres of C_1 and C_2 and C be a circle touching circles C_1 and C_2 externally. If a common tangent to C_1 and C passing through P is also a common tangent to C_2 and C, then the radius of the circle C is-

(A) 8 (B) 4 (C) 16 (D) 2

[IIT 2009]

Q.18 The circle passing through the point (–1, 0) and touching the y-axis at (0, 2) also passes through the point - **[IIT 2011]**

(A) $\left(-\frac{3}{2}, 0\right)$ (B) $\left(-\frac{5}{2}, 2\right)$

(C) $\left(-\frac{3}{2}, \frac{5}{2}\right)$ (D) (– 4, 0)

Q.19 The straight line $2x - 3y = 1$ divides the circular region $x^2 + y^2 \le 6$ into two parts. If $S = \left\{\left(2, \frac{3}{4}\right), \left(\frac{5}{2}, \frac{3}{4}\right), \left(\frac{1}{4}, -\frac{1}{4}\right), \left(\frac{1}{8}, \frac{1}{4}\right)\right\}$, then the number of point(s) in S lying inside the smaller part is - **[IIT 2011]**

(A) 8 (B) 2 (C) 4 (D) 16

ANSWER KEY

LEVEL- 1

Qus.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	D	D	A	B	B	A	A	A	C	C	C	D	C	A	B	C	D	A	B	C
Qus.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	D	B	B	C	B	A	D	A	B	C	B	C	D	A	C	A	B	B	C	A
Qus.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	D	A	B	A	B	A	C	A	A	C	B	C	C	C	C	C	A	C	D	A
Qus.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	B	A	B	A	A	A	D	B	C	C	D	B	B	B	D	D	B	C	B	D
Qus.	81	82	83	84	85	86														
Ans.	C	C	B	C	C	C														

LEVEL- 2

Qus.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	B	A	A	D	A	B	B	A	A	B	C	C	A	A	C	B	B	D	A	C
Qus.	21	22	23	24	25	26	27	28	29											
Ans.	B	B	D	A	D	D	B	B	C											

LEVEL- 3

Qus.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	C	D	A	C	A	C	C	A	B	A	B	A	B	B	B	A	D	C	C	A
Qus.	21	22	23	24	25	26	27	28	29	30	31	32								
Ans.	D	A	B	C	B	B	C	B	B	A	D	A								

LEVEL- 4

SECTION-A

Qus.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Ans.	C	B	D	B	A	A	B	D	D	D	C	C	B	B	A	B	B	B

SECTION-B

Qus.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Ans.	D	A	C	C	A	B	B, C	A	C	A	C	A	C	A	A	B	A	D	B