CIRCLE

AIEEE Syllabus

1. *Definition*
2. *Standard form of equation of a circle*
3. *Equation of circle in some special cases*
4. *Position of a point with respect to a circle*
5. *Line and circle*
6. *Equation of tangent and normal*
7. *Chord of contact*
8. *Director circle*
9. *Position of two circles*
10. *Equation of a chord whose middle point is given*
11. *Circle through the point of intersection*
12. *Common chord of two circles*
13. *Angle of intersection of two circles*

Total No. of questions in **Circle** are:

Solved examples……….......…………………..…30

Level # 1 ……………………………………… 86

Level # 2 ……………………………………..…29

Level # 3 ……………………………………..…32

Level # 4 …………………………………….…37

Total No. of questions…..………..214

1. Students are advised to solve the questions of exercises (Levels # 1, 2, 3, 4) in the same sequence or as directed by the faculty members.
2. Level #3 is not for foundation course students, it will be discussed in fresher and target courses.

Index : Preparing your own list of Important/Difficult Questions

Instruction to fill

(A) Write down the Question Number you are unable to solve in **column A** below, by Pen.

(B) After discussing the Questions written in **column A** with faculties, strike off them in the manner so that you can see at the time of Revision also, to solve these questions again.

(C) Write down the Question Number you feel are important or good in the **column B**.

EXERCISE NO.	COLUMN :A	COLUMN :B
	Questions I am unable to solve in first attempt	Good/Important questions
Level # 1		
Level # 2		
Level # 3		
Level # 4		

Advantages

1. It is advised to the students that they should prepare a question bank for the revision as it is very difficult to solve all the questions at the time of revision.

2. Using above index you can prepare and maintain the questions for your revision

KEY CONCEPTS

1. Definition

A circle is the locus of a point which moves in a plane in such a way that its distance from a fixed point is always constant. The fixed point is called the centre and constant distance is called the radius of the circle.

NOTE :

(i) If r (r > 0) is the radius of a circle, the diameter d = 2r is the maximum distance between any two points on the given circle

(ii) The length of the curve or perimeter (also called circumference) of circle is = $2\pi r$ or πd

(iii) The area of circle = πr^2 or

(iv) Lines joining any two points of a circle is called chord of circle

(v) Curved section of any two point of a circle is called arc of circle.

(vi) Angle subtended at the centre of a circle by any arc is given by = arc/radius.

(vii) Angle subtended at the centre of a circle by an arc is double of angle subtended at the circumference of a circle.

2. Standard forms of Equation of a Circle

2.1 General Equation of a Circle :

The general equation of a circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$, Where g, f, c are constants.

(i) Centre of a general equation of a circle is $(-g, -f)$

i.e. $(-\frac{1}{2}$ coefficient of x, $-\frac{1}{2}$ coefficient of y)

(ii) Radius of a general equation of a circle is

$$\sqrt{g^2 + f^2 - c}$$

NOTE :

(i) The general equation of second degree $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$ represents a circle if $a = b \neq 0$ and $h = 0$.

(ii) Locus of a point P represent a circle if its distance from two points A and B is not equal i.e. PA = kPB represent a circle if $k \neq 1$

(iii) General equation of a circle represents –

(a) A real circle if $g^2 + f^2 - c > 0$

(b) A point circle if $g^2 + f^2 - c = 0$

(c) An imaginary circle if $g^2 + f^2 - c < 0$

(iv) In General equation of a circle –

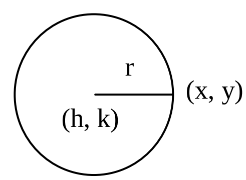
(a) If $c = 0 \Rightarrow$ The circle passes through origin

(b) If $f = 0 \Rightarrow$ The centre is on x – axis

(c) If $g = 0 \Rightarrow$ The centre is on y – axis

2.2 Central Form of Equation of a circle :

The equation of a circle having centre (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$



NOTE :

(i) If the centre is origin, then the equation of the circle is $x^2 + y^2 = r^2$

(ii) If r = 0 than circle is called point circle and its equation is

$(x - h)^2 + (y - k)^2 = 0$

2.3 Diametral Form :

If (x_1, y_1) and (x_2, y_2) be the extremities of a diameter, then the equation of circle is

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

2.4 Parametric Equation of a Circle :

(i) The parametric equations of a circle $x^2 + y^2 = r^2$ are $x = r\cos\theta$, $y = r\sin\theta$. Hence parametric coordinates of any point lying on the circle $x^2 + y^2 = r^2$ are $(r\cos\theta, r\sin\theta)$.

(ii) The parametric equations of the circle $(x - h)^2 + (y - k)^2 = r^2$ are $x = h + r\cos\theta$, $y = k + r\sin\theta$. Hence parametric coordinates of any point lying on the circle are $(h + r\cos\theta, k + r\sin\theta)$

(iii) Parametric equations of the circle

$x^2 + y^2 + 2gx + 2fy + c = 0$ is

$x = -g + \cos\theta$

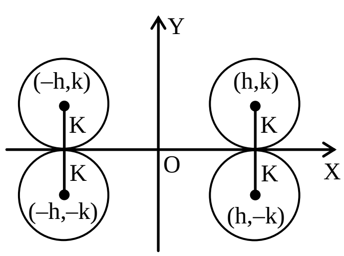
$y = -f + \sin\theta$

3. Equation of a Circle in some special cases

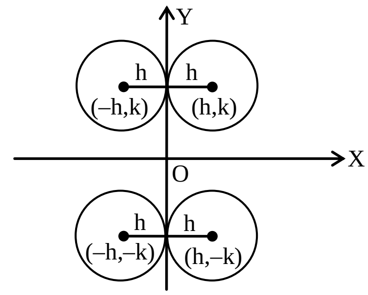
(i) If centre of circle is (h, k) and passes through origin then its equation is $(x - h)^2 + (y - k)^2 = h^2 + k^2$

$\Rightarrow x^2 + y^2 - 2hx - 2ky = 0$

(ii) If the circle touches x axis then its equation is (Four cases) $(x \pm h)^2 + (y \pm k)^2 = k^2$



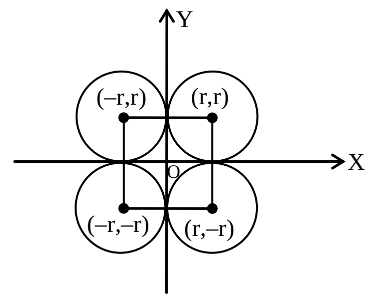
(iii) If the circle touches y axis then its equation is (Four cases)



$(x \pm h)^2+(y \pm k)^2 = h^2$

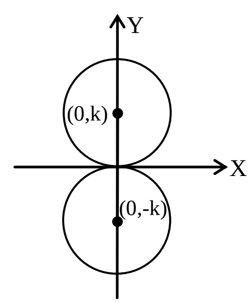
(iv) If the circle touches both the axis then its equation is (Four cases)

$(x \pm r)^2 + (y \pm r)^2 = r^2$



(v) If the circle touches x axis at origin (Two cases)

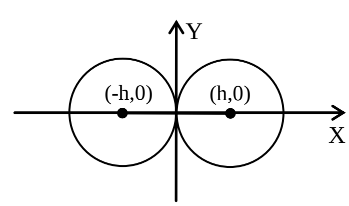
$x^2 + (y \pm k)^2 = k^2 \Rightarrow x^2 + y^2 \pm 2ky = 0$



(vi) If the circle touches y axis at origin (Two cases)

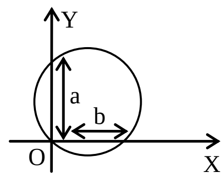
$(x \pm h)^2 + y^2 = h^2$

$\Rightarrow x^2 + y^2 \pm 2xh = 0$



(vii) If the circle passes through origin and cut intercept of a and b on axes, the equation of circle is (Four cases)

$x^2 + y^2 – ax – by = 0$ and centre is $(a/2, b/2)$



4. Position of a Point with respect to a Circle

A point (x_1, y_1) lies outside, on or inside a circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ according as

$S_1 \equiv x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$ is positive, zero or negative i.e.

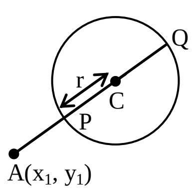
$S_1 > 0 \Rightarrow$ Point is outside the circle.

$S_1 = 0 \Rightarrow$ Point is on the circle.

$S_1 < 0 \Rightarrow$ Point is inside the circle.

4.1 The least and greatest distance of a point from a circle :

Let $S = 0$ be a circle and $A (x_1, y_1)$ be a point. If the diameter of the circle which is passing through the circle at P and Q. then



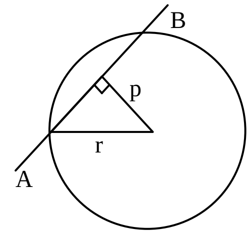
AP = AC – r = least distance

AQ = AC + r = greatest distance,

where 'r' is the radius and C is the centre of circle

5. Line and Circle

Let $L = 0$ be a line and $S = 0$ be a circle, if 'r' be the radius of a circle and p be the length of perpendicular from the centre of circle on the line, then if



$p > r \Rightarrow$ Line is outside the circle

$p = r \Rightarrow$ Line touches the circle

$p < r \Rightarrow$ Line is the chord of circle

$p = 0 \Rightarrow$ Line is diameter of circle

NOTE :

(i) Length of the intercept made by the circle on the line is $= 2\sqrt{r^2 - p^2}$

(ii) The length of the intercept made by line $y = mx + c$ with the circle $x^2 + y^2 = a^2$ is

$$2\sqrt{\frac{a^2(1+m^2)-c^2}{1+m^2}}$$

5.1 Condition of Tangency :

A line $L = 0$ touches the circle $S = 0$, if length of perpendicular drawn from the centre of the circle to the line is equal to radius of the circle i.e. $p = r$. This is the condition of tangency for the line $L = 0$

Circle $x^2 + y^2 = a^2$ will touch the line $y = mx + c$ if $c = \pm a\sqrt{1+m^2}$

Again

(a) If $a^2 (1 + m^2) – c^2 > 0$ line will meet the circle at real and different points.

(b) If $c^2 = a^2 (1 + m^2)$ line will touch the circle.

(c) If $a^2 (1 + m^2) – c^2 < 0$ line will meet circle at two imaginary points.

5.2 Intercepts made on coordinate axes by the circle :

The intercept made by the circle

$x^2 + y^2 + 2gx + 2fy + c = 0$ on -

(i) x axis $= 2\sqrt{g^2 - c}$

(ii) y axis = $2\sqrt{f^2 - c}$

NOTE : Circle $x^2 + y^2 + 2gx + 2fy + c = 0$ cuts

(i) x axis in two real, coincident or imaginary points according as $g^2 >, =, < c$

(ii) y axis in two real, coincident or imaginary points according as $f^2 >, = , < c$

6. Equation of Tangent & Normal

6.1 Equation of Tangent :

The equation of tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at a point (x_1, y_1) is

$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$ or $T = 0$

NOTE :

(i) The equation of tangent to circle $x^2 + y^2 = a^2$ at point (x_1, y_1) is $xx_1 + yy_1 = a^2$

(ii) **Slope Form:** From condition of tangency for every value of m, the line $y = mx \pm a\sqrt{1+m^2}$ is a tangent of the circle $x^2 + y^2 = a^2$ and its point of contact is $\left(\frac{\mp am}{\sqrt{1+m^2}}, \frac{\pm a}{\sqrt{1+m^2}}\right)$

6.2 Equation of Normal :

Normal to a curve at any point P of a curve is the straight line passes through P and is perpendicular to the tangent at P.

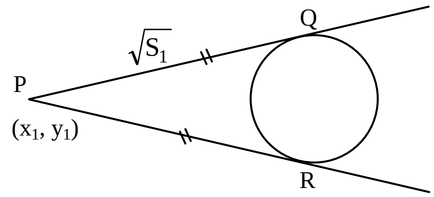
The equation of normal to the circle

$x^2 + y^2 + 2gx + 2fy + c = 0$ at any point (x_1, y_1) is

$$y - y_1 = \frac{y_1 + f}{x_1 + g}(x - x_1)$$

6.3 Length of Tangent :

From any point, say $P(x_1, y_1)$ two tangents can be drawn to a circle which are real, coincident or imaginary according as P lies outside, on or inside the circle.



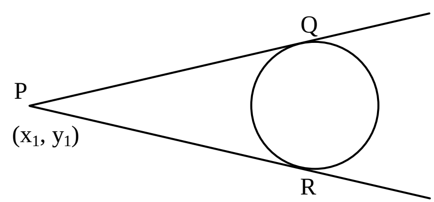
Let PQ and PR be two tangents drawn from $P(x_1, y_1)$ to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$. Then PQ = PR is called the length of tangent drawn from point P and is given by $PQ = QR = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c} = \sqrt{S_1}$.

6.4.1 Pair of Tangents :

From a given point $P(x_1, y_1)$ two tangents PQ and PR can be drawn to the circle $S = x^2 + y^2 + 2gx + 2fy + c = 0$. Their combined equation is $SS_1 = T^2$.

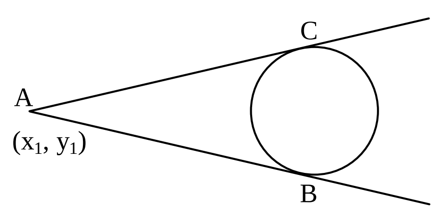
Where

$S = 0$ is the equation of circle $T = 0$ is the equation of tangent at (x_1, y_1) and S_1 is obtained by replacing x by x_1 and y by y_1 in S.



7. Chord of Contact

The chord joining the two points of contact of tangents to a circle drawn from any point A is called chord of contact of A with respect to the given circle.



Let the given point is $A(x_1, y_1)$ and the circle is $S = 0$ then equation of the chord of contact is

$T = xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

NOTE :

(i) It is clear from the above that the equation to the chord of contact coincides with the equation of the tangent, if the point (x_1, y_1) lies on the circle.

(ii) The length of chord of contact = $2\sqrt{r^2 - p^2}$

(iii) Area of Δ ABC is given by

$$\frac{a(x_1^2 + y_1^2 - a^2)^{3/2}}{x_1^2 + y_1^2}$$

8. Director Circle

The locus of the point of intersection of two perpendicular tangents to a circle is called the **Director circle**.

Let the circle be $x^2 + y^2 = a^2$, then equation of the pair of tangents to a circle from a point (x_1, y_1) is $(x^2 + y^2 - a^2)(x_1^2 + y_1^2 - a^2) = (xx_1 + yy_1 - a^2)^2$. If this represents a pair of perpendicular lines then coefficient of x^2 + coefficient of $y^2 = 0$

i.e.$(x_1^2 + y_1^2 - a^2 - x_1^2) + (x_1^2 + y_1^2 - a^2 - y_1^2) = 0$

$\Rightarrow \ x_1^2 + y_1^2 = 2a^2$

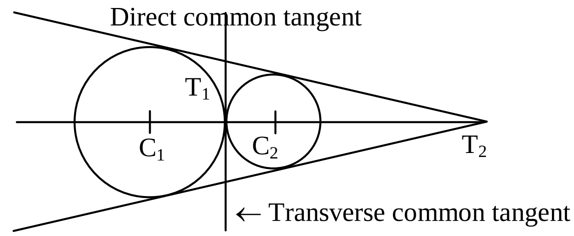
Hence the equation of director circle is $x^2 + y^2 = 2a^2$

Obviously director circle is a concentric circle whose radius is times the radius of the given circle.

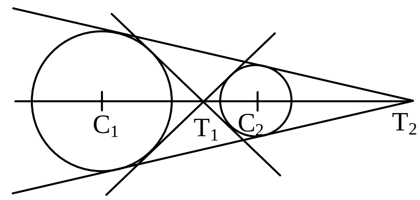
9. Position of Two Circles

Let $C_1 (h_1, k_1)$ and $C_2 (h_2, k_2)$ be the centre of two circle and r_1, r_2 be their radius then

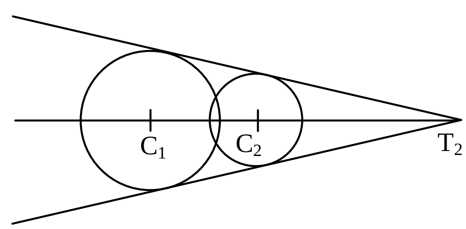
Case-I: When $C_1C_2 = r_1 + r_2$ i.e. the distance between the centres is equal to the sum of their radii. In this case, two direct tangents are real and distinct while the transverse tangents are coincident. The point T_1 divides c_1 and c_2 in the ratio of $r_1 : r_2$.



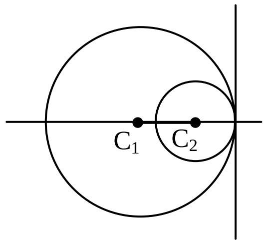
Case-II: When $C_1C_2 > r_1 + r_2$ i.e. the distance between the centres is greater than the sum of their radii. In this case , the two circles do not intersect with each other and four common tangents be drawn. Two common tangents intersects at T_2 called the direct common tangents and other two intersect at T_1 called the transverse common tangents.



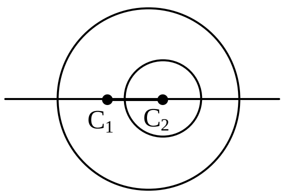
Case-III: When $(r_1 - r_2) < C_1C_2 < r_1 + r_2$ i.e. the distance between the centre is less than the sum of their radii. In this case, the two direct common tangents are real while the transverse tangents are imaginary.



Case-IV: When $C_1C_2 = | r_1 - r_2 |$ i.e. the distance between the centre is equal to the difference of their radii. In this case, two tangents are real and coincident while the other two are imaginary.



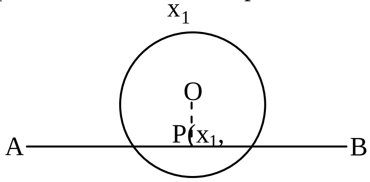
Case-V: When $C_1C_2 < |r_1 - r_2|$ i.e. the distance between centre is less than the difference of their radii. In this case, all the four common tangents are imaginary.



10. Equation of a chord whose middle point is given

The equation of the chord of the circle $x^2 + y^2 = a^2$ whose middle point $P(x_1, y_1)$ is given is

Slope of line OP = $\dfrac{y_1}{x_1}$; slope of AB = $\dfrac{x_1}{y_1}$



So equation of chord is $(y - y_1) = \dfrac{x_1}{y_1}(x - x_1)$

or $xx_1 + yy_1 = x_1^2 + y_1^2$.

Which can be represent by $T = S_1$

11. Circle through the Point of intersection

(i) The equation of the circle passing through the points of intersection of the circle S = 0 and line L = 0 is $S + \lambda L = 0$.

(ii) The equation of the circle passing through the points of intersection of the two circle S = 0 and $S' = 0$ is $S + \lambda S' = 0$. Where $(\lambda \neq -1)$

In the above both cases λ can be find out according to the given problem.

12. Common Chord of two Circles

The line joining the points of intersection of two circles is called the common chord. If the equation of two circle.

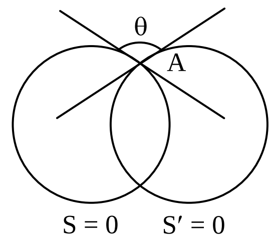
$S_1 = x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and

$S_2 = x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$, then equation of common chord is $S_1 - S_2 = 0$

$\Rightarrow\ 2x(g_1 - g_2) + 2y(f_1 - f_2) + c_1 - c_2 = 0$

13. Angle of Intersection of two Circles

The angle of intersection between two circles S = 0 and S' = 0 is defined as the angle between their tangents at their point of intersection.



If $S \equiv x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$

$S' \equiv x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$

are two circles with radii r_1, r_2 and d be the distance between their centres then the angle of intersection θ between them is given by

$$\cos\theta = \frac{r_1^2 + r_2^2 + d^2}{2r_1r_2}$$

or $$\cos\theta = \frac{2(g_1g_2 + f_1f_2) - (c_1 + c_2)}{2\sqrt{g_1^2 + f_1^2 - c_1}\sqrt{g_2^2 + f_2^2 - c_2}}$$

13.1 Condition of Orthogonality :

If the angle of intersection of the two circle is a right angle (θ = 90º) then such circle are called Orthogonal circle and conditions for their orthogonality is $2g_1g_2 + 2f_1f_2 = c_1 + c_2$