ELLIPSE

JEE MAINS Syllabus

1. *Definition*
2. *Equation of an Ellipse*
3. *Second form of Ellipse*
4. *General equation of the Ellipse*
5. *Parametric forms of the Ellipse*
6. *Point and Ellipse*
7. *Ellipse and Line*
8. *Equation of the Tangent*

Total No. of questions in **Ellipse** are:

Solved examples……….....………………..…16

Level # 1 ……………………………………… 40

Level # 2 ………………………………………41

Level # 3 ………………………………………17

Total No. of questions…..………..114

1. Students are advised to solve the questions of exercises (Levels # 1, 2, 3, 4) in the same sequence or as directed by the faculty members.

2. Level # 3 is not for foundation course students, it will be discussed in fresher and target courses.

Index : Preparing your own list of Important/Difficult Questions

Instruction to fill

(A) Write down the Question Number you are unable to solve in **column A** below, by Pen.

(B) After discussing the Questions written in **column A** with faculties, strike off them in the manner so that you can see at the time of Revision also, to solve these questions again.

(C) Write down the Question Number you feel are important or good in the **column B**.

EXERCISE NO.	COLUMN :A	COLUMN :B
	Questions I am unable to solve in first attempt	Good/Important questions
Level # 1		
Level # 2		
Level # 3		
Level # 4		

Advantages

1. It is advised to the students that they should prepare a question bank for the revision as it is very difficult to solve all the questions at the time of revision.

2. Using above index you can prepare and maintain the questions for your revision.

KEY CONCEPTS

1. Definition

An ellipse is the locus of a point which moves in such a way that its distance form a fixed point is in constant ratio to its distance from a fixed line. The fixed point is called the **focus** and fixed line is called the **directrix** and the constant ratio is called the **eccentricity of a ellipse** denoted by (e).

In other word, we can say an ellipse is the locus of a point which moves in a plane so that the sum of it distances from fixed points is constant.

2. Equation of an Ellipse

2.1 Standard Form of the equation of ellipse

$$\boxed{\frac{x^2}{a^2}+\frac{y^2}{b^2}=1}\quad (a > b)$$

Let the distance between two fixed points S and S' be 2ae and let C be the mid point of SS′.

Taking CS as x- axis, C as origin.

Let P(h, k) be the moving point Let SP+ SP′ = 2a (fixed distance) then

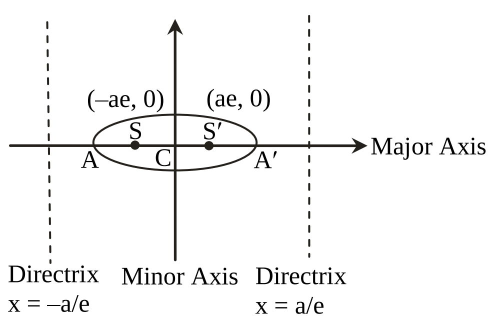
SP+S'P = $\sqrt{\{(h-ae)^2+k^2\}}+\sqrt{\{(h+ae)^2+k^2\}}$ = 2a

$h^2(1-e^2) + k^2 = a^2(1-e^2)$

Hence Locus of P(h, k) is given by.

$x^2(1-e^2) + y^2 = a^2(1-e^2)$

$\Rightarrow \frac{x^2}{a^2}+\frac{y^2}{a^2(1-e^2)}=1$



Let us assume that $a^2(1-e^2) = b^2$

∴ The standard equation will be given by

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

2.1.1 Various parameter related with standard ellipse :

Let the equation of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (a > b)

(i) **Vertices of an ellipse :**

The points of the ellipse where it meets with the line joining its two foci are called its vertices.

For above standard ellipse A. A′ are vertices

A ≡ (a, 0), A′ ≡ (– a, 0)

(ii) **Major axis :**

The chord AA′ joining two vertices of the ellipse is called its major axis.

Equation of major axis : y = 0

Length of major axis = 2a

(iii) **Minor axis :**

The chord BB′ which bisects major axis AA′ perpendicularly is called minor axis of the ellipse.

Equation of minor axis x = 0

Length of minor axis = 2b

(iv) **Centre :**

The point of intersection of major axis and minor axis of an ellipse is called its centre.

For above standard ellipse

centre = C(0, 0)

(v) **Directrix :**

Equation of directrices are x= a/e and x = – a/e.

(vi) **Focus :** S (ae, 0) and S′ (– ae, 0) are two foci of an ellipse.

(vii) **Latus Rectum :** Such chord which passes through either focus and perpendicular to the major axis is called its latus rectum.

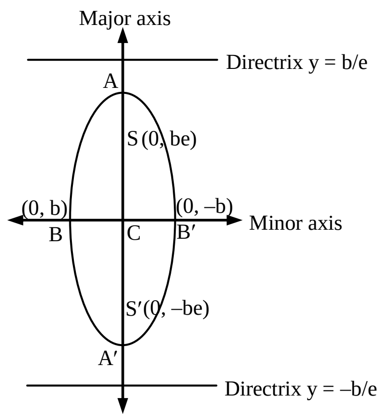
(viii) **Length of Latus Rectum :**

Length of Latus rectum is given by $\frac{2b^2}{a}$.

(ix) **Relation between constant a, b, and e**

$$b^2 = a^2(1 - e^2) \Rightarrow e = \sqrt{1 - \frac{b^2}{a^2}}$$

3. Second form of Ellipse



$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$ when a < b.

For this ellipse

(i) centre : (0, 0)

(ii) vertices : (0, b) ; (0, – b)

(iii) foci : (0, be) ; (0, – be)

(iv) major axis : equation x = 0, length = 2b

(v) minor axis : equation y = 0, length = 2a

(vi) directrices : y = b/e, y = – b/e

(vii) length of latus ractum = $2a^2/b$

(viii) eccentricity : $e = \sqrt{1 - \frac{a^2}{b^2}}$

4. General equation of the ellipse

The general equation of an ellipse whose focus is (h,k) and the directrix is the line ax + by + c = 0 and the eccentricity will be e. Then let $P(x_1,y_1)$ be any point on the ellipse which moves such that SP = ePM

$$\Rightarrow (x_1 - h)^2 + (y_1 - k)^2 = \frac{e^2(ax_1 + by_1 + c)^2}{a^2 + b^2}$$

Hence the locus of (x_1,y_1) will be given by

$$(a^2 + b^2)\,[(x - h)^2 + (y - k)^2] = e^2(ax + by + c)^2$$

Which is the equation of second degree from which we can say that any equation of second degree represent equation of an ellipse.

Note : Condition for second degree in X & Y to represent an ellipse is that if $h^2 = ab < 0$ & $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 \neq 0$

5. Parametric forms of the Ellipse

Let the equation of ellipse in standard form will be given by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Then the equation of ellipse in the parametric form will be given by $x = a\cos\phi$, $y = b\sin\phi$ where ϕ is the eccentric angle whose value vary from $0 \le \phi < 2\pi$. Therefore coordinate of any point P on the ellipse will be given by $(a\cos\phi, b\sin\phi)$.

6. Point and Ellipse

Let $P(x_1, y_1)$ be any point and let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is the equation of an ellipse.

The point lies outside, on or inside the ellipse as if $S_1 = \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1 > 0, = 0, < 0$

7. Ellipse and a Line

Let the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the given line be y = mx + c.

Solving the line and ellipse we get

$$\frac{x^2}{a^2} + \frac{(mx + c)^2}{b^2} = 1$$

i.e. $(a^2m^2 + b^2)\,x^2 + 2\,mca^2\,x + a^2\,(c^2 - b^2) = 0$

above equation being a quadratic in x.

$\therefore$ discriminant $= 4m^2c^2a^4 - 4a^2\,(a^2m^2 + b^2)\,(c^2 - b^2)$

$= b^2\,\{(a^2m^2 + b^2) - c^2\}$

Hence the line intersects the ellipse in
(i) two distinct points if $a^2m^2 + b^2 > c^2$

(ii) in one point if $c^2 = a^2m^2 + b^2$

(iii) does not intersect if $a^2m^2 + b^2 < c^2$

$\therefore\ y = mx \pm \sqrt{(a^2m^2 + b^2)}$ touches the ellipse and condition for tangency $c^2 = a^2m^2 + b^2$.

Hence $y = mx \pm \sqrt{(a^2m^2 + b^2)}$, touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at $\left(\frac{\pm a^2m}{\sqrt{a^2m^2 + b^2}}, \frac{\pm b^2}{\sqrt{a^2m^2 + b^2}}\right)$.

8. Equation of the Tangent

(i) The equation of the tangent at any point (x_1, y_1) on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$.

(ii) The equation of tangent at any point 'ϕ' is

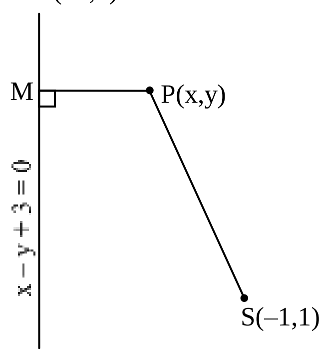
$\frac{x}{a}\cos\phi + \frac{y}{b}\sin\phi = 1$.

SOLVED EXAMPLES

Ex.1 The equation of an ellipse whose focus is (–1, 1), eccentricity is 1/2 and the directrix is x – y + 3 = 0 is.

(A) $7x^2 + 7y^2 + 2xy + 10x - 10y + 7 = 0$
(B) $7x^2 + 7y^2 + 2xy - 10x - 10y + 7 = 0$
(C) $7x^2 + 7y^2 + 2xy - 10x + 10y + 7 = 0$
(D) None of these

Sol.[A] Let P (x,y) be any point on the ellipse whose focus is S (–1,1) and the directrix is x – y + 3 = 0.



PM is perpendicular from P (x,y) on the directrix x –y + 3 = 0.

Then by definition

SP = ePM

$\Rightarrow (SP)^2 = e^2 (PM)^2$

$\Rightarrow (x + 1)^2 + (y - 1)^2 = \frac{1}{4}\left\{\frac{x-y+3}{\sqrt{2}}\right\}^2$

$\Rightarrow 8 (x^2 + y^2 + 2x - 2y + 2)$

$= x^2 + y^2 + 9 - 2xy + 6x - 6y$

$\Rightarrow 7x^2 + 7y^2 + 2xy + 10x - 10y + 7 = 0$

which is the required equation of the ellipse.

Ex.2 The foci of an ellipse are (±2, 0) and its eccentricity is 1/2, the equation of ellipse is.

(A) $\frac{x^2}{16}+\frac{y^2}{9}=1$ (B) $\frac{x^2}{16}+\frac{y^2}{12}=1$

(C) $\frac{x^2}{4}+\frac{y^2}{2}=1$ (D) None of these

Sol.[B] Let the equation of the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$,

Then coordinates of foci are (± ae, 0).

$\therefore ae = 2 \Rightarrow a \times \frac{1}{2} = 2$ $\left[\because e = \frac{1}{2}\right]$

$\Rightarrow a = 4$

We have $b^2 = a^2 (1 - e^2)$

$\therefore b^2 = 16\left(1-\frac{1}{4}\right) = 12$

Thus, the equation of the ellipse is $\frac{x^2}{16} + \frac{y^2}{12} = 1$.

Ex.3 The equation of the ellipse which passes through origin and has its foci at the points (1, 0) and (3, 0) is -

(A) $3x^2 + 4y^2 = x$ (B) $3x^2 + y^2 = 12x$
(C) $x^2 + 4y^2 = 12x$ (D) $3x^2 + 4y^2 = 12x$

Sol.[D] Centre being mid point of the foci is

$\left(\frac{1+3}{2},0\right) = (2, 0)$

Distance between foci 2ae = 2

ae = 1 or $b^2 = a^2 (1 - e^2)$

$b^2 = a^2 - a^2e^2 \Rightarrow a^2 - b^2 = 1$ …(i)

If the ellipse $\frac{(x-2)^2}{a^2} + \frac{y^2}{b^2} = 1$, then as it passes from (0, 0)

$\frac{4}{a^2} = 1 \Rightarrow a^2 = 4$

from (i) $b^2 = 3$

Hence $\frac{(x-2)^2}{4} + \frac{y^2}{3} = 1$

or $3x^2 + 4y^2 - 12x = 0$

Ex.4 A man running round a racecourse notes that the sum of the distance of two flag posts from him is always 10 meters and the distance between the flag posts is 8 meters. The area of the path he encloses -

(A) 10π (B) 15π
(C) 5π (D) 20π

Sol.[B] The race course will be an ellipse with the flag posts as its foci. If a and b are the semi major and minor axes of the ellipse, then sum of focal distances 2a = 10 and 2ae = 8

a = 5, e = 4/5

$\therefore \quad b^2 = a^2(1 - e^2) = 25 \left(1 - \frac{16}{25}\right) = 9$

Area of the ellipse = πab

$= \pi.5.3 = 15\pi$

Ex.5 The distance of a point on the ellipse $\frac{x^2}{6} + \frac{y^2}{2} = 1$ from the centre is 2. Then eccentric angle of the point is -

(A) $\pm \frac{\pi}{2}$ (B) $\pm \pi$

(C) $\frac{\pi}{4}, \frac{3\pi}{4}$ (D) $\pm \frac{\pi}{4}$

Sol.[C] Any point on the ellipse is

$(\sqrt{6}\cos\phi, \sqrt{2}\sin\phi)$, where ϕ is an eccentric angle.

It's distance from the center (0, 0) is given 2.

$6\cos^2\phi + 2\sin^2\phi = 4$

or $3\cos^2\phi + \sin^2\phi = 2$

$2\cos^2\phi = 1$

$\Rightarrow \cos\phi = \pm \frac{1}{\sqrt{2}}$; $\phi = \frac{\pi}{4}, \frac{3\pi}{4}$

Ex.6 The equation of tangents to the ellipse $9x^2 + 16y^2 = 144$ which pass through the point (2, 3) -

(A) y = 3 (B) x + y = 2

(C) x – y = 3 (D) y = 3 ; x + y = 5

Sol.[D] Ellipse $9x^2 + 16y^2 = 144$

or $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Any tangent is $y = mx + \sqrt{16m^2 + 9}$ it passes through (2, 3)

$3 = 2m + \sqrt{16m^2 + 9}$

$(3 - 2m)^2 = 16m^2 + 9$

m = 0, –1

Hence the tangents are y = 3, x + y = 5

Ex.7 The line $x = at^2$ meets the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in the real points if -

(A) | t | < 2 (B) | t | ≤ 1

(C) | t | >1 (D) None of these

Sol.[B] Putting $x = at^2$ in the equation of the ellipse, we get

$\frac{a^2t^4}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow y^2 = b^2(1 - t^4)$

$y^2 = b^2(1 - t^2)(1 + t^2)$

This will give real values of y if

$(1 - t^2) \geq 0$ | t | ≤ 1

Ex.8 The equation $x^2 + 4y^2 + 2x + 16y + 13 = 0$ represents a ellipse -

(A) whose eccentricity is $\sqrt{3}$

(B) whose focus is $(\pm\sqrt{3}, 0)$

(C) whose directrix is $x = \pm \frac{4}{\sqrt{3}} - 1$

(D) None of these

Sol.[C] We have $x^2 + 4y^2 + 2x + 16y + 13 = 0$

$(x^2 + 2x + 1) + 4(y^2 + 4y + 4) = 4$

$(x + 1)^2 + 4(y + 2)^2 = 4$

$\frac{(x+1)^2}{2^2} + \frac{(y+2)^2}{1^2} = 1$

Comparing with $\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1$

where X = x + 1, Y = y + 2

and a = 2, b = 1

eccentricity of the ellipse

$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$

Focus of the ellipse (± ae, 0)

X = ± ae and Y = 0

$x + 1 = \pm 2.\frac{\sqrt{3}}{2}$ and y + 2 = 0

$\Rightarrow x = -1 \pm \sqrt{3}$ and y = – 2

$\therefore$ Focus $(-1 \pm \sqrt{3}, -2)$

Directrix of the ellipse X = ± a/e

$x + 1 = \pm \frac{2}{\sqrt{3}/2}$; $x = \pm \frac{4}{\sqrt{3}} - 1$

Ex.9 Product of the perpendiculars from the foci upon any tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is -

(A) b (B) a

(C) a^2 (D) b^2

Sol.[D] The equation of any tangent to the ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $y = mx + \sqrt{a^2m^2 + b^2}$

$\Rightarrow mx - y + \sqrt{a^2m^2 + b^2} = 0$ ...(i)

The two foci of the given ellipse are S(ae, 0) and S′ (–ae, 0). let p_1 and p_2 be the lengths of perpendicular from S and S′ respectively on (i), Then

p_1 = length of perpendicular from S(ae, 0) on (i)

$$p_1 = \frac{mae + \sqrt{a^2m^2 + b^2}}{\sqrt{m^2+1}}$$

p_2 = length of perpendicular from S′(–ae, 0) on (i)

$$p_2 = \frac{-mae + \sqrt{a^2m^2 + b^2}}{\sqrt{m^2+1}}$$

Now p_1p_2

$$\left(\frac{mae + \sqrt{a^2m^2 + b^2}}{\sqrt{m^2+1}}\right)\left(\frac{-mae + \sqrt{a^2m^2 + b^2}}{\sqrt{m^2+1}}\right)$$

$$= \frac{a^2m^2(1-e^2) + b^2}{1+m^2} \quad \because b^2 = a^2(1-e^2)$$

$$= \frac{m^2b^2 + b^2}{1+m^2} = \frac{b^2(m^2+1)}{m^2+1} = b^2$$

Ex.10 The equation of the ellipse whose axes are along the coordinate axes, vertices are (± 5,0) and foci at (± 4,0) is.

(A) $\frac{x^2}{25} + \frac{y^2}{9} = 1$ (B) $\frac{x^2}{25} + \frac{y^2}{16} = 1$

(C) $\frac{x^2}{25} + \frac{y^2}{12} = 1$ (D) None of these

Sol.[A] Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad ...(1)$$

The coordinates of its vertices and foci are (± a, 0) and (± ae,0) respectively.

a = 5 and ae = 4 $\Rightarrow$ e = 4/5.

Now, $b^2 = a^2(1-e^2) \Rightarrow b^2 = 25\left(1-\frac{16}{25}\right) = 9$.

Substituting the values of a^2 and b^2 in (1), we get

$$\frac{x^2}{25} + \frac{y^2}{9} = 1,$$

which is the equation of the required ellipse.

Ex.11 Find the centre, the length of the axes and the eccentricity of the ellipse $2x^2+3y^2-4x-12y+13 = 0$.

Sol. The given equation can be rewritten as $2[x^2 - 2x] + 3[y^2 - 4y] + 13 = 0$

or $2(x-1)^2 + 3(y-2)^2 = 1$

or $\frac{(x-1)^2}{(1/\sqrt{2})^2} + \frac{(y-2)^2}{(1/\sqrt{3})^2} = 1$,

Comparing with $\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1$

$\therefore$ Centre X = 0, Y = 0 i.e. (1,2)

Length of major axis = 2a = $\sqrt{2}$

Length of minor axis = 2b = $2/\sqrt{3}$ and

$$e = \sqrt{1-\frac{b^2}{a^2}} = \frac{1}{\sqrt{3}}$$

Ex.12 Find the equations of the tangents to the ellipse $4x^2 + 3y^2 = 5$ which are inclined at an angle of 60º to the axis of x. Also, find the point of contact.

Sol. The slope of the tangent = tan 60º = $\sqrt{3}$

Now, $4x^2 + 3y^2 = 5 \Rightarrow \frac{x^2}{5/4} + \frac{y^2}{5/3} = 1$

This is of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where

$a^2 = \frac{5}{4}$ and $b^2 = \frac{5}{3}$. We know that the equations of the tangents of slope m to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are given by $y = mx \pm \sqrt{a^2m^2 + b^2}$ and the coordinates of the points of contact are $\left(\pm\frac{a^2m}{\sqrt{a^2m^2+b^2}}, \mp\frac{b^2}{\sqrt{a^2m^2+b^2}}\right)$

Here, m = $\sqrt{3}$, $a^2 = 5/4$ and $b^2 = 5/3$.

So, the equations of the tangents are

$$y = \sqrt{3}\,x \pm \sqrt{\left(\frac{5}{4}\times 3\right)+\frac{5}{3}} \quad \text{i.e. } y = \sqrt{3}\,x \pm \sqrt{\frac{65}{12}}$$

The coordinates of the points of contact are $\left(\pm\frac{5\sqrt{3}/4}{\sqrt{65/12}}, \mp\frac{5/3}{\sqrt{65/12}}\right)$ i.e .

$$\left(\pm\frac{3\sqrt{65}}{26}, \mp\frac{2\sqrt{195}}{39}\right)$$

Ex.13 The radius of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, and having its centre (0, 3) is -

(A) 4 (B) 3

(C) $\sqrt{12}$ (D) 7/2

Sol.[A] $e = \sqrt{1-\frac{b^2}{a^2}} = \sqrt{1-\frac{9}{16}} \quad \therefore e = \frac{\sqrt{7}}{4}$

$\therefore$ Foci are ($\pm$ ae, 0) or ($\pm\sqrt{7}$, 0)

Centre of circle is (0, 3) and passes through foci ($\pm\sqrt{7}$, 0)

$\therefore$ Radius = $\sqrt{7+9} = 4$

Ex.14 The eccentricity of the ellipse represented by the equation $25x^2 + 16y^2 - 150x - 175 = 0$ is-

(A) 2/5 (B) 3/5

(C) 4/5 (D) None of these

Sol.[B] $25(x^2 - 6x + 9) + 16y^2 = 175 + 225$

or $25(x-3)^2 + 16y^2 = 400$ or $\frac{X^2}{16} + \frac{Y^2}{25} = 1$. (b > a)

Form $\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1$

$\therefore$ Major axis lies along y- axis. ;

$\therefore e = \sqrt{1-\frac{a^2}{b^2}} = 1 - \sqrt{\frac{16}{25}}$;

$\therefore e = \frac{3}{5}$

Ex.15 For what value of λ does the line $y = x + \lambda$ touches the ellipse $9x^2 + 16y^2 = 144$.

Sol. $\because$ Equation of ellipse is

$9x^2 + 16y^2 = 144$ or $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Comparing this with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

then we get $a^2 = 16$ and $b^2 = 9$

& comparing the line $y = x + \lambda$ with $y = mx + c$

$\therefore$ m = 1 and c = λ

If the line $y = x + \lambda$ touches the ellipse

$9x^2 + 16y^2 = 144$, then

$c^2 = a^2m^2 + b^2$

$\Rightarrow \lambda^2 = 16 \times 1^2 + 9$

$\Rightarrow \lambda^2 = 25$

$\therefore \lambda = \pm 5$

Ex.16 Find the equations of the tangents to the ellipse $3x^2 + 4y^2 = 12$ which are perpendicular to the line $y + 2x = 4$.

Sol. Let m be the slope of the tangent, since the tangent is perpendicular to the line $y + 2x = 4$.

$\therefore m \times -2 = -1 \quad \Rightarrow m = \frac{1}{2}$

Since $3x^2 + 4y^2 = 12$

or $\frac{x^2}{4} + \frac{y^2}{3} = 1$

Comparing this with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\therefore a^2 = 4$ and $b^2 = 3$

So the equation of the tangents are

$$y = \frac{1}{2}x \pm \sqrt{4x\frac{1}{4}+3}$$

$\Rightarrow y = \frac{1}{2}x \pm 2$ or $x - 2y \pm 4 = 0$

LEVEL- 1

Question based on **Equation and properties of the ellipse**

Q.1 The equation to the ellipse (referred to its axes as the axes of x and y respectively) whose foci are (± 2, 0) and eccentricity 1/2, is-

(A) $\frac{x^2}{12}+\frac{y^2}{16} = 1$ (B) $\frac{x^2}{16}+\frac{y^2}{12} = 1$

(C) $\frac{x^2}{16}+\frac{y^2}{8} = 1$ (D) None of these

Q.2 The eccentricity of the ellipse $9x^2 + 5y^2 – 30 y = 0$ is-

(A) 1/3 (B) 2/3

(C) 3/4 (D) None of these

Q.3 If the latus rectum of an ellipse be equal to half of its minor axis, then its eccentricity is-

(A) 3/2 (B) $\sqrt{3}/2$ (C) 2/3 (D) $\sqrt{2}/3$

Q.4 If distance between the directrices be thrice the distance between the foci, then eccentricity of ellipse is-

(A) 1/2 (B) 2/3 (C) $1/\sqrt{3}$ (D) 4/5

Q.5 The equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents an ellipse if-

(A) $\Delta = 0, h^2 < ab$ (B) $\Delta \neq 0, h^2 < ab$

(C) $\Delta \neq 0, h^2 > ab$ (D) $\Delta \neq 0, h^2 = ab$

Q.6 Equation of the ellipse whose focus is (6, 7) directrix is x + y + 2 = 0 and e = $1/\sqrt{3}$ is-

(A) $5x^2 + 2xy + 5y^2 – 76x – 88y + 506 = 0$

(B) $5x^2 – 2xy + 5y^2 – 76x – 88y + 506 = 0$

(C) $5x^2 – 2xy + 5y^2 + 76x + 88y – 506 = 0$

(D) None of these

Q.7 The eccentricity of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose latus rectum is half of its major axis is-

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{\frac{2}{3}}$

(C) $\frac{\sqrt{3}}{2}$ (D) None of these

Q.8 The equation of the ellipse whose centre is at origin and which passes through the points (– 3,1) and (2,–2) is-

(A) $5x^2 + 3y^2 = 32$ (B) $3x^2 + 5y^2 = 32$

(C) $5x^2 – 3y^2 = 32$ (D) $3x^2 + 5y^2 + 32 = 0$

Q.9 The equation of the ellipse (referred to its axes as the axes of x and y respectively) which passes through the point (– 3, 1) and has eccentricity $\sqrt{\frac{2}{5}}$, is-

(A) $3x^2 + 6y^2 = 33$ (B) $5x^2 + 3y^2 = 48$

(C) $3x^2 + 5y^2 –32 = 0$ (D) None of these

Q.10 Latus rectum of ellipse $4x^2 + 9 y^2 – 8x – 36 y + 4 = 0$ is-

(A) 8/3 (B) 4/3

(C) $\frac{\sqrt{5}}{3}$ (D) 16/3

Q.11 The latus rectum of an ellipse is 10 and the minor axis is equal to the distance between the foci. The equation of the ellipse is-

(A) $x^2 + 2y^2 = 100$ (B) $x^2 + \sqrt{2}\, y^2 =10$

(C) $x^2 – 2y^2 = 100$ (D) None of these

Q.12 If the distance between the foci of an ellipse be equal to its minor axis, then its eccentricity is-

(A) 1/2 (B) $1/\sqrt{2}$

(C) 1/3 (D) $1/\sqrt{3}$

Q.13 The equation $2x^2 + 3y^2 = 30$ represents-

(A) A circle (B) An ellipse
(C) A hyperbola (D) A parabola

Q.14 The equation of the ellipse whose centre is (2, –3), one of the foci is (3, –3) and the corresponding vertex is (4, – 3) is-

(A) $\frac{(x-2)^2}{3} + \frac{(y+3)^2}{4} = 1$

(B) $\frac{(x-2)^2}{4} + \frac{(y+3)^2}{3} = 1$

(C) $\frac{x^2}{3} + \frac{y^2}{4} = 1$

(D) None of these

Q.15 Eccentricity of the ellipse
$4x^2 + y^2 - 8x + 2y + 1 = 0$ is-
(A) $1/\sqrt{3}$ (B) $\sqrt{3}/2$
(C) 1/2 (D) None of these

Q.16 The equation of ellipse whose distance between the foci is equal to 8 and distance between the directrix is 18, is-
(A) $5x^2 - 9y^2 = 180$ (B) $9x^2 + 5y^2 = 180$
(C) $x^2 + 9y^2 = 180$ (D) $5x^2 + 9y^2 = 180$

Q.17 In an ellipse the distance between its foci is 6 and its minor axis is 8. Then its eccentricity is-

(A) $\frac{4}{5}$ (B) $\frac{1}{\sqrt{52}}$

(C) $\frac{3}{5}$ (D) $\frac{1}{2}$

Q.18 The eccentricity of an ellipse is 2/3, latus rectum is 5 and centre is (0, 0). The equation of the ellipse is -

(A) $\frac{x^2}{81}+\frac{y^2}{45}=1$ (B) $\frac{4x^2}{81}+\frac{4y^2}{45}=1$

(C) $\frac{x^2}{9}+\frac{y^2}{5}=1$ (D) $\frac{x^2}{81}+\frac{y^2}{45}=5$

Q.19 The length of the latus rectum of the ellipse $\frac{x^2}{36} + \frac{y^2}{49} = 1$ is -
(A) 98/6 (B) 72/7
(C) 72/14 (D) 98/12

Q.20 For the ellipse $\frac{x^2}{64} + \frac{y^2}{28} = 1$, the eccentricity is

(A) $\frac{3}{4}$ (B) $\frac{4}{3}$

(C) $\frac{2}{\sqrt{7}}$ (D) $\frac{1}{3}$

Q.21 The equation of the ellipse whose one of the vertices is (0, 7) and the corresponding directrix is y = 12, is-
(A) $95x^2 + 144y^2 = 4655$
(B) $144x^2 + 95y^2 = 4655$
(C) $95x^2 + 144y^2 = 13680$
(D) None of these

Q.22 The foci of the ellipse,
$25(x + 1)^2 + 9(y + 2)^2 = 225$, are at-
(A) (–1, 2) and (–1, –6)
(B) (–2, 1) and (– 2, 6)
(C) (– 1, – 2) and (–2, – 1)
(D) (–1, –2) and (–1, – 6)

Q.23 The eccentricity of the ellipse represented by the equation $25x^2 + 16y^2 - 150x - 175 = 0$ is -
(A) 2/5 (B) 3/5
(C) 4/5 (D) None of these

Q.24 The equation of the ellipse whose foci are (± 5, 0) and one of its directrix is 5x = 36, is -

(A) $\frac{x^2}{36}+\frac{y^2}{11}=1$ (B) $\frac{x^2}{6}+\frac{y^2}{\sqrt{11}}=1$

(C) $\frac{x^2}{6}+\frac{y^2}{11}=1$ (D) None of these

Q.25 If S and S′ are two foci of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (a < b) and P (x_1, y_1) a point on it, then SP + S′ P is equal to-
(A) 2a (B) 2b
(C) $a + ex_1$ (D) $b + ey_1$

Q.26 Let P be a variable point on the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ with foci S and S′. If A be the area of triangle PSS′, then maximum value of A is–

(A) 12 sq. units (B) 24 sq. units
(C) 36 sq. units (D) 48 sq. units

Question based on Paramatric equation

Q.27 The parametric representation of a point on the ellipse whose foci are (– 1, 0) and (7, 0) and eccentricity 1/2 is-
(A) $(3 + 8\cos\theta, 4\sqrt{3}\sin\theta)$
(B) $(8\cos\theta, 4\sqrt{3}\sin\theta)$
(C) $(3 + 4\sqrt{3}\cos\theta, 8\sin\theta)$
(D) None of these

Question based on Ellipse and a point, Ellipse and a line

Q.28 The position of the point (4,– 3) with respect to the ellipse $2x^2 + 5y^2 = 20$ is-
(A) outside the ellipse
(B) on the ellipse
(C) on the major axis
(D) None of these

Q.29 If $\frac{x}{a} + \frac{y}{b} = \sqrt{2}$ touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then its eccentric angle θ is equal to-
(A) 0 (B) 90º
(C) 45º (D) 60º

Q.30 Find the equation of the tangent to the ellipse $x^2 + 2y^2 = 4$ at the points where ordinate is 1.
(A) $x + \sqrt{2}y - 2\sqrt{2} = 0$ & $x - \sqrt{2}y + 2\sqrt{2} = 0$
(B) $x - \sqrt{2}y - 2\sqrt{2} = 0$ & $x - \sqrt{2}y + 2\sqrt{2} = 0$
(C) $x + \sqrt{2}y + 2\sqrt{2} = 0$ & $x + \sqrt{2}y + 2\sqrt{2} = 0$
(D) None of these

Q.31 Find the equation of the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ which make equal intercepts on the axes.
(A) $y = x \pm \sqrt{a^2+b^2}$ & $y = -x \pm \sqrt{a^2+b^2}$
(B) $y = x + \sqrt{a^2+b^2}$ & $y = -x \pm \sqrt{a^2+b^2}$
(C) $y = x + \sqrt{a^2+b^2}$ & $y = x \pm \sqrt{a^2+b^2}$
(D) None of these

Q.32 Find the equations of tangents to the ellipse $9x^2 + 16y^2 = 144$ which pass through the point (2,3).
(A) y = 3 and y = –x + 5
(B) y = 5 and y = –x + 3
(C) y = 3 and y = x – 5
(D) None of these

Q.33 If any tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ intercepts lengths h and k on the axes, then-
(A) $\frac{h^2}{a^2} + \frac{k^2}{b^2} = 1$ (B) $\frac{h^2}{a^2} + \frac{k^2}{b^2} = 2$
(C) $\frac{a^2}{h^2} + \frac{b^2}{k^2} = 1$ (D) $\frac{a^2}{h^2} + \frac{b^2}{k^2} = 2$

Q.34 The equation of the tangent at the point (1/4, 1/4) of the ellipse $\frac{x^2}{4} + \frac{y^2}{12} = 1$, is-
(A) 3x + y = 48 (B) 3x + y = 3
(C) 3x + y = 16 (D) None of these

Q.35 The line $x\cos\alpha + y\sin\alpha = p$ will be a tangent to the conic $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if-
(A) $p^2 = a^2\sin^2\alpha + b^2\cos^2\alpha$
(B) $p^2 = a^2 + b^2$
(C) $p^2 = b^2\sin^2\alpha + a^2\cos^2\alpha$
(D) None of these

Q.36 If y = mx + c is tangent on the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$, then the value of c is-
(A) 0 (B) 3/m
(C) $\pm\sqrt{9m^2+4}$ (D) $\pm 3\sqrt{1+m^2}$

Q.37 The ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the straight line y = mx + c intersect in real points only if-
(A) $a^2m^2 < c^2 - b^2$
(B) $a^2m^2 > c^2 - b^2$
(C) $a^2m^2 \ge c^2 - b^2$
(D) $c \ge b$

Q.38 If the straight line y = 4x + c is a tangent to the ellipse $\frac{x^2}{8} + \frac{y^2}{4} = 1$, then c will be equal to-

(A) ± 4 (B) ± 6
(C) ± 1 (D) None of these

Q.39 The equation of the tangents to the ellipse $4x^2 + 3y^2 = 5$ which are parallel to the line $y = 3x + 7$ are

(A) $y = 3x \pm \sqrt{\frac{155}{3}}$ (B) $y = 3x \pm \sqrt{\frac{155}{12}}$

(C) $y = 3x \pm \sqrt{\frac{95}{12}}$ (D) None of these

Q.40 The equation of tangent to the ellipse $x^2 + 3y^2 = 3$ which is $\perp^r$ to line $4y = x - 5$ is-

(A) $4x + y + 7 = 0$ (B) $4x + y - 7 = 0$
(C) $4x + y - 3 = 0$ (D) None of these

LEVEL- 2

Q.1 The area of quadrilateral formed by tangents at the ends of latus-rectum of the ellipse $x^2 + 2y^2 = 2$ is-

(A) $\frac{8}{\sqrt{2}}$ (B) $8\sqrt{2}$
(C) 8 (D) None of these

Q.2 The equation $\frac{x^2}{10-a} + \frac{y^2}{4-a} = 1$ represents an ellipse if -

(A) $a < 4$ (B) $a > 4$
(C) $4 < a < 10$ (D) $a > 10$

Q.3 If the focal distance of an end of the minor axis of an ellipse (referred to its axes as the axes of x and y respectively) is k and the distance between its foci is 2h, then its equation is-

(A) $\frac{x^2}{k^2} + \frac{y^2}{h^2} = 1$

(B) $\frac{x^2}{k^2} + \frac{y^2}{k^2 - h^2} = 1$

(C) $\frac{x^2}{k^2} + \frac{y^2}{h^2 - k^2} = 1$

(D) $\frac{x^2}{k^2} + \frac{y^2}{k^2 + h^2} = 1$

Q.4 The locus of the mid-points of the portion of the tangents to the ellipse intercepted between the axes
is -

(A) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 4$ (B) $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 4$

(C) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 4$ (D) None of these

Q.5 If S and T are foci of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and B is an end of the minor axis. If STB is an equilateral triangle the eccentricity of ellipse is-

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{\sqrt{3}}{2}$

Q.6 The sum of the squares of the perpendicular on any tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ from two points on the minor axis each distance $\sqrt{a^2 - b^2}$ from the centre is -

(A) a^2 (B) b^2
(C) $2a^2$ (D) $2b^2$

Q.7 If (5, 12) and (24, 7) are the focii of an ellipse passing through origin, then the eccentricity of ellipse is -

(A) $\frac{\sqrt{386}}{38}$ (B) $\frac{\sqrt{386}}{12}$

(C) $\frac{\sqrt{386}}{13}$ (D) $\frac{\sqrt{386}}{25}$

Q.8 The common tangent of $x^2 + y^2 = 4$ and $2x^2 + y^2 = 2$ is-

(A) $x + y + 4 = 0$ (B) $x - y + 7 = 0$
(C) $2x + 3y + 8 = 0$ (D) None

Q.9 The eccentric angles of the extremities of latus rectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are given by-

(A) $\tan^{-1}\left(\pm \frac{ae}{b}\right)$ (B) $\tan^{-1}\left(\pm \frac{be}{a}\right)$

(C) $\tan^{-1}\left(\pm\frac{b}{ae}\right)$ (D) $\tan^{-1}\left(\pm\frac{a}{be}\right)$

Q.10 A point, ratio of whose distance from a fixed point and line x = 9/2 is always 2 : 3. Then locus of the point will be -

(A) Hyperbola (B) Ellipse
(C) Parabola (D) Circle

Q.11 If the minor axis of an ellipse subtends an angle 60° at each focus then the eccentricity of the ellipse is -

(A) $\sqrt{3}/2$ (B) $1/\sqrt{2}$
(C) $2/\sqrt{3}$ (D) None

Q.12 LL′ is the latus rectum of an ellipse and ΔSLL′ is an equilateral triangle. The eccentricity of the ellipse is -

(A) $1/\sqrt{5}$ (B) $1/\sqrt{3}$
(C) $1/\sqrt{2}$ (D) $\sqrt{2}/\sqrt{3}$

Q.13 If the latus rectum of the ellipse $x^2\tan^2\alpha + y^2\sec^2\alpha = 1$ is 1/2 then α =

(A) π/12 (B) π/6
(C) 5π/12 (D) None

Q.14 If P is a point on the ellipse of eccentricity e and A, A′ are the vertices and S, S′ are the focii then ΔSPS′ : Δ APA′ =

(A) e^3 (B) e^2
(C) e (D) 1/e

Q.15 The tangent at P on the ellipse meets the minor axis in Q, and PR is drawn perpendicular to the minor axis and C is the centre. Then CQ . CR =

(A) b^2 (B) $2b^2$
(C) a^2 (D) $2a^2$

Q.16 The circle on SS′ as diameter touches the ellipse then the eccentricity of the ellipse is
(where S and S′ are the focus of the ellipse)

(A) $2/\sqrt{3}$ (B) $\sqrt{3}/2$
(C) $1/\sqrt{2}$ (D) None of these

Q.17 The tangent at any point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the tangents at the vertices A, A′ in L and L′. Then AL. A′L′ =

(A) a + b (B) $a^2 + b^2$
(C) a^2 (D) b^2

Q.18 The tangent at any point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ to meets the major and minor axes in P and Q respectively, then $\frac{a^2}{CP^2}+\frac{b^2}{CQ^2}=$

(A) 4 (B) 3
(C) 2 (D) 1

Q.19 The locus of extremities of the latus rectum of the family of ellipses $b^2x^2 + a^2y^2 = a^2b^2$ is

(A) $x^2 - ay = a^2b^2$ (B) $x^2 - ay = b^2$
(C) $x^2 + ay = a^2$ (D) $x^2 + ay = b^2$

Q.20 The length of the common chord of the ellipse $\frac{(x-1)^2}{9}+\frac{(y-2)^2}{4}=1$ and the circle $(x-1)^2 + (y-2)^2 = 1$ is

(A) 0 (B) 1 (C) 3 (D) 8

Q.21 If any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ intercepts equal lengths ℓ on the axes, then ℓ =

(A) $\sqrt{a^2+b^2}$ (B) $a^2 + b^2$
(C) $(a^2 + b^2)^2$ (D) None of these

Q.22 If C is the centre of the ellipse $9x^2 + 16y^2 = 144$ and S is one focus. The ratio of CS to major axis, is

(A) $\sqrt{7}$: 16 (B) $\sqrt{7}$: 4
(C) $\sqrt{5}$: $\sqrt{7}$ (D) None of these

Q.23 P is a variable point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with AA′ as the major axis. Then, the maximum value of the area of the triangle APA′ is-

(A) ab (B) 2ab
(C) ab/2 (D) None of these

Q.24 If PSQ is a focal chord of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $a > b$, then the harmonic mean of SP and SQ is

(A) $\frac{b^2}{a}$ (B) $\frac{a^2}{b}$ (C) $\frac{2b^2}{a}$ (D) $\frac{2a^2}{b}$

Q.25 If the eccentricity of the ellipse $\frac{x^2}{a^2+1}+\frac{y^2}{a^2+2}=1$ be $\frac{1}{\sqrt{6}}$, then latus rectum of ellipse is -

(A) $\frac{5}{\sqrt{6}}$ (B) $\frac{10}{\sqrt{6}}$

(C) $\frac{8}{\sqrt{6}}$ (D) None of these

Q.26 Locus of the point which divides double ordinate of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in the ratio 1 : 2 internally, is

(A) $\frac{x^2}{a^2}+\frac{9y^2}{b^2}=1$ (B) $\frac{x^2}{a^2}+\frac{9y^2}{b^2}=\frac{1}{9}$

(C) $\frac{9x^2}{a^2}+\frac{9y^2}{b^2}=1$ (D) None of these

Q.27 A tangent having slope of –4/3 to the ellipse $\frac{x^2}{18}+\frac{y^2}{32}=1$ intersect the major and minor axes at A and B respectively. If C is the centre of ellipse then area of triangle ABC is-

(A) 12 (B) 24

(C) 36 (D) 48

Q.28 If F_1 and F_2 are the feet of the perpendiculars from the foci S_1 & S_2 of an ellipse $\frac{x^2}{5}+\frac{y^2}{3}=1$ on the tangent at any point P on the ellipse, then $(S_1 F_1).(S_2 F_2)$ is equal to-

(A) 2 (B) 3

(C) 4 (D) 5

Q.29 Equation of one of the common tangent of $y^2 = 4x$ and $\frac{x^2}{4}+\frac{y^2}{3}=1$ is equal to-

(A) $x + 2y + 4 = 0$ (B) $x + 2y - 4 = 0$

(C) $x - 2y - 4 = 0$ (D) None of these

Q.30 The eccentricity of ellipse which meets straight line $2x - 3y = 6$ on the X axis and $4x + 5y = 20$ on the Y axis and whose principal axes lie along the co-ordinate axes is equal to-

(A) $\frac{1}{2}$ (B) $\frac{4}{5}$

(C) $\frac{\sqrt{3}}{4}$ (D) $\frac{\sqrt{7}}{4}$

Q.31 If a circle of radius r is concentric with ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then common tangent is inclined to the major axis at an angle-

(A) $\tan^{-1}\sqrt{\frac{r^2-b^2}{a^2-r^2}}$ (B) $\tan^{-1}\sqrt{\frac{r^2-b^2}{r^2-a^2}}$

(C) $\tan^{-1}\sqrt{\frac{a^2-r^2}{r^2-b^2}}$ (D) None of these

Q.32 If the ellipse $\frac{x^2}{4}+\frac{y^2}{1}=1$ meet the ellipse $\frac{x^2}{1}+\frac{y^2}{a^2}=1$ in four distinct points and $a = b^2 - 10b + 25$ then which of the following is true

(A) $b < 4$ (B) $4 < b < 6$

(C) $b > 6$ (D) $b \in R - [4, 6]$

Q.33 An ellipse and a hyperbola have the same centre "origin", the same foci. The minor-axis of the one is the same as the conjugate axis of the other. If e_1, e_2 be their eccentricities respectively, then $\frac{1}{e_1^2}+\frac{1}{e_2^2}$ is equal to

(A) 1 (B) 2

(C) 4 (D) 3

Q.34 A parabola is drawn whose focus is one of the foci of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (where $a > b$) and whose directrix passes through the other focus and perpendicular to the major axes of the ellipse. Then the eccentricity of the ellipse for which the latus-rectum of the ellipse and the parabola are same, is

(A) $\sqrt{2}-1$ (B) $2\sqrt{2}+1$

(C) $\sqrt{2}+1$ (D) $2\sqrt{2}-1$

Assertion-Reason Type Question

The following questions given below consist of an "Assertion" (1) and "Reason "(2) Type questions. Use the following key to choose the appropriate answer.

(A) Both (1) and (2) are true and (2) is the correct explanation of (1)

(B) Both (1) and (2) are true but (2) is not the correct explanation of (1)

(C) (1) is true but (2) is false

(D) (1) is false but (2) is true

Q.35 **Statement- (1) :** From a point $(5, \lambda)$ perpendicular tangents are drawn to the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ then $\lambda = \pm 4$.

Statement- (2) : The locus of the point of intersection of perpendicular tangent to the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ is $x^2 + y^2 = 41$.

Passage : 1 (Q.36 to 38)

Variable tangent drawn to ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(a > b)$ intersects major and minor axis at points A & B in first quadrant then (where, O is the centre of the ellipse)

Q.36 Area of ΔOAB is minimum when $\theta =$

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$

(C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

Q.37 Minimum value of OA. OB is

(A) 2b (B) 2ab

(C) ab (D) b

Q.38 Locus of centroid of ΔOAB is $\frac{a^2}{x^2}+\frac{b^2}{y^2}=k^2$ then k =

(A) 1 (B) 2

(C) 3 (D) 4

Passage : 2 (Q.39 & 40)

A parabola P : $y^2 = 8x$, ellipse E : $\frac{x^2}{4}+\frac{y^2}{15}=1$.

Q.39 Equation of a tangent common to both the parabola P and the ellipse E is

(A) $x - 2y + 8 = 0$ (B) $2x - y + 8 = 0$

(C) $x + 2y - 8 = 0$ (D) $2x - y - 8 = 0$

Q.40 Point of contact of a common tangent to P and E on the ellipse is

(A) $\left(\frac{1}{2}, \frac{15}{4}\right)$ (B) $\left(-\frac{1}{2}, \frac{15}{4}\right)$

(C) $\left(\frac{1}{2}, -\frac{15}{2}\right)$ (D) $\left(-\frac{1}{2}, -\frac{15}{2}\right)$

COLUMN MATCHING QUESTIONS

Q.41

Column I	**Column II**
(A) eccentricity of $\frac{x^2}{64}+\frac{y^2}{39}=1$	(P) 10
(B) Length of latus-rectum of $\frac{x^2}{9}+\frac{y^2}{4}=1$	(Q) 8
(C) Length of major axis of $25x^2 + 16y^2 = 400$	(R) 5/8
(D) The length of minor axis of $16x^2 + 9y^2 = 144$	(S) 8/3
	(T) 6

LEVEL- 3

(Question asked in previous AIEEE and IIT-JEE)

SECTION -A

Q.1 If distance between the foci of an ellipse is equal to its minor axis, then eccentricity of the ellipse is- **[AIEEE-2002]**

(A) $e = \frac{1}{\sqrt{2}}$ (B) $e = \frac{1}{\sqrt{3}}$

(C) $e = \frac{1}{\sqrt{4}}$ (D) $e = \frac{1}{\sqrt{6}}$

Q.2 The equation of an ellipse, whose major axis = 8 and eccentricity = 1/2, is **[AIEEE-2002]**

(A) $3x^2 + 4y^2 = 12$ (B) $3x^2 + 4y^2 = 48$

(C) $4x^2 + 3y^2 = 48$ (D) $3x^2 + 9y^2 = 12$

Q.3 The eccentricity of an ellipse, with its centre at the origin, is $\frac{1}{2}$. If one of the directrices is x = 4, then the equation of the ellipse is- **[AIEEE- 2004]**

(A) $3x^2 + 4y^2 = 1$ (B) $3x^2 + 4y^2 = 12$

(C) $4x^2 + 3y^2 = 12$ (D) $4x^2 + 3y^2 = 1$

Q.4 In an ellipse, the distance between its foci is 6 and minor axis is 8. Then its eccentricity is- **[AIEEE- 2006]**

(A) $\frac{1}{2}$ (B) $\frac{4}{5}$

(C) $\frac{1}{\sqrt{5}}$ (D) $\frac{3}{5}$

Q.5 A focus of an ellipse is at the origin. The directrix is the line x = 4 and the eccentricity is $\frac{1}{2}$. Then the length of the semi-major axis is- **[AIEEE- 2008]**

(A) $\frac{2}{3}$ (B) $\frac{4}{3}$

(C) $\frac{5}{3}$ (D) $\frac{8}{3}$

Q.6 The ellipse $x^2 + 4y^2 = 4$ is inscribed in a rectangle aligned with the coordinate axes, which in turn is inscribed in another ellipse that passes through the point (4, 0), then the equation of the ellipse is- **[AIEEE- 2009]**

(A) $x^2 + 16y^2 = 16$ (B) $x^2 + 12y^2 = 16$

(C) $4x^2 + 48y^2 = 48$ (D) $4x^2 + 64y^2 = 48$

Q.7 Equation of the ellipse whose axes are the axes of coordinates and which passes through the point (–3, 1) and has eccentricity $\sqrt{\frac{2}{5}}$ is – **[AIEEE- 2011]**

(A) $3x^2 + 5y^2 - 32 = 0$

(B) $5x^2 + 3y^2 - 48 = 0$

(C) $3x^2 + 5y^2 - 15 = 0$

(D) $5x^2 + 3y^2 - 32 = 0$

SECTION -B

Q.1 Let P be a variable point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with foci F_1 and F_2. If A is the area of the triangle $PF_1\ F_2$, then the maximum value of A is- **[IIT-1994]**

(A) 2abe (B) abe

(C) $\frac{1}{2}$abe (D) None of these

Q.2 If P(x, y), F_1= (3,0), F_2= (– 3, 0) and $16x^2 + 25\ y^2 = 400$, then $P\ F_1 + P\ F_2$ = **[IIT-1996]**

(A) 8 (B) 6

(C) 10 (D) 12

Q.3 An ellipse has OB as semi - minor axis. F and F' are its foci and the angle FBF' is a right angle. Then the eccentricity of the ellipse is- **[IIT- 97/AIEEE-2005]**

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$

(C) $\frac{2}{3}$ (D) $\frac{1}{3}$

Q.4 The number of values of c such that the straight line y = 4x + c touches the curve $\frac{x^2}{4} + y^2 = 1$ is **[IIT-1998]**

(A) 0 (B) 1 (C) 2 (D) infinite

Q.5 Locus of middle point of segment of tangent to ellipse $x^2 + 2y^2 = 2$ which is intercepted between the coordinate axes, is- **[IIT Scr. 2004]**

(A) $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$ (B) $\frac{1}{4x^2} + \frac{1}{2y^2} = 1$

(C) $\frac{x^2}{2} + \frac{y^2}{4} = 1$ (D) $\frac{x^2}{4} + \frac{y^2}{2} = 1$

Q.6 A tangent is drawn at some point P of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is intersecting to the coordinate axes at points A & B then minimum area of the Δ OAB is- **[IIT Scr. 2005]**
(where O is the centre of ellipse.)

(A) ab (B) $\frac{a^2+b^2}{2}$

(C) $\frac{a^2+b^2}{4}$ (D) $\frac{a^2+b^2-ab}{3}$

Q.7 The line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse $x^2 + 9y^2 = 9$ meets its auxiliary circle at the point M. Then the area of the triangle with vertices at A, M and the origin O is **[IIT -2009]**

(A) $\frac{31}{10}$ (B) $\frac{29}{10}$

(C) $\frac{21}{10}$ (D) $\frac{27}{10}$

Passage : (Q8 to Q.10)

Tangents are drawn from the point P(3, 4) to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$, touching the ellipse at points A and B. **[IIT 2010]**

Q.8 The coordinates of A and B are

(A) (3, 0) and (0, 2)

(B) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ and $\left(-\frac{9}{5}, \frac{8}{5}\right)$

(C) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ and (0, 2)

(D) (3, 0) and $\left(-\frac{9}{5}, \frac{8}{5}\right)$

Q.9 The orthocentre of the triangle PAB is

(A) $\left(5, \frac{8}{7}\right)$ (B) $\left(\frac{7}{5}, \frac{25}{8}\right)$

(C) $\left(\frac{11}{5}, \frac{8}{5}\right)$ (D) $\left(\frac{8}{25}, \frac{7}{5}\right)$

Q.10 The equation of the locus of the point whose distances from the point P and the line AB are equal, is

(A) $9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$

(B) $x^2 + 9y^2 + 6xy - 54x + 62y - 241 = 0$

(C) $9x^2 + 9y^2 - 6xy - 54x - 62y - 241 = 0$

(D) $x^2 + y^2 - 2xy + 27x + 31y - 120 = 0$

ANSWER KEY

LEVEL- 1

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	B	B	B	C	B	B	A	B	C	A	A	B	B	B	B	D	C	B	B	A
Q.No.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
Ans.	B	A	B	A	B	A	A	A	C	A	A	A	C	D	C	C	C	D	B	A,B

LEVEL- 2

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	A	A	B	B	B	C	A	D	C	B	A	B	A	C	A	C	D	D	C	A
Q.No.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
Ans.	A	D	A	A	B	A	B	B	A	D	A	A,C,D	B	A	A	C	B	C	A	B

41. **(A) $\rightarrow$ R ; (B) $\rightarrow$ S; (C) $\rightarrow$ P; (D) $\rightarrow$ T**

LEVEL- 3

SECTION-A

Qus.	1	2	3	4	5	6	7
Ans.	A	B	B	D	D	B	A,B

SECTION-B

Q.No.	1	2	3	4	5	6	7	8	9	10
Ans.	B	C	B	C	A	A	D	D	C	A