SOLVED EXAMPLES

Ex.1 Which of the following is a function?

(A) {(2,1), (2,2), (2,3), (2,4)}

(B) {(1,4), (2,5), (1,6) , (3,9)}

(C) {(1,2), (3,3), (2,3), (1,4)}

(D) { (1,2), (2,2), (3,2), (4,2)}

Sol. We know that for a relation to be function every element of first set should be associated with one and only one element of second set but elements of first set can have same f-image in second set which is given in (D).

Ans.[D]

Ex.2 If $f(x) = \frac{x}{x-1} = \frac{1}{y}$, then f (y) equals

(A) x (B) x – 1

(C) x + 1 (D) 1 – x

Sol. $f(y) = \frac{y}{y-1} = \frac{(x-1)/x}{\frac{x-1}{x}-1} = \frac{x-1}{x-1-x} = 1 - x.$

Ans.[D]

Ex.3 The domain of $f(x) = \frac{1}{x^3 - x}$ is -

(A) R – {–1,0,1} (B) R

(C) R – {0,1} (D) None of these

Sol. Domain = $\{x; x \in R; x^3 - x \neq 0\}$

= R – {–1, 0,1} **Ans.[A]**

Ex.4 The range of $f(x) = \cos \frac{\pi[x]}{2}$ is -

(A) {0,1} (B) {–1,1}

(C) {–1,0,1} (D) [–1,1]

Sol. [x] is an integer, cos (–x) = cos x and

$\cos\left(\frac{\pi}{2}\right) = 0, \cos 2\left(\frac{\pi}{2}\right) = -1.$

$\cos 0\left(\frac{\pi}{2}\right) = 1, \cos 3\left(\frac{\pi}{2}\right) = 0,.....$

Hence range = {–1,0,1} **Ans.[C]**

Ex.5 If $f : R^+ \to R^+$, $f(x) = x^2 + 2$ and

$g : R^+ \to R^+$, $g(x) = \sqrt{x+1}$

then (f + g) (x) equals -

(A) $\sqrt{x^2+3}$ (B) x + 3

(C) $\sqrt{x^2+2} + (x+1)$ (D) $x^2 + 2 + \sqrt{(x+1)}$

Sol. (f + g) (x) = f(x) + g(x)

$= x^2 + 2 + \sqrt{x+1}$ **Ans. [D]**

Ex.6 Function $f(x) = x^{-2} + x^{-3}$ is -

(A) a rational function

(B) an irrational function

(C) an inverse function

(D) None of these

Sol. $f(x) = \frac{1}{x^2} + \frac{1}{x^3} = \frac{x+1}{x^3}$

= ratio of two polynomials

$\therefore$ f(x) is a rational function. **Ans.[A]**

Ex.7 The period of | sin 2x | is-

(A) π/4 (B) π/2 (C) π (D) 2π

Sol. Here $|\sin 2x| = \sqrt{\sin^2 2x}$

$= \sqrt{\frac{(1 - \cos 4x)}{2}}$

Period of cos 4 x is π/2

Period of |sin 2x | will be π/2. **Ans.[B]**

Ex.8 If $f(x) = \frac{x-3}{x+1}$, then f [f {f (x)}] equals -

(A) x (B) 1/x (C) –x (D) –1/x

Sol. Here $f\{f(x)\} = f\left(\frac{x-3}{x+1}\right) = \frac{\left(\frac{x-3}{x+1}\right) - 3}{\left(\frac{x-3}{x+1}\right) + 1} = \frac{x+3}{1-x}$

$\therefore$ $f[f\{f(x)\}] = \frac{\frac{x+3}{1-x} - 3}{\frac{x+3}{1-x} + 1} = \frac{4x}{4} = x$ **Ans. [A]**

Ex.9 If $f(x) = 2|x - 2| - 3|x - 3|$, then the value of $f(x)$ when $2 < x < 3$ is -

(A) $5 - x$ (B) $x - 5$

(C) $5x - 13$ (D) None of these

Sol. $2 < x < 3 \Rightarrow |x - 2| = x - 2$

$|x - 3| = 3 - x$

$f(x) = 2(x - 2) - 3(3 - x) = 5x - 13$. **Ans. [C]**

Ex.10 Which of the following functions defined from R to R are one-one -

(A) $f(x) = |x|$ (B) $f(x) = \cos x$

(C) $f(x) = e^x$ (D) $f(x) = x^2$

Sol. $x_1 \neq x_2 \Rightarrow e^{x_1} \neq e^{x_2}$

$\Rightarrow f(x_1) \neq f(x_2)$

$\therefore$ $f(x) = e^x$ is one-one. **Ans. [C]**

Ex.11 The function $f : R \to R$, $f(x) = x^2$ is -

(A) one-one but not onto

(B) onto but not one-one

(C) one-one onto

(D) None of these

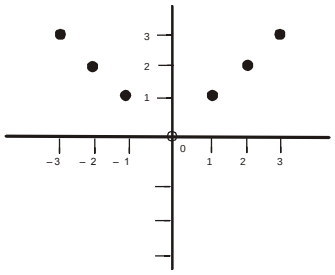
Sol. $\because$ $4 \neq -4$, but $f(4) = f(-4) = 16$

$\therefore$ f is many one function.

Again $f(R) = R^+ \cup \{0\} \subset R$, therefore f is into.

Ans. [D]

Ex.12 If $f : I_0 \to N$, $f(x) = |x|$, then f is -



(A) one-one (B) onto

(C) one-one onto (D) none of these

Sol. Observing the graph of this function, we find that every line parallel to x-axis meets its graph at more than one point so it is not one-one.

Now range of f = N = Co-domain, so it is onto.

Ans. [B]

Ex.13 If $f : R - \{3\} \to R - \{1\}$, $f(x) = \frac{x - 2}{x - 3}$ then function f(x) is -

(A) Only one-one (B) one-one into

(C) Many one onto (D) one-one onto

Sol. $\because$ $f(x) = \frac{x - 2}{x - 3}$

$\therefore$ $f'(x) = \frac{(x - 3).1 - (x - 2).1}{(x - 3)^2} = \frac{-1}{(x - 3)^2}$

$\therefore$ $f'(x) < 0 \ \forall \ x \in R - \{3\}$

$\therefore$ f(x) is monotonocally decreasing function

$\Rightarrow$ f is one-one function.

onto/ into : Let $y \in R - \{1\}$ (co-domain)

Then one element $x \in R - \{3\}$ is domain is such that

$f(x) = y \Rightarrow \frac{x - 2}{x - 3} = y \Rightarrow x - 2 = xy - 3y$

$\Rightarrow x = \left(\frac{3y - 2}{y - 1}\right) = x \in R - \{3\}$

$\therefore$ the pre-image of each element of co-domain $R - \{1\}$ exists in domain $R - \{3\}$.

$\Rightarrow$ f is onto. **Ans. [D]**

Ex.14 Function $f : N \to N$, $f(x) = 2x + 3$ is -

(A) one-one onto (B) one-one into

(C) many one onto (D) many one into

Sol. f is one-one because for any $x_1, x_2 \in N$

$x_1 \neq x_2 \Rightarrow 2x_1 + 3 \neq 2x_2 + 3 \Rightarrow f(x_1) \neq f(x_2)$

Further $f^{-1}(x) = \frac{x - 3}{2} \notin N$ (domain) when

$x = 1, 2, 3$ etc.

$\therefore$ f is into which shows that f is one- one into.

Alter

$f(x) = 2x + 3$

$f'(x) = 2 > 0 \ \forall \ x \in N$

$\therefore$ f(x) is increasing function

$\therefore$ f(x) is one-one function

& $\because$ $x = 1, 2, 3, ...$

$\therefore$ min value of f(x) is $2.1 + 3 = 5$

$\therefore$ $f(x) \neq \{1, 2, 3, 4\}$

$\therefore$ Co Domain $\neq$ Range

$\therefore$ f(x) is into function **Ans. [B]**

Ex.15 Function f : R $\to$ R, $f(x) = x^3 - x$ is -

(A) one-one onto (B) one-one into

(C) many-one onto (D) many-one into

Sol. Since $-1 \neq 1$, but $f(-1) = f(1)$, therefore f is many-one.

Also let, $f(x) = x^3 - x = \alpha \Rightarrow x^3 - x - \alpha = 0$. This is a cubic equation in x which has at least one real root because complex roots always occur in pairs. Therefore each element of co-domain R has pre-image in R. Thus function f in onto .

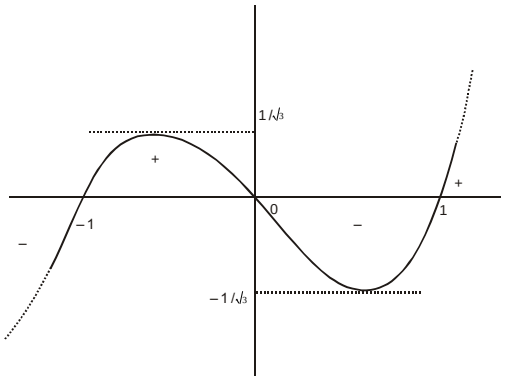
$\therefore$ function f is many-one onto.

Alter

$f(x) = x^3 - x$

$= x(x-1)(x+1)$

graph of f(x) is



from graph function is many one- onto function

Ans. [C]

Ex.16 If f : R $\to$ R, $f(x) = 2x - 1$ and g : R $\to$ R, $g(x) = x^2 + 2$, then (gof) (x) equals-

(A) $2x^2 - 1$ (B) $(2x-1)^2$

(C) $2x^2 + 3$ (D) $4x^2 - 4x + 3$

Sol. Here (gof) (x) = g [f(x)] = g (2x – 1)

$= (2x-1)^2 + 2 = 4x^2 - 4x + 3$. **Ans. [D]**

Ex.17 If f : R $\to$ R, $f(x) = 4x^3 + 3$, then $f^{-1}(x)$ equals-

(A) $\left(\frac{x-3}{4}\right)^{1/3}$ (B) $\left(\frac{x^{1/3}-3}{4}\right)$

(C) $\frac{1}{4}(x-3)^{1/3}$ (D) None of these

Sol. Since f is a bijection, therefore f^{-1} exists. Now if f-image of x is y, then f^{-1}: R $\to$ R defined as follows :

$f^{-1}(y) = x \Rightarrow f(x) = y$

But $f(x) = 4x^3 + 3 \Rightarrow y = 4x^3 + 3 \Rightarrow x = \left(\frac{y-3}{4}\right)^{1/3}$

Therefore $f^{-1}(y) = \left(\frac{y-3}{4}\right)^{1/3}$

$\Rightarrow f^{-1}(x) = \left(\frac{x-3}{4}\right)^{1/3}$ **Ans. [A]**

Ex.18 $f(x) = \sqrt{|x-1|}$ and g (x) = sin x then (fog) (x) equals -

(A) $\sin\left\{\sqrt{|x-1|}\right\}$

(B) $|\sin x/2 - \cos x/2|$

(C) $|\sin x - \cos x|$

(D) None of these

Sol. (fog) (x) = f [g(x)] = f [sin x]

$= \sqrt{|\sin x - 1|}$

$= \sqrt{|1 - \sin x|}$

$= \sqrt{|\sin^2 x/2 + \cos^2 x/2 - 2\sin x/2 \cos x/2|}$

$= \sqrt{|(\sin x/2 - \cos x/2)^2|}$

$= |\sin x/2 - \cos x/2|$ **Ans.[B]**

Ex.19 If f : R $\to$ R, $f(x) = 2x + 1$ and g : R $\to$ R, $g(x) = x^3$,then $(gof)^{-1}(27)$ equals -

(A) –1 (B) 0 (C) 1 (D) 2

Sol. Here $f(x) = 2x + 1$ $f^{-1}(x) = \frac{x-1}{2}$

and $g(x) = x^3 \Rightarrow g^{-1}(x) = x^{1/3}$

$\because (gof)^{-1}(27) = (f^{-1}og^{-1})(27)$

$= f^{-1}[g^{-1}(27)] = f^{-1}[(27)^{1/3}]$

$= f^{-1}(3) = \frac{3-1}{2} = 1$ **Ans.[C]**

Ex.20 The domain of function $f(x) = \sqrt{2^x - 3^x}$ is -

(A) $(-\infty, 0]$ (B) R

(C) $[0, \infty)$ (D) No value of x

Sol. Domain = $\{x ; 2^x - 3^x \geq 0\} = \{x ; (2/3)^x \geq 1\}$

$= x \in (-\infty, 0]$ **Ans.[A]**

Ex.21 The domain of the function

$f(x) = \sin^{-1}\left(\log_2 \frac{x^2}{2}\right)$ is -

(A) $[-2, 2] - (-1, 1)$ (B) $[-1,2] - \{0\}$

(C) $[1, 2]$ (D) $[-2,2] - \{0\}$

Sol. We know that the domain of $\sin^{-1}x$ is $[-1,1]$. So for f(x) to be meaningful , we must have

$$-1 \le \log_2 \frac{x^2}{2} \le 1$$

$\Rightarrow 2^{-1} \le x^2/2 \le 2 \quad x \ne 0$

$\Rightarrow 1 \le x^2 \le 4, x \ne 0$

$\Rightarrow x \in [-2, -1] \cup [1,2]$

$\Rightarrow x \in [-2, 2] - (-1, 1)$ **Ans.[A]**

Ex.22 The range of function $f(x) = \frac{x^2}{1 + x^2}$ is -

(A) $R - \{1\}$ (B) $R^+ \cup \{0\}$

(C) $[0, 1]$ (D) None of these

Sol. Range is containing those real numbers y for which f(x) = y where x is real number.

Now $f(x) = y = \Rightarrow \frac{x^2}{1 + x^2} = y$

$$\Rightarrow x = \sqrt{\frac{y}{(1 - y)}} \qquad ....(1)$$

by (1) clearly $y \ne 1$, and for x to be real

$\frac{y}{1 - y} \ge 0 \Rightarrow \quad y \ge 0$ and $y < 1$.

($\because$ If y = 2 then $\frac{y}{1 - y} = \frac{2}{1 - 2} = (-2)$ and

$\sqrt{\frac{y}{(1 - y)}} = \sqrt{-2} \notin R$)

$\therefore \quad 0 \le y < 1$

$\therefore$ Range of function = $(0 \le y < 1) = [0,1)$

Ans.[D]

Ex.23 If f(x) = cos (log x), then

f(x) f(y) – 1/2 [f (x/y) + f(xy)] is equal to

(A) –1 (B) 1/2

(C) –2 (D) 0

Sol. cos (log x) cos (log y)

$$-\frac{1}{2}[\cos(\log x/y) + \cos(\log xy)]$$

$$= \frac{1}{2}[\cos(\log x + \log y) + \cos(\log x - \log y)]$$

$$-\frac{1}{2}[\cos(\log x - \log y) + \cos(\log x + \log y)]$$

= 0 **Ans.[D]**

Ex.24 If $f(x) = \frac{2^x + 2^{-x}}{2}$, then f (x + y) . f (x – y) is equal to -

(A) $\frac{1}{2}$ [f (x+ y) + f(x – y)]

(B) $\frac{1}{2}$ [f (2x) + f (2y)]

(C) $\frac{1}{2}$ [f(x+ y). f(x – y)]

(D) None of these

Sol. $f(x + y). f(x - y) = \frac{2^{x+y} + 2^{-x-y}}{2} \cdot \frac{2^{x+y} + 2^{-x-y}}{2}$

$$= \frac{2^{2x} + 2^{2y} + 2^{-2x} + 2^{-2y}}{4}$$

$$= \frac{1}{2}\left[\frac{2^{2x} + 2^{-2x}}{2} \cdot \frac{2^{2y} + 2^{-2y}}{2}\right]$$

$= \frac{1}{2}$ [f (2x) + f(2y)] **Ans.[B]**

Ex.25 If f : R $\to$ R. f (x) = 2x + |x| , then f (3x) – f (–x) – 4x equals -

(A) f(x) (B) – f(x)

(C) f (–x) (D) 2f(x)

Sol. f (3x) – f (–x) – 4x

= 6x + |3x| – {–2x + | –x|} – 4x

= 6x + 3 |x| + 2x – |x| – 4x

= 4x + 2 |x| = 2 f(x). **Ans.[D]**

Ex.26 If $g(x) = x^2 + x - 2$ and $\frac{1}{2}$ (gof) (x) = $2x^2 - 5x + 2$, then f(x) is equal to -

(A) 2x – 3 (B) 2x + 3

(C) $2x^2 + 3x + 1$ (D) $2x^2 - 3x - 1$

Sol. $g(x) = x^2 + x - 2$

$\Rightarrow (gof)(x) = g[f(x)] = [f(x)]^2 + f(x) - 2$

Given, $\frac{1}{2}(gof)(x) = 2x^2 - 5x + 2$

$\therefore \frac{1}{2}[f(x)]^2 + \frac{1}{2}f(x) - 1 = 2x^2 - 5x + 2$

$\Rightarrow [f(x)]^2 + f(x) = 4x^2 - 10x + 6$

$\Rightarrow f(x)[f(x) + 1] = (2x - 3)[(2x - 3) + 1]$

$\Rightarrow f(x) = 2x - 3$ **Ans.[A]**

Ex.27 If $f(x) = |x|$ and $g(x) = [x]$, then value of

$fog\left(-\frac{1}{4}\right) + gof\left(-\frac{1}{4}\right)$ is -

(A) 0 (B) 1

(C) –1 (D) 1/4

Sol. $fog = f\left[g\left(-\frac{1}{4}\right)\right]$ $f(-1) = 1$

and $gof\left(-\frac{1}{4}\right) = g\left[f\left(-\frac{1}{4}\right)\right] = g\left(\frac{1}{4}\right) = [1/4] = 0$

Required value = 1 + 0 = 1. **Ans.[B]**

LEVEL- 1

Question based on **Inequation**

Q.1 The inequality $\frac{2}{x} < 3$ is true, when x belongs to-

(A) $\left[\frac{2}{3}, \infty\right)$ (B) $\left(-\infty, \frac{2}{3}\right]$

(C) $\left(\frac{2}{3}, \infty\right) \cup (-\infty, 0)$ (D) none of these

Q.2 $\frac{x+4}{x-3} < 2$ is satisfied when x satisfies-

(A) $(-\infty, 3) \cup (10, \infty)$ (B) (3, 10)

(C) $(-\infty, 3) \cup [10, \infty)$ (D) none of these

Q.3 Solution of $\frac{2x-3}{3x-5} \geq 3$ is -

(A) $\left[1, \frac{12}{7}\right)$ (B) $\left(\frac{5}{3}, \frac{12}{7}\right]$

(C) $\left(-\infty, \frac{5}{3}\right)$ (D) $\left[\frac{12}{7}, \infty\right)$

Q.4 Solution of $(x-1)^2 (x+4) < 0$ is-

(A) $(-\infty, 1)$ (B) $(-\infty, -4)$

(C) (–1, 4) (D) (1, 4)

Q.5 Solution of $(2x+1)(x-3)(x+7) < 0$ is-

(A) $(-\infty, -7) \cup \left(-\frac{1}{2}, 3\right)$

(B) $(-\infty, -7) \cup \left(\frac{1}{2}, 3\right)$

(C) $(-\infty, 7) \cup \left(-\frac{1}{2}, 3\right)$

(D) $(-\infty, -7) \cup (3, \infty)$

Q.6 If $x^2 + 6x - 27 > 0$ and $x^2 - 3x - 4 < 0$, then-

(A) $x > 3$ (B) $x < 4$

(C) $3 < x < 4$ (D) $x = \frac{7}{2}$

Q.7 If $x^2 - 1 \leq 0$ and $x^2 - x - 2 \geq 0$, then x line in the interval/set

(A) (–1, 2) (B) (–1, 1)

(C) (1, 2) (D) {– 1}

Question based on **Definition of function**

Q.8 Which of the following relation is a function ?

(A) {(1,4), (2,6), (1,5), (3,9)}

(B) {(3,3), (2,1), (1,2), (2,3)}

(C) {(1,2), (2,2,), (3,2), (4,2)}

(D) {(3,1), (3,2), (3,3), (3,4)}

Q.9 If $x, y \in R$, then which of the following rules is not a function-

(A) $y = 9 - x^2$ (B) $y = 2x^2$

(C) $y = \sqrt{x} - |x|$ (D) $y = x^2 + 1$

Question based on **Even and Odd function**

Q.10 Which one of the following is not an odd function-

(A) sin x (B) tan x

(C) tan h x (D) None of these

Q.11 The function $f(x) = \frac{\sin^4 x + \cos^4 x}{x + \tan x}$ is -

(A) odd

(B) Even

(C) neither even nor odd

(D) odd and periodic

Q.12 $f(x) = \cos \log (x + \sqrt{1 + x^2})$ is

(A) even function

(B) odd function

(C) neither even nor odd

(D) constant

Q.13 A function whose graph is symmetrical about the y-axis is given by-

(A) $f(x) = \log_e (x + \sqrt{x^2 + 1})$

(B) $f(x + y) = f(x) + f(y)$ for all $x, y \in R$

(C) $f(x) = \cos x + \sin x$

(D) None of these

Q.14 Which of the following is an even function?

(A) $x\frac{a^x-1}{a^x+1}$ (B) tan x

(C) $\frac{a^x-a^{-x}}{2}$ (D) $\frac{a^x+1}{a^x-1}$

Q.15 In the following, odd function is -

(A) $\cos x^2$ (B) $(e^x+1)/(e^x-1)$

(C) $x^2-|x|$ (D) None of these

Q.16 The function $f(x) = x^2 - |x|$ is-

(A) an odd function

(B) a rational function

(C) an even function

(D) None of these

Question based on **Periodic function**

Q.17 The period of $\sin^4 x + \cos^4 x$ is -

(A) π (B) $\pi/2$

(C) 2π (D) None of these

Q.18 The period of function $|\cos 2x|$ is -

(A) π (B) $\pi/2$ (C) 4π (D) 2π

Q.19 The period of function $\sin\left(\frac{\pi x}{2}\right) + \cos\left(\frac{\pi x}{2}\right)$ is-

(A) 4 (B) 6 (C) 12 (D) 24

Q.20 The period of the function

$f(x) = \log \cos 2x + \tan 4x$ is-

(A) $\pi/2$ (B) π

(C) 2π (D) $2\pi/5$

Q.21 The period of the function $f(x) = 2\cos\frac{1}{3}(x-\pi)$ is -

(A) 6π (B) 4π (C) 2π (D) π

Q.22 In the following which function is not periodic-

(A) tan 4x (B) cos 2πx

(C) $\cos x^2$ (D) $\cos^2 x$

Question based on **Domain, Co-domain and Range of function**

Q.23 Domain of the function $f(x) = \frac{1}{\sqrt{x+2}}$ is-

(A) R (B) $(-2, \infty)$

(C) $[2, \infty]$ (D) $[0, \infty]$

Q.24 The domain where function $f(x) = 2x^2 - 1$ and $g(x) = 1 - 3x$ are equal, is-

(A) {1/2} (B) {2}

(C) {1/2, 2} (D) {1/2, –2}

Q.25 The domain of the function $\log\sqrt{\frac{3-x}{2}}$ is-

(A) $(3, \infty)$ (B) $(-\infty, 3)$

(C) (0, 3) (D) (–3, 3)

Q.26 Domain of the function $\cos^{-1}(4x-1)$ is-

(A) (0,1/2) (B) [0,1/2]

(C) [1/2,2] (D) None of these

Q.27 Domain of the function $\log|x^2 - 9|$ is-

(A) R (B) R– [–3, 3]

(C) R – {–3, 3} (D) None of these

Q.28 The domain of the function-

$f(x) = \sqrt{x-1} + \sqrt{6-x}$ is-

(A) (1, 6) (B) [1, 6]

(C) $[1, \infty)$ (D) $(-\infty, 6]$

Q.29 The domain of the function

$f(x) = \sqrt{(2-2x-x^2)}$ is -

(A) $-\sqrt{3} \le x \le \sqrt{3}$ (B) $-1-\sqrt{3} \le x \le -1+\sqrt{3}$

(C) $-2 \le x \le 2$ (D) $-2+\sqrt{3} \le x \le -2-\sqrt{3}$

Q.30 Domain of a function $f(x) = \sin^{-1} 5x$ is-

(A) $\left(-\frac{1}{5}, \frac{1}{5}\right)$ (B) $\left[-\frac{1}{5}, \frac{1}{5}\right]$

(C) R (D) $\left(0, \frac{1}{5}\right)$

Q.31 The range of the function f : R→R, f(x) = $\tan^{-1}$ x is-

(A) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (B) $\left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$

(C) R (D) None of these

Q.32 The range of f(x) = sin $\frac{\pi}{2}$ [x] is -

(A) {–1, 1} (B) {–1, 0, 1}

(C) {0, 1} (D) [–1, 1]

Q.33 Domain and range of f(x) = $\frac{|x-3|}{x-3}$ are respectively-

(A) R, [–1, 1] (B) R– {3}, {1, –1}

(C) R^+, R (D) None of these

Q.34 The domain of the function f(x) = sin 1/x is -

(A) R (B) R^+ (C) R_0 (D) R^-

Q.35 Range of the function f(x) = 9 – 7 sin x is-

(A) (2, 16) (B) [2, 16]

(C) [–1, 1] (D) (2, 16]

Q.36 For real values of x, range of function

y = $\frac{1}{2-\sin 3x}$ is -

(A) $\frac{1}{3} \le y \le 1$ (B) $-\frac{1}{3} \le y \le 1$

(C) $-\frac{1}{3} > y > -1$ (D) $\frac{1}{3} > y > 1$

Q.37 If f : R → R, f(x) = $\begin{cases} 1, & \text{when } x \in Q \\ -1, & \text{when } x \notin Q \end{cases}$, then image set of R under f is -

(A) {1, 1} (B) (–1, –1)

(C) {1, –1} (D) None of these

Q.38 If f : R→R, f(x) = x^2, then {x|f(x) = –1} equals-

(A) {–1, 1} (B) {1}

(C) ϕ (D) None of these

Q.39 The range of f(x) = cos 2x – sin 2x contains the set -

(A) [2, 4] (B) [–1, 1]

(C) [–2, 2] (D) [–4, 4]

Q.40 If the domain of the function f(x) = $\frac{|x|}{x}$ be [3, 7] then its range is-

(A) [–1, 1] (B) {–1, 1}

(C) {1} (D) {–1}

Q.41 The domain of the function f(x) = $\frac{1}{\sqrt{x-[x]}}$ is-

(A) R (B) R–Z

(C) Z (D) None of these

Q.42 The range of the function f(x) = 2 + x – [x–3] is-

(A) [5, 6] (B) [5, 6)

(C) R (D) None of these

Question based on Value of function

Q.43 If f is a real function satisfying the relation f(x + y) = f (x) f(y) for all x, y ∈ R and f(1) = 2, then a ∈ N, for which $\sum_{k=1}^{n} f(a+k) = 16(2^n - 1)$, is given by -

(A) 2 (B) 4

(C) 3 (D) None of these

Q.44 If f : R → R, f(x) = $\begin{cases} 1, & \text{when } x \in Q \\ -1, & \text{when } x \notin Q \end{cases}$, then which of the following statement is wrong ?

(A) f ($\sqrt{2}$)= –1 (B) f(π) = –1

(C) f(e) = 1 (D) f($\sqrt{4}$)= 1

Q.45 If f (x) = $\frac{x(x-1)}{2}$, then the value of f (x + 2) is-

(A) f(x) + f(x + 1) (B) $\frac{x+2}{x}$ f(x + 1)

(C) $\frac{(x+1)}{2}$ f(x +1) (D) $\frac{(x+2)}{2}$ f(x +1)

Q.46 If f(x + ay, x – ay) = axy, then f(x, y) equals-

(A) $\frac{x^2+y^2}{4}$ (B) $\frac{x^2-y^2}{4}$

(C) x^2 (D) y^2

Q.47 If f(x) = cos (log x), then $\frac{f(xy)+f(x/y)}{f(x)f(y)}$ equals-

(A) 1 (B) –1 (C) 0 (D) 2

Q.48 If f(x) = |x| + |x – 1|, then for 0 < x < 1, f(x) equals-

(A) 1 (B) –1 (C) 2x + 1 (D) 2x – 1

Q.49 f(2x + 3y, 2x – 7y) = 20 x then f(x , y) equals to –

(A) 7x – 3y (B) 7x + 3y

(C) 3x – 7y (D) x – 10y

Q.50 If f(x) = $\log_a x$, then f(ax) equals-

(A) f(a) f(x) (B) 1+ f(x)

(C) f(x) (D) a f(x)

Q.51 If f(x) = (ax – c)/(cx – a) = y, then f(y) equals-

(A) x (B) 1/x (C) 1 (D) 0

Question based on **Mapping**

Q.52 If f : $I \to I$, $f(x) = x^3 + 1$, then *f* is -

(A) one-one but not onto

(B) onto but not one-one

(C) One-one onto

(D) None of these

Q.53 Function f : R → R, f(x) = x |x| is -

(A) one-one but not onto

(B) onto but not one-one

(C) one-one onto

(D) neither one-one nor onto

Q.54 f : R → R, $f(x) = \frac{x^2}{1+x^2}$, is -

(A) many-one function

(B) odd function

(C) one-one function

(D) None of these

Q.55 If f : $R_0 \to R_0$, $f(x) = \frac{1}{x}$, then f is -

(A) one-one but not onto

(B) onto but not one-one

(C) neither one-one nor onto

(D) both one-one and onto

Q.56 Function f : R → R, f(x) = x + |x| is

(A) one-one (B) onto

(C) one-one onto (D) None of these

Q.57 Function f : $\left]\frac{\pi}{2}, \frac{3\pi}{2}\right[\to$ R, f(x) = tan x is

(A) one-one (B) onto

(C) one-one onto (D) None of these

Q.58 Function f : $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right] \to [-1,1]$, f(x) = sin x is -

(A) one-one (B) onto

(C) one-one onto (D) None of these

Q.59 f : N → N where f(x) = x – $(-1)^x$ then f ' is -

(A) one-one and into (B) many-one and into

(C) one-one and onto (D) many-one and onto

Q.60 If f : R→R, f(x) = $e^x + e^{-x}$, then f is -

(A) one-one but not onto

(B) onto but not one-one

(C) neither one-one nor onto

(D) both one-one and onto

Q.61 If f : R → [–1,1], f(x) = sin x, then f is-

(A) one-one onto (B) one-one into

(C) many-one onto (D) many-one into

Q.62 If f : R→R , f(x) = $\sin^2 x + \cos^2 x$, then f is -

(A) one-one but not onto

(B) onto but not one-one

(C) neither one-one nor onto

(D) both one-one onto

Q.63 Which of the following functions from Z to itself are bijections ?

(A) $f(x) = x^3$ (B) f(x) = x + 2

(C) f(x) = 2x + 1 (D) $f(x) = x^2 + x$

Q.64 Which of the following functions from A = {x: –1≤ x ≤1} to itself are bijections ?

(A) $f(x) = \frac{x}{2}$ (B) $g(x) = \sin\left(\frac{\pi x}{2}\right)$

(C) h(x) = |x| (D) $k(x) = x^2$

Q.65 Which of the following function is onto ?
(A) $f : R \to R$; $f(x) = 3^x$
(B) $f : R \to R^+$; $f(x) = e^{-x}$
(C) $f: [0,\pi/2] \to [-1,1]$; $f(x) = \sin x$
(D) $f : R \to R$: $f(x) = \cosh x$

Q.66 Which of the following function defined from R to R is onto ?
(A) $f(x) = |x|$ (B) $f(x) = e^{-x}$
(C) $f(x) = x^3$ (D) $f(x) = \sin x$.

Q.67 If $f : I \to I$, $f(x) = x^2 - x$, then f is -
(A) one-one onto (B) one-one into
(C) many-one onto (D) many-one into

Question based on **Composite function**

Q.68 If $f(x) = 2x$ and g is identity function, then-
(A) $(fog)(x) = g(x)$
(B) $(g + g)(x) = g(x)$
(C) $(fog)(x) = (g + g)(x)$
(D) None of these

Q.69 gof exists, when-
(A) domain of f = domain of g
(B) co-domain of f = domain of g
(C) co-domain of g = domain of g
(D) co-domain of g = co-domain of f

Q.70 If $f : R \to R$, $f(x) = x^2 + 2x - 3$ and $g : R \to R$, $g(x) = 3x - 4$, then the value of fog (x) is-
(A) $3x^2 + 6x - 13$ (B) $9x^2 - 18x + 5$
(C) $(3x - 4)^2 + 2x - 3$ (D) None of these

Q.71 If $f : R \to R$, $f(x) = x^2 - 5x + 4$ and $g : R \to R$, $g(x) = \log x$, then the value of (gof) (2) is -
(A) 0 (B) ∞
(C) $-\infty$ (D) Undefined

Q.72 If $f : R^+ \to R^+$, $f(x) = x^2 + 1/x^2$ and $g : R^+ \to R^+$, $g(x) = e^x$ then (gof) (x) equals-
(A) $e^{x^2} + e^{x^{-2}}$ (B) $e^{x^2} + \frac{1}{e^{x^{-2}}}$
(C) $e^{2x} + e^{-2x}$ (D) $e^{x^2} . e^{x^{-2}}$

Q.73 If $f : R \to R$, $g : R \to R$ and $f(x) = 3x + 4$ and $(gof)(x) = 2x - 1$, then the value of g(x) is-
(A) $2x - 1$ (B) $2x - 11$
(C) $\frac{1}{3}(2x - 11)$ (D) None of these

Q.74 If $f : R \to R$, $g : R \to R$ and $g(x) = x + 3$ and $(fog)(x) = (x + 3)^2$, then the value of f(–3) is -
(A) –9 (B) 0
(C) 9 (D) None of these

Q.75 If $f(x) = ax + b$ and $g(x) = cx + d$, then $f(g(x)) = g(f(x))$ is equivalent to-
(A) f(a) = g(c) (B) f(b) = g(b)
(C) f(d) = g(b) (D) f(c) = g(a)

Q.76 If $f:[0,1] \to [0,1]$, $f(x) = \frac{1 - x}{1 + x}$. $g : [0,1] \to [0,1]$, $g(x) = 4x(1-x)$, then (fog) (x) equals-
(A) $\frac{1 - 4x + 4x^2}{1 + 4x - 4x^2}$ (B) $\frac{8x(1 - x)}{(1 + x)^2}$
(C) $\frac{1 - 4x - 4x^2}{1 + 4x - 4x^2}$ (D) None of these

Q.77 If f, g, h are three functions in any set, then wrong statement is -
(A) $(fog)^{-1} = g^{-1}of^{-1}$ (B) $gof \neq fog$
(C) (fog)oh = fo(goh) (D) $(gof)^{-1} = g^{-1}of^{-1}$

Q.78 If $f(x) = \frac{1 - x}{1 + x}$, then f [f (sinθ)] equals -
(A) sin θ (B) tan (θ/2)
(C) cot (θ/2) (D) cosec θ

Q.79 If $f(x) = (a - x^n)^{1/n}$, $n \in N$, then f [f(x)] =
(A) 0 (B) x
(C) x^n (D) $(a^n - x)^n$

Q.80 If $f(x) = \log\left(\frac{1 + x}{1 - x}\right)$ and $g(x) = \left(\frac{3x + x^3}{1 + 3x^2}\right)$, then f[g(x)] is equal to-
(A) –f(x) (B) 3f(x)
(C) $[f(x)]^3$ (D) None of these

Q.81 If function $f(x) = \begin{cases} 1, & \text{when } x \in Q \\ 0, & \text{when } x \notin Q \end{cases}$, (fof) $(\sqrt{4})$ the value will be-

(A) 0 (B) 2

(C) 1 (D) None of these

Q.82 If $f(y) = \frac{y}{\sqrt{1-y^2}}$, $g(y) = \frac{y}{\sqrt{1+y^2}}$, then (fog)(y) equals -

(A) $\frac{y}{\sqrt{1-y^2}}$ (B) $\frac{y}{\sqrt{1+y^2}}$

(C) y (D) $\frac{1-y^2}{1+y^2}$

Q.83 If f(x) = [x] and g(x) = cos (πx), then the range of gof is -

(A) {0} (B) {–1, 1}

(C) {–1, 0, 1} (D) [–1, 1]

Question based on **Inverse function**

Q.84 If f : R→ R, $f(x) = x^2 + 3$, then pre-image of 2 under f is –

(A) {1,–1} (B) {1} (C) {–1} (D) ϕ

Q.85 Which of the following functions has its inverse-

(A) f : R →R, $f(x) = a^x$

(B) f : R→R, f(x) = |x| + |x – 1|

(C) f : $R_0 \to R^+$, f(x) = |x|

(D) f : [π, 2π]→ [–1,1], f(x) = cos x

Q.86 If function f : R→R^+, $f(x) = 2^x$, then $f^{-1}(x)$ will be equal to-

(A) $\log_x 2$ (B) $\log_2 (1/x)$

(C) $\log_2 x$ (D) None of these

Q.87 The inverse of the function $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} + 2$ is given by -

(A) $\log \left(\frac{x-2}{x-1}\right)^{1/2}$ (B) $\log \left(\frac{x-1}{x+1}\right)^{1/2}$

(C) $\log \left(\frac{x}{2-x}\right)^{1/2}$ (D) $\log \left(\frac{x-1}{3-x}\right)^{1/2}$

Q.88 If f : [1, ∞) → [2, ∞) is given by $f(x) = x + \frac{1}{x}$ then $f^{-1}(x)$ equals -

(A) $\frac{x + \sqrt{x^2-4}}{2}$ (B) $\frac{x}{1+x^2}$

(C) $\frac{x - \sqrt{x^2-4}}{2}$ (D) $1 + \sqrt{x^2-4}$

Q.89 If $f(x) = \log_e(x + \sqrt{1+x^2})$, then $f^{-1}(x)$ equals-

(A) $\log (x - \sqrt{1+x^2})$ (B) $\frac{e^x + e^{-x}}{2}$

(C) $\frac{e^x - e^{-x}}{2}$ (D) $\frac{e^x - e^{-x}}{e^x + e^{-x}}$

Q.90 If $f(x) = x^3 - 1$ and domain of f = {0, 1, 2, 3}, then domain of f^{-1} is -

(A) {0, 1, 2, 3} (B) {1, 0, –7, –26}

(C) {–1, 0, 7, 26} (D) {0, –1, –2, –3}

Q.91 If $f(x) = \{4 - (x-7)^3\}^{1/5}$, then its inverse is-

(A) $7 - (4 - x^5)^{1/3}$ (B) $7 - (4 + x^5)^{1/3}$

(C) $7 + (4 - x^5)^{1/3}$ (D) None of these

Q.92 If f : R→ R, $f(x) = e^x$ & g : R→R, g(x) = 3x – 2, then the value of $(fog)^{-1}(x)$ is equal to -

(A) log (x – 2) (B) $\frac{2 + \log x}{3}$

(C) $\log \left(\frac{x+3}{2}\right)$ (D) None of these

LEVEL- 2

Q.1 The range of f (x) = $\sin^{-1}\sqrt{x^2 + x + 1}$ is -

(A) $(0, \pi/2]$ (B) $(0, \pi/3]$

(C) $[\pi/3, \pi/2]$ (D) $[\pi/6, \pi/3]$

Q.2 If $f(x) = \frac{1}{x+1}$ and $g(x) = \frac{1}{\sqrt{x-1}}$, then common domain of function is -

(A) $\{x \mid x < 1, x \in R\}$

(B) $\{x \mid x \geq 0, x \neq 1, x \in R\}$

(C) $\{1\}$

(D) $\{-1\}$

Q.3 If $f(x) = \left(\frac{x}{1-|x|}\right)^{1/12}$, $x \in R$ then domain of the function f(x) is -

(A) $(-1,0]$ (B) $(-\infty, -1) \cup [0, 1)$

(C) $(-1, \infty) - \{1\}$ (D) None of these

Q.4 If $f : R \to R$, $f(x) = \tan x$, then pre-image of -1 under f is-

(A) $\left\{n\pi - \frac{\pi}{4} \middle| n \in I\right\}$ (B) $\left\{n\pi + \frac{\pi}{4} \middle| n \in I\right\}$

(C) $\{n\pi | n \in I\}$ (D) None of these

Q.5 The domain of

$$f(x) = \sqrt{[\cos(\sin x)]} + (1-x)^{-1} + \sin^{-1}\left(\frac{x^2+1}{2x}\right)$$

equal to -

(A) $R - \{1\}$ (B) $\{-1\}$

(C) $(1, \infty)$ (D) None of these

Q.6 If $f : R \to R$, $f(x) = x^3 + 3$, and $g : R \to R$, $g(x) = 2x + 1$, then $f^{-1}og^{-1}(23)$ equals-

(A) 2 (B) 3 (C) $(14)^{1/3}$ (D) $(15)^{1/3}$

Q.7 The period of $f(x) = \frac{|\sin x| + |\cos x|}{|\sin x - \cos x|}$ is -

(A) $\pi/2$ (B) π

(C) 2π (D) None of these

Q.8 The function $f(x) = \frac{\sec^{-1} x}{\sqrt{x - [x]}}$, where [x] denotes the greatest integer less than or equal to x, is defined for all x belonging to -

(A) R

(B) $R - \{(-1, 1) \cup \{n : n \in Z\}\}$

(C) $R^+ - (0, 1)$

(D) $R^+ - \{n : n \in N\}$

Q.9 The interval for which $\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x} = \frac{\pi}{2}$ holds-

(A) $[0, \infty)$ (B) $[0, 3]$

(C) $[0, 1]$ (D) $[0, 2]$

Q.10 The function $f(x) = \cos^{-1}\left(\frac{|x|-3}{2}\right) + [\log_e (4-x)]^{-1}$ is defined for -

(A) $[-1, 0] \cup [1, 5]$

(B) $[-5, -1] \cup [1, 4]$

(C) $[-5, -1] \cup ([1, 4) - \{3\})$

(D) $[1, 4] - \{3\}$

Q.11 Function $f : R \to R^+$, $f(x) = x^2 + 2$ & $g : R^+ \to R$, $g(x) = \left(1 - \frac{1}{1-x}\right)$ then the value of gof (2) is -

(A) 5/6 (B) 8/7

(C) 1/6 (D) 6/5

Q.12 Period of function $2^{\{x\}} + \sin \pi x + 3^{\{x/2\}} + \cos 2\pi x$ is (where { } represent fractional part of x)

(A) 2 (B) 1

(C) 3 (D) None of these

Q.13 Let $f : (4, 6) \to (6, 8)$ be a function defined by $f(x) = x + [x/2]$ where [ ] represent G.I.F. then $f^{-1}(x)$ is equal to -

(A) $x - 2$ (B) $x - [x/2]$

(C) $-x - 2$ (D) None of these

Q.14 If $f(x) = \log \frac{1+x}{1-x}$, when $-1 < x_1, x_2 < 1$, then $f(x_1) + f(x_2)$ equals -

(A) $f\left(\frac{x_1 + x_2}{1 + x_1 x_2}\right)$ (B) $f\left(\frac{x_1 + x_2}{1 - x_1 x_2}\right)$

(C) $f\left(\frac{x_1 - x_2}{1 + x_1 x_2}\right)$ (D) $f\left(\frac{x_1 - x_2}{1 - x_1 x_2}\right)$

Q.15 Period of the function $f(x) = |\sin \pi x| + e^{3(x-[x])}$ (where [ ] represent G.I.F.) is -

(A) 1 (B) 2

(C) 1/3 (D) None of these

Q.16 If the domain of function $f(x) = x^2 - 6x + 7$ is $(-\infty, \infty)$, then the range of function is -

(A) $(-\infty, \infty)$ (B) $[-2, \infty)$

(C) $(-2, 3)$ (D) $(-\infty, -2)$

Q.17 Period of $f(x) = \sin 3\pi \{x\} + \tan \pi [x]$ where [ ] and {} represent of G.I.F and fractional part of x

(A) 1 (B) 2 (C) 3 (D) π

Q.18 If S be the set of all triangles and $f : S \to R^+$, $f(\Delta)$ = Area of Δ, then f is -

(A) One-one onto (B) one-one into

(C) many-one onto (D) many-one into

Q.19 If $f : C \to R$, $f(z) = |z|$, then f is -

(A) one-one but not onto

(B) onto but not one-one

(C) neither one-one nor onto

(D) both one-one and onto

Q.20 If period of $\frac{\cos(\sin nx)}{\tan(x/n)}$ $(n \in N)$ is 6π then n is equal to -

(A) 3 (B) 2 (C) 6 (D) 1

Q.21 If [x] and {x} represent the integral and fractional part of x respectively then value of $\sum_{r=1}^{2000} \frac{\{x + r\}}{2000}$ is

(A) x (B) [x]

(C) {x} (D) x + 2001

Q.22 The period of $f(x) = \cos(\sin x) + \cos(\cos x)$ is -

(A) $\pi/3$ (B) $\pi/6$

(C) π (D) $\pi/2$

Q.23 If f be the greatest integer function and g be the modulus function, then

$(gof)\left(-\frac{5}{3}\right) - (fog)\left(-\frac{5}{3}\right) =$

(A) 1 (B) –1 (C) 2 (D) 4

Q.24 The domain of function $f(x) = \log |\log x|$ is-

(A) $(0, \infty)$ (B) $(1, \infty)$

(C) $(0, 1) \cup (1, \infty)$ (D) $(-\infty, 1)$

Q.25 Domain of the function $\tan^{-1} x + \cos^{-1} x^2$ is -

(A) R– [–1, 1] (B) R– (–1, 1)

(C) (–1, 1) (D) [–1, 1]

Q.26 Which of the following functions are equal?

(A) $f(x) = x$, $g(x) = \sqrt{x^2}$

(B) $f(x) = \log x^2$, $g(x) = 2 \log x$

(C) $f(x) = 1$, $g(x) = \sin^2 x + \cos^2 x$

(D) $f(x) = x/x$, $g(x) = 1$

Q.27 $f : N \to N$ defined by $f(x) = x^2 + x + 1$, $x \in N$ then f is

(A) one-one onto

(B) many-one onto

(C) one-one but not onto

(D) none of these

Q.28 Let $f(x) = \sin^2 (x/2) + \cos^2 (x/2)$ and $g(x) = \sec^2 x - \tan^2 x$. The two function are equal over the set -

(A) ϕ

(B) $R - \left\{x : x = (2n+1)\frac{\pi}{2}, n \in Z\right\}$

(C) R

(D) None of these

Q.29 The domain of the function

$f(x) = \sin^{-1}\left(\frac{2 - |x|}{4}\right) + \cos^{-1}\left(\frac{2 - |x|}{4}\right) + \tan^{-1}\left(\frac{2 - |x|}{4}\right)$ is given by

(A) [– 3, 3] (B) [–6, 6]

(C) [0, 6] (D) None of these

Q.30 The domain of function

$f(x) = \frac{1}{\log_{10}(3-x)} + \sqrt{x+2}$ is -

(A) [–2, 3) (B) [–2, 3) – {2}

(C) [–3, 2] (D) [–2, 3] – {2}

Q.31 Domain of the function $f(x) = \frac{x-3}{(x-1)\sqrt{x^2-4}}$ is -

(A) (1, 2) (B) $(-\infty, -2) \cup (2, \infty)$

(C) $(-\infty, -2) \cup (1, \infty)$ (D) $(-\infty, \infty) - \{1, \pm 2\}$

Q.32 Domain and range of $\sin\left(\log\left(\frac{\sqrt{4-x^2}}{1-x}\right)\right)$ is -

(A) [–2, 1), (–1, 1) (B) (–2, 1), [–1, 1]

(C) (–2, 1), R (D) None of these

Q.33 Let f : R $\rightarrow$ R be a function defined by $f(x) = x + \sqrt{x^2}$, then f is-

(A) injective (B) surjective

(C) bijective (D) None of these

Q.34 If $f(x) = e^{3x}$ and $g(x) = \ell n\, x$, $x > 0$, then (fog) (x) is equal to-

(A) 3x (B) x^3

(C) log 3x (D) 3 log x

Q.35 If f : R$\rightarrow$ R f(x) = cos (5x + 2) then the value of $f^{-1}(x)$ is -

(A) $\frac{\cos^{-1}(x) - 2}{5}$ (B) $\cos^{-1}(x) - 2$

(C) $\frac{\cos^{-1}(x)}{5} - 2$ (D) Does not exist

Q.36 Let $f(x) = \frac{\sin([x]\pi)}{x^2 + 2x + 4}$, [.] = G.I.F., then which one is not true -

(A) f is periodic (B) f is even

(C) f is many-one (D) f is onto

Q.37 The domain of function

f(x) = log (3x –1) + 2 log (x +1) is -

(A) [1/3, ∞) (B) [–1, 1/3]

(C) (–1, 1/3) (D) None of these

Q.38 If $f(x) = \frac{x}{\sqrt{1+x^2}}$, then (fofof) (x) is equal to-

(A) $\frac{3x}{\sqrt{1+x^2}}$ (B) $\frac{x}{\sqrt{1+3x^2}}$

(C) $\frac{3x}{\sqrt{1-x^2}}$ (D) None of these

Q.39 If f (x) be a polynomial satisfying f (x). f(1/x)= f (x) + f (1/x) and f(4) = 65 then f(6) = ?

(A) 176 (B) 217

(C) 289 (D) None of these

Q.40 If $f(x) = x^3 - x$ and g(x) = sin 2x, then-

(A) g [f(1)] = 1 (B) f (g (π/12)) = – 3/8

(C) g {f(2)} = sin 2 (D) None of these

Q.41 f : R $\rightarrow$ R is defined by $f(x) = \cos^2 x + \sin^4 x$ for $x \in R$ then the range of f (x) is -

(A) (3/4, 1) (B) [3/4, 1)

(C) [3/4, 1] (D) (3/4, 1)

Q.42 The natural domain of the real valued function defined by f (x) = $\sqrt{x^2 - 1} + \sqrt{x^2 + 1}$ is-

(A) $1 < x < \infty$ (B) $-\infty < x < \infty$

(C) $-\infty < x < -1$ (D) $(-\infty, \infty) - (-1, 1)$

Q.43 If $f(x) = \frac{\sqrt{9-x^2}}{\sin^{-1}(3-x)}$, then domain of f is -

(A) [2, 3] (B) [2, 3)

(C) (2, 3] (D) None of these

Q.44 Let f $\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$ $(x \neq 0)$, then f(x) equals -

(A) $x^2 - 2$ (B) $x^2 - 1$

(C) x^2 (D) None of these

Q.45 Let $f(x) = \sqrt{(2 + x - x^2)}$ and

$g(x) = \sqrt{-x} + \frac{1}{\sqrt{x+2}}$. Then domain of f + g is given by -

(A) (–2 0] (B) [0, 1]

(C) [–1, 0] (D) (0, 1)

Q.46 The range of $\sin^{-1}[x^2 + 1/2] + \cos^{-1}[x^2 - 1/2]$ where [ ] represent G.I.F.

(A) $\{\pi/2, \pi\}$ (B) $\{\pi\}$

(C) $\{\pi/2\}$ (D) None of these

Q.47 If $x = \log_a bc$, $y = \log_b ca$, and $z = \log_c ab$, then $\frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z}$ equals-

(A) 1 (B) x + y + z

(C) abc (D) ab + bc + ca

Q.48 The range of $5\cos x - 12\sin x + 7$ is-

(A) [–6,20] (B) [–3,18]

(C) [–6,15] (D) None of these

Q.49 The domain of the function $\log_2\log_3\log_4(x)$ is-

(A) $(1, \infty)$ (B) $(2, \infty)$

(C) $(3, \infty)$ (D) $(4, \infty)$

Q.50 Let $f(x) = \frac{x - [x]}{1 - [x] + x}$, then range of f(x) is ([.] = G.I.F.) -

(A) [0, 1] (B) [0, 1/2]

(C) [1/2, 1] (D) [0, 1/2)

Q.51 $f(x) = \log(\sqrt{x-3} + \sqrt{5-x})$, $x \in R$ then domain of f(x) is

(A) [3, 5] (B) $[-\infty, 3] \cup [5, \infty]$

(C) {3, 5} (D) None of these

Q.52 The range of the function $f(x) = |x - 1| + |x - 2|$, $-1 \le x \le 3$ is

(A) [1, 3] (B) [1, 5]

(C) [3, 5] (D) None of these

Q.53 The range of the function $y = \log_3(5 + 4x - x^2)$ is -

(A) (0, 2] (B) $(-\infty, 2]$

(C) (0, 9] (D) None of these

Q.54 Let $f(x) = \frac{9^x}{9^x + 3}$ and $f(x) + f(1-x) = 1$ then find value of $f\left(\frac{1}{1996}\right) + f\left(\frac{2}{1996}\right) + \ldots\ldots + f\left(\frac{1995}{1996}\right)$ is -

(A) 998 (B) 997 (C) 997.5 (D) 998.5

Q.55 The range of $f(x) = \sqrt{(1-\cos x)\sqrt{(1-\cos x)\sqrt{1-\cos x \ldots\ldots \infty}}}$ is -

(A) [0, 1] (B) [0, 1/2]

(C) [0, 2] (D) None of these

LEVEL- 3

Q.1 The domain of definition of

$f(x) = \sqrt{\log_{0.4}\left(\frac{x-1}{x+5}\right)} \times \frac{1}{x^2-36}$ is –

(A) $(x : x < 0, x \neq -6\}$

(B) $(x : x > 0, x \neq 1, x \neq 6\}$

(C) $(x : x > 1, x \neq 6\}$

(D) $(x : x \geq 1, x \neq 6\}$

Q.2 The function $f : R \to R$ defined by $f(x) = (x-1)(x-2)(x-3)$ is -

(A) one-one but not onto

(B) onto but not one-one

(C) both one and onto

(D) neither one-one nor onto

Q.3 The domain of f(x) is (0, 1) therefore domain of $f(e^x) + f(\ell n\,|x|)$ is -

(A) (–1, e) (B) (1, e)

(C) (–e, – 1) (D) (– e, 1)

Q.4 If $g : [-2, 2] \to R$ where $f(x) = x^3 + \tan x + \left\lfloor \frac{x^2+1}{p} \right\rfloor$ is a odd function then the value of p where [ ] represent G.I.F. -

(A) $-5 < p < 5$ (B) $p < 5$

(C) $p > 5$ (D) None of these

$$\frac{a^x - 1}{x^n(a^x + 1)}$$

Q.6 Let $f : R \to R$ be a function defined by

$f(x) = \frac{e^{|x|} - e^{-x}}{e^x + e^{-x}}$. Then -

(A) f is a bijection

(B) f is an injection only

(C) f is a surjection only

(D) f is neither an injection nor a surjection

Q.7 The value of $n \in I$ for which the function $f(x) = \frac{\sin nx}{\sin\left(\frac{x}{n}\right)}$ has 4π as its period is-

(A) 2 (B) 3 (C) 4 (D) 5

Q.8 If f(x) is an odd periodic function with period 2, then f (4) equals to-

(A) 0 (B) 2 (C) 4 (D) –4

Q.9 Domain of the function

$f(x) = \sin^{-1}\left(\log_5 \frac{x^2}{5}\right)$ is-

(A) $[-5, -1] \cup [1, 5]$ (B) $[-5, 5]$

(C) $(-5, -1) \cup (1, 5)$ (D) None of these

Q.10 Domain of $f(x) = \sqrt{\frac{1-|x|}{2-|x|}}$ is -

(A) R – [–2, 2]

(B) R – [–1, 1]

(C) $[-1, 1] \cup (-\infty, -2) \cup (2, \infty)$

(D) None of these

Q.11 If $f(x) = 3^{\sin\sqrt{\frac{\pi^2}{16} - x^2}}$, then values of f(x) lie in

(A) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ (B) [–2, 2]

(C) $\left[0, \frac{3}{\sqrt{2}}\right]$ (D) None of these

Q.12 The period of $f(x) = \sin\frac{x}{n!} + \cos\frac{x}{(n+1)!}$ is -

(A) non-periodic

(B) periodic with period (2π) n!

(C) periodic with period 2π (n + 1)!

(D) periodic with period 2 (n + 1) π

Q.13 The function f(x) = max. [1 – x, 1+x, 2] ; x ∈ R is equivalent to -

(A) $f(x) = \begin{cases} 1-x, & x \le -1 \\ 2, & -1 < x < 1 \\ 1+x, & x \ge 1 \end{cases}$

(B) $f(x) = \begin{cases} 1+x, & x \le -1 \\ 2, & -1 < x < 1 \\ 1-x, & x \ge 1 \end{cases}$

(C) $f(x) = \begin{cases} 1-x, & x \le -1 \\ 1, & -1 < x < 1 \\ 1+x, & x \ge 1 \end{cases}$

(D) None of these

Q.14 The domain of the function f(x) = ${}^{9-x}P_{x-5}$ is-

(A) [5, 7] (B) {5, 6, 7}

(C) {3, 4, 5, 6, 7} (D) None of these

Q.15 The range of the function f(x) = ${}^{9-x}P_{x-5}$ is -

(A) {1, 2, 3} (B) [1, 2]

(C) {1, 2, 3, 4, 5} (D) None of these

Q.16 Domain of the function

$f(x) = \log_2\left(-\log_{1/2}\left(1+\frac{1}{\sqrt[4]{x}}\right)-1\right)$ is-

(A) (0, 1) (B) (0, 1]

(C) [1, ∞) (D) (1, ∞)

Q.17 The period of f(x) = [sin 5x] + |cos 6x| is -

(A) $\frac{\pi}{2}$ (B) π (C) 2π (D) $\frac{2\pi}{5}$

Q.18 Period of $f(x) = \sin x + \tan\frac{x}{2} + \sin\frac{x}{2^2} + \tan\frac{x}{2^3} + ... + \sin\frac{x}{2^{n-1}} + \tan\frac{x}{2^n}$ is -

(A) π (B) 2π (C) $2^n\pi$ (D) $\frac{\pi}{2^n}$

Q.19 The period of f(x) = [x] + [2x] + … + [nx] – $\frac{n(n+1)}{2}$ x where n ∈ N and [ ] represent G.I.F. is

(A) n (B) 1

(C) $\frac{1}{n}$ (D) None of these

Q.20 The function f : [–1/2, 1/2] → [–π/2, π/2] defined by $f(x) = \sin^{-1}(3x - 4x^3)$ is–

(A) both one-one and onto

(B) neither one-one nor onto

(C) onto but not one-one

(D) one-one but not onto

Q.21 The function f satisfies the equation

$3f(x) + 2f\left(\frac{x+59}{x-1}\right) = 10x + 30$ for all real x ≠ 1.

The value of f(7) is -

(A) 8 (B) 4

(C) –8 (D) 11

Q.22 The domain of the function

$f(x) = \log_{3+x}(x^2 - 1)$ is -

(A) (–3, –1) ∪ (1, ∞)

(B) [–3, –1) ∪ [1, ∞)

(C) (–3, –2) ∪ (–2, –1) ∪ (1, ∞)

(D) [–3, –2) ∪ (–2, –1) ∪ [1, ∞)

➢ Statement type Questions

Each of the questions given below consists of Statement-I and Statement-II. Use the following key to choose the appropriate answer.

(A) Statement-I and Statement-II are true but Statement-II is the correct explanation of Statement-I

(B) Statement-I and Statement-II are true but Statement-II is not the correct explanation of Statement-I.

(C) Statement-I is true but Statement-II is false

(D) Statement-I is false but Statement-II is true.

Q.23 **Statement- I :** The period of

$f(x) = \sin 2x \cos[2x] - \cos 2x \sin[2x]$ is $\frac{1}{2}$

Statement- II : The period of x – [x] is 1

Where [.] = G.I.F.

Q.24 **Statement- I :** If $f(x) = |x - 1| + |x - 2| + |x - 3|$ Where $2 < x < 3$ is an identity function.
Statement- II : $f : A \to A$ defined by $f(x) = x$ is an identity function.

Q.25 **Statement- I :** $f : R \to R$ defined by $f(x) = \sin x$ is a bijection
Statement- II : If f is both one and onto it is bijection

Q. 26 **Statement- I :** $f : R \to R$ is a function defined by $f(x) = \frac{2x + 1}{3}$.

Then $f^{-1}(x) = \frac{3x - 1}{2}$

Statement- II : f(x) is not a bijection.

Q.27 **Statement- I :** If f is even function, g is odd function then $\frac{f}{g}$, $(g \neq 0)$is an odd function.

Statement- II : If $f(-x) = -f(x)$ for every x of its domain, then f(x) is called an odd function and if $f(-x) = f(x)$ for every x of its domain, then f(x) is called an even function.

Q.28 **Statement I :** Function $f(x) = \sin x + \{x\}$ is periodic with period 2π
Statement II : sinx and {x} are both periodic with period 2π and 1 respectively.

Q.29 **Statement I :** $y = f(x) = \frac{x^2 - 2x + 4}{x^2 - 2x + 5}$, $x \in R$
Range of f(x) is [3/4, 1)

Statement II : $(x - 1)^2 = \frac{4y - 3}{1 - y}$.

➢ Passage Based Questions

Passage :-

Let here we define $f : R \to [-1, 1]$ and $g : R \to [-1, 1]$. Now $f(x) = 2\cos^2 x - 1$, $g(x) = \cos 2x$, $h(x) = f(x) + g(x)$, $I(x) = f(x) - g(x)$, $j(x) = \frac{f(x)}{g(x)}$ are 5 functions.

On the basis of above information, answer the following questions-

Q.30 Which statement is correct-
(A) Period of f(x), g(x) and h(x) are same and value is $\frac{2\pi}{3}$
(B) Period of f(x), g(x) and h(x) makes the A.P. with common difference $\frac{\pi}{4}$
(C) Sum of periods of f(x), g(x) and h(x) is 3π
(D) None of these

Q.31 Which statement is correct regarding function j(x) and I(x)-
(A) The domain of j(x) and I(x) are the same
(B) Range of j(x) and I(x) are the same
(C) The union of domain of j(x) and I(x) are all real numbers
(D) None of these

Q.32 If the solution of equation $I(x) - g(x) = 0$ are $x_1, x_2, x_3, ... x_n$ when $x \in [0, 10\pi]$ then which option is correct-
(A) $x_1, x_2, x_3 ... x_n$ makes the A.P. with common difference π
(B) Total no. of solutions of $I(x) - g(x) = 0$ is 20 for $x \in [0, 10\pi]$
(C) Sum of all solutions of the given equation is 100π in the interval $[0, 10\pi]$
(D) (B) and (C) are correct

Q.33 If $h : R \to [-2, 2]$, then -
(A) h(x) is one-one function
(B) h(x) is one-one and onto function
(C) h(x) is onto function
(D) h(x) is many one and into function

Q.34 Domain and range of j(x) respectively -
(A) R and {1}
(B) R and {0, 1}
(C) $R - \{(2n + 1)\pi/4\}$, $n \in I$ and {1}
(D) $R - \{(2n + 1)\pi/2\}$, $n \in I$ and {1}

➢ Column Matching Questions

Match the entry in Column 1 with the entry in Column 2.

Q.35 Match the column

Column 1	**Column 2**
(A) $f(x) = \{x\}$, the fractional part of x	(P) $f^{-1}(x) = \frac{1}{2}(4^x - 4^{-x})$
(B) $f(x) = \frac{16^x - 1}{4^x}$	(Q) f is an even function
(C) $f(x) = \log_4 (x + \sqrt{x^2 + 1})$	(R) f is a periodic function
(D) $f(x) = x\frac{3^x - 1}{3^x + 1}$	(S) f is odd function

LEVEL- 4

(Question asked in previous AIEEE and IIT-JEE)

SECTION –A

Q.1 Which of the following is not a periodic function - **[AIEEE 2002]**

(A) $\sin 2x + \cos x$ (B) $\cos \sqrt{x}$

(C) $\tan 4x$ (D) $\log \cos 2x$

Q.2 The period of $\sin^2 x$ is- **[AIEEE 2002]**

(A) $\pi/2$ (B) π (C) $3\pi/2$ (D) 2π

Q.3 The function $f : R \to R$ defined by $f(x) = \sin x$ is- **[AIEEE-2002]**

(A) into (B) onto

(C) one-one (D) many-one

Q.4 The range of the function $f(x) = \frac{2+x}{2-x}$, $x \neq 2$ is - **[AIEEE-2002]**

(A) R (B) R – {–1}

(C) R – {1} (D) R – {2}

Q.5 The function $f(x) = \log (x + \sqrt{x^2+1})$, is- **[AIEEE 2003]**

(A) neither an even nor an odd function

(B) an even function

(C) an odd function

(D) a periodic function

Q.6 Domain of definition of the function $f(x) = \frac{3}{4-x^2} + \log_{10}(x^3 - x)$, is- **[AIEEE 2003]**

(A) $(-1, 0) \cup (1, 2) \cup (2, \infty)$ (B) (1, 2)

(C) $(-1, 0) \cup (1, 2)$ (D) $(1, 2) \cup (2, \infty)$

Q.7 If $f : R \to R$ satisfies $f(x + y) = f(x) + f(y)$, for all $x, y \in R$ and $f(1) = 7$, then $\sum_{r=1}^{n} f(r)$ is- **[AIEEE 2003]**

(A) $\frac{7n(n+1)}{2}$ (B) $\frac{7n}{2}$

(C) $\frac{7(n+1)}{2}$ (D) $7n(n+1)$

Q.8 A function f from the set of natural numbers to integers defined by

$$f(n) = \begin{cases} \frac{n-1}{2}, & \text{when } n \text{ is odd} \\ -\frac{n}{2}, & \text{when } n \text{ is even} \end{cases}$$ is

[AIEEE 2003]

(A) neither one-one nor onto

(B) one-one but not onto

(C) onto but not one-one

(D) one-one and onto both

Q.9 The range of the function $f(x) = {}^{7-x}P_{x-3}$ is- **[AIEEE 2004]**

(A) {1, 2, 3} (B) {1, 2, 3, 4, 5, 6}

(C) {1, 2,3,4} (D) {1, 2, 3, 4, 5}

Q.10 If $f : R \to S$, defined by $f(x) = \sin x - \sqrt{3}\cos x + 1$, is onto, then the interval of S is- **[AIEEE 2004]**

(A) [0, 3] (B) [–1, 1]

(C) [0, 1] (D) [–1, 3]

Q.11 The graph of the function $y = f(x)$ is symmetrical about the line $x = 2$, then- **[AIEEE 2004]**

(A) $f(x+2) = f(x-2)$ (B) $f(2+x) = f(2-x)$

(C) $f(x) = f(-x)$ (D) $f(x) = -f(-x)$

Q.12 The domain of the function $f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is- **[AIEEE 2004]**

(A) [2, 3] (B) [2, 3) (C) [1, 2] (D) [1, 2)

Q.13 Let $f : (-1, 1) \to B$, be a function defined by $f(x) = \tan^{-1}\frac{2x}{1-x^2}$, then f is both one-one and onto when B is the interval - **[AIEEE-2005]**

(A) $\left(0, \frac{\pi}{2}\right)$ (B) $\left[0, \frac{\pi}{2}\right)$

(C) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (D) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Q.14 A real valued function f(x) satisfies the functional equation f(x – y) = f(x) f(y) – f (a – x) f(a + y) where a is a given constant and f(0) = 1, then f(2a – x) is equal to - **[AIEEE-2005]**

(A) –f(x) (B) f(x)

(C) f(a) + f(a – x) (D) f(–x)

Q.15 The largest interval lying in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ for which the function $f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)$ defined, is- **[AIEEE 2007]**

(A) $[0, \pi]$ (B) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

(C) $\left[-\frac{\pi}{4}, \frac{\pi}{2}\right)$ (D) $\left[0, \frac{\pi}{2}\right)$

Q.16 Let $f : N \to Y$ be a function defined as $f(x) = 4x + 3$ where $Y = \{y \in N : y = 4x + 3 \text{ for some } x \in N\}$. inverse of f is - **[AIEEE 2008]**

(A) $g(y) = 4 + \frac{y + 3}{4}$ (B) $g(y) = \frac{y + 3}{4}$

(C) $g(y) = \frac{y - 3}{4}$ (D) $g(y) = \frac{3y + 4}{3}$

Q.17 For real x, let $f(x) = x^3 + 5x + 1$, then - **[AIEEE 2009]**

(A) f is one - one but not onto R

(B) f is onto R but not one - one

(C) f is one - one and onto on R

(D) f is neither one - one nor onto R

Q.18 Let $f(x) = (x + 1)^2 - 1$, $x \geq -1$

Statement - 1 :

The set $\{x : f(x) = f^{-1}(x)\} = \{0, -1\}$.

Statement - 2 :

f is a bijection. **[AIEEE 2009]**

(A) Statement -1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement -1

(B) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement -1.

(C) Statement -1 is true, Statement-2 is false.

(D) Statement -1 is false, Statement-2 is true.

Q.19 The domain of the function $f(x) = \frac{1}{\sqrt{|x| - x}}$ is : **[AIEEE 2011]**

(A) $(-\infty, \infty)$ (B) $(0, \infty)$

(C) $(-\infty, 0)$ (D) $(-\infty, \infty) - \{0\}$

SECTION-B

Q.1 If function $f(x) = \frac{1}{2} - \tan\left(\frac{\pi x}{2}\right)$; $(-1 < x < 1)$ and $g(x) = \sqrt{3 + 4x - 4x^2}$, then the domain of gof is – **[IIT-1990]**

(A) (–1, 1) (B) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(C) $\left[-1, \frac{1}{2}\right]$ (D) $\left[-\frac{1}{2}, -1\right]$

Q.2 If $f(x) = \cos[\pi^2]x + \cos[-\pi^2]x$, where [x] stands for the greatest integer function, then **[IIT- 1991]**

(A) $f\left(\frac{\pi}{2}\right) = -1$ (B) $f(\pi) = 1$

(C) $f\left(\frac{\pi}{4}\right) = 2$ (D) None of these

Q.3 The value of b and c for which the identity $f(x + 1) - f(x) = 8x + 3$ is satisfied, where $f(x) = bx^2 + cx + d$, are- **[IIT- 1992]**

(A) b = 2, c = 1 (B) b = 4, c = –1

(C) b = –1, c = 4 (D) None

Q.4 Let f(x) = sin x and g(x) = ln |x|. If the ranges of the compositie functions fog and gof are R_1 and R_2 respectively, then – **[IIT- 1994]**

(A) $R_1 = \{u : -1 < u < 1\}$, $R_2 = \{v : -\infty < v < 0\}$

(B) $R_1 = \{u : -\infty < u < 0\}$, $R_2 = \{v : -1 < v < 1\}$

(C) $R_1 = \{u : -1 < u < 2\}$, $R_2 = \{v : -\infty < v < 0\}$

(D) $R_1 = \{u : -1 \leq u \leq 1\}$, $R_2 = \{v : -\infty < v < 0\}$

Q.5 Let $2\sin^2 x + 3\sin x - 2 > 0$ and $x^2 - x - 2 < 0$ (x is measured in radians). Then x lies in the interval **[IIT- 1994]**

(A) $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ (B) $\left(-1, \frac{5\pi}{6}\right)$

(C) (–1, 2) (D) $\left(\frac{\pi}{6}, 2\right)$

Q.6 Let $f(x) = (x + 1)^2 - 1$, $(x \geq -1)$. Then the set $S = \{x : f(x) = f^{-1}(x)\}$ is - **[IIT- 1995]**

(A) Empty

(B) $\{0, -1\}$

(C) $\{0, 1, -1\}$

(D) $\left\{0, -1, \frac{-3 + i\sqrt{3}}{2}, \frac{-3 - i\sqrt{3}}{2}\right\}$

Q.7 If $f(1) = 1$ and $f(n + 1) = 2f(n) + 1$ if $n \geq 1$, then $f(n)$ is- **[IIT- 1995]**

(A) 2^{n+1} (B) 2^n

(C) $2^n - 1$ (D) $2^{n-1} - 1$

Q.8 If f is an even function defined on the interval $(-5, 5)$, then the real values of x satisfying the equation $f(x) = f\left(\frac{x+1}{x+2}\right)$ are- **[IIT- 1996]**

(A) $\frac{-1 \pm \sqrt{5}}{2}, \frac{-3 \pm \sqrt{5}}{2}$

(B) $\frac{-1 \pm \sqrt{3}}{2}, \frac{-3 \pm \sqrt{3}}{2}$

(C) $\frac{-2 \pm \sqrt{5}}{2}$

(D) None of these

Q.9 Let $f(x) = [x] \sin\left(\frac{\pi}{[x+1]}\right)$, where [.] denotes the greatest integer function. The domain of f is ..... **[IIT 1996]**

(A) $\{x \in R | x \in [-1, 0)\}$

(B) $\{x \in R | x \notin [1, 0)\}$

(C) $\{x \in R | x \notin [-1, 0)\}$

(D) None of these

Q.10 If $f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x \cos\left(x + \frac{\pi}{3}\right)$ and $g\left(\frac{5}{4}\right) = 1$, then (gof) (x) = **[IIT 1996]**

(A) –2 (B) –1 (C) 2 (D) 1

Q.11 If $g(f(x)) = |\sin x|$ and $f(g(x)) = (\sin\sqrt{x})^2$, then **[IIT 1998]**

(A) $f(x) = \sin^2 x$, $g(x) = \sqrt{x}$

(B) $f(x) = \sin x$, $g(x) = |x|$

(C) $f(x) = x^2$, $g(x) = \sin\sqrt{x}$

(D) f and g cannot be determined

Q.12 If $f(x) = 3x - 5$, then $f^{-1}(x)$ **[IIT 1998]**

(A) is given by $\frac{1}{3x - 5}$

(B) is given by $\frac{x + 5}{3}$

(C) does not exist because f is not one-one

(D) does not exist because f is not onto

Q.13 If the function $f : [1, \infty) \to [1, \infty)$ is defined by $f(x) = 2^{x(x-1)}$, then $f^{-1}(x)$ is **[IIT 1999]**

(A) $\left(\frac{1}{2}\right)^{x(x-1)}$

(B) $\frac{1}{2}(1 + \sqrt{1 + 4\log_2 x})$

(C) $\frac{1}{2}(1 - \sqrt{1 + 4\log_2 x})$

(D) not defined

Q.14 The domain of definition of the function y(x) given by the equation $2^x + 2^y = 2$ is – **[IIT Scr. 2000]**

(A) $0 < x < 1$ (B) $0 < x < 1$

(C) $-\infty < x < 0$ (D) $-\infty < x < 1$

Q.15 Let $f(\theta) = \sin\theta(\sin\theta + \sin 3\theta)$, then $f(\theta)$ **[IIT 2000]**

(A) ≥ 0 only when $\theta \geq 0$

(B) ≤ 0 for all θ

(C) ≥ 0 for all real θ

(D) ≤ 0 only when $\theta \leq 0$

Q.16 The number of solutions of $\log_4(x-1) = \log_2(x-3)$ is – **[IIT Scr. 2001]**

(A) 3 (B) 1 (C) 2 (D) 0

Q.17 Let $f(x) = \frac{\alpha x}{x+1}$, $x \neq -1$, then for what value of α, $f\{f(x)\} = x$. **[IIT Scr. 2001]**

(A) $\sqrt{2}$ (B) $-\sqrt{2}$ (C) 1 (D) –1

Q.18 The domain of definition of $f(x) = \frac{\log_2(x+3)}{x^2+3x+2}$ is **[IIT Scr. 2001]**

(A) R / { –2, –2} (B) (– 2, ∞)

(C) R/ {–1, –2, –3} (D) (–3, ∞) / {–1, –2}

Q.19 If $f : [1, \infty) \to [2, \infty)$ is given by $f(x) = x + \frac{1}{x}$ then $f^{-1}(x)$ equals – **[IIT Scr. 2001]**

(A) $\frac{x + \sqrt{x^2-4}}{2}$ (B) $\frac{x}{1+x^2}$

(C) $\frac{x - \sqrt{x^2-4}}{2}$ (D) $1 + \sqrt{x^2-4}$

Q.20 Let $g(x) = 1 + x - [x]$ and

$f(x) = \begin{cases} -1 & ; \ x < 0 \\ 0 & ; \ x = 0 \\ 1 & ; \ x > 0 \end{cases}$. Then for all x, f(g(x)) is equal to :

(where [.] denotes the greatest integer function): **[IIT Scr. 2001]**

(A) x (B) 1

(C) f(x) (D) g(x)

Q.21 Suppose $f(x) = (x+1)^2$ for $x \geq -1$. If g(x) is the function whose graph is the reflection of the graph of f(x) with respect to the line y = x, then g(x) equals– **[IIT Scr. 2002]**

(A) $-\sqrt{x} - 1, x \geq 0$ (B) $\frac{1}{(x+1)^2}, x > -1$

(C) $\sqrt{x+1}, x \geq -1$ (D) $\sqrt{x} - 1, x \geq 0$

Q.22 Let function $f : R \to R$ be defined by $f(x) = 2x + \sin x$ for $x \in R$. Then f is– **[IIT Scr. 2002]**

(A) one to one and onto

(B) one to one but NOT onto

(C) onto but NOT one to one

(D) neither one to one nor onto

Q.23 Let $f(x) = \frac{x}{1+x}$ defined as $[0, \infty) \to [0, \infty)$, f(x) is– **[IIT Scr.2003]**

(A) one-one & onto

(B) one-one but not onto

(C) not one-one but onto

(D) neither one-one nor onto

Q.24 Find the range of $f(x) = \frac{x^2+x+2}{x^2+x+1}$ is– **[IIT Scr.2003]**

(A) (1, ∞) (B) $\left(1, \frac{11}{7}\right)$

(C) $\left(1, \frac{7}{3}\right)$ (D) $\left(1, \frac{7}{5}\right)$

Q.25 Domain of $f(x) = \sqrt{\sin^{-1}(2x) + \pi/6}$ is– **[IIT Scr.2003]**

(A) $\left[-\frac{1}{4}, \frac{1}{2}\right]$ (B) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(C) $\left[-\frac{1}{4}, \frac{1}{4}\right]$ (D) $\left[-\frac{1}{2}, \frac{1}{4}\right]$

Q.26 Let $f(x) = \sin x + \cos x$ & $g(x) = x^2 - 1$, then g(f (x)) will be invertible for the domain- **[IIT Scr.2004]**

(A) $x \in [0, \pi]$ (B) $x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

(C) $x \in \left[0, \frac{\pi}{2}\right]$ (D) $x \in \left[-\frac{\pi}{2}, 0\right)$

Q.27 $f(x) = \begin{cases} x & x \in Q \\ 0 & x \notin Q \end{cases}$; $g(x) = \begin{cases} 0 & x \in Q \\ x & x \notin Q \end{cases}$

then (f – g) is **[IIT Scr.2005]**

(A) one-one, onto

(B) neither one-one, nor onto

(C) one-one but not onto

(D) onto but not one-one

Q.28 Let $f(x) = x^2$ and g (x) = sin x for all $x \in R$. Then the set of all x satisfying (fogogof) (x) = (gogof) (x), where (fog)(x) = f(g(x)), is - **[IIT 2011]**

(A) $\pm\sqrt{n\pi}, n \in \{0, 1, 2, \ldots\}$

(B) $\pm\sqrt{n\pi}, n \in \{1, 2, \ldots\}$

(C) $\frac{\pi}{2} + 2n\pi, n \in \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

(D) $2n\pi, n \in \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

ANSWER KEY

LEVEL- 1

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	C	A	B	B	A	C	D	C	C	D	A	A	D	A	B	C	B	B	A	B
Q.No.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
Ans.	A	C	B	D	B	B	C	B	B	B	B	B	B	C	B	A	C	C	B	C
Q.No.	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
Ans.	B	B	C	C	B	B	D	A	B	B	A	A	C	A	D	D	C	C	C	C
Q.No.	**61**	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
Ans.	C	C	B	B	B	C	D	C	B	B	D	D	C	C	C	A	D	A	B	B
Q.No.	**81**	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**	**91**	**92**								
Ans.	C	C	B	D	D	C	D	A	C	C	C	B								

LEVEL- 2

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	C	B	B	A	B	A	B	B	C	C	D	A	A	A	A	B	A	C	C	C
Q.No.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
Ans.	C	D	A	C	D	C	C	B	B	B	B	B	D	B	D	D	D	B	B	B
Q.No.	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**					
Ans.	B	D	B	A	C	B	A	A	D	D	A	B	B	C	C					

LEVEL- 3

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	C	B	C	C	D	D	A	A	A	C	C	C	A	B	A	A	C	C	B	A
Q.No.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**						
Ans.	B	C	A	A	D	C	A	D	A	C	C	D	C	C						

Q.35 A $\rightarrow$ R , B $\rightarrow$ P , C $\rightarrow$ S, D $\rightarrow$ Q

LEVEL-4

SECTION-A

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Ans.	B	B	A,D	B	C	A	A	D	A	D	B	B	D	A	D	C	B	B	C

SECTION-B

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	A	B	D	D	B	C	A	C	D	A	B	B	D	C
Q.No.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**		
Ans.	B	D	D	A	B	D	A	B	C	A	B	A	A		