HYPERBOLA

AIEEE Syllabus

1. *Standard equation and definitions*
2. *Conjugate Hyperbola*
3. *Parametric equation of the Hyperbola*
4. *Position of a point $P(x_1, y_1)$ with respect to Hyperbola*
5. *Line and Hyperbola*
6. *Equation of the Tangent*

Total No. of questions in Ellipse are:

Solved examples……..……..…10

Level # 1 …….…… 34

Level # 2 …….……..…31

Level # 3 …….……..…13

Total No. of questions…..……..88

1. Students are advised to solve the questions of exercises (Levels # 1, 2, 3, 4) in the same sequence or as directed by the faculty members.
2. Level # 3 is not for foundation course students, it will be discussed in fresher and target courses.

Index : Preparing your own list of Important/Difficult Questions

Instruction to fill

(A) Write down the Question Number you are unable to solve in **column A** below, by Pen.

(B) After discussing the Questions written in **column A** with faculties, strike off them in the manner so that you can see at the time of Revision also, to solve these questions again.

(C) Write down the Question Number you feel are important or good in the **column B**.

EXERCISE NO.	COLUMN :A	COLUMN :B
	Questions I am unable to solve in first attempt	Good/Important questions
Level # 1		
Level # 2		
Level # 3		
Level # 4		

Advantages

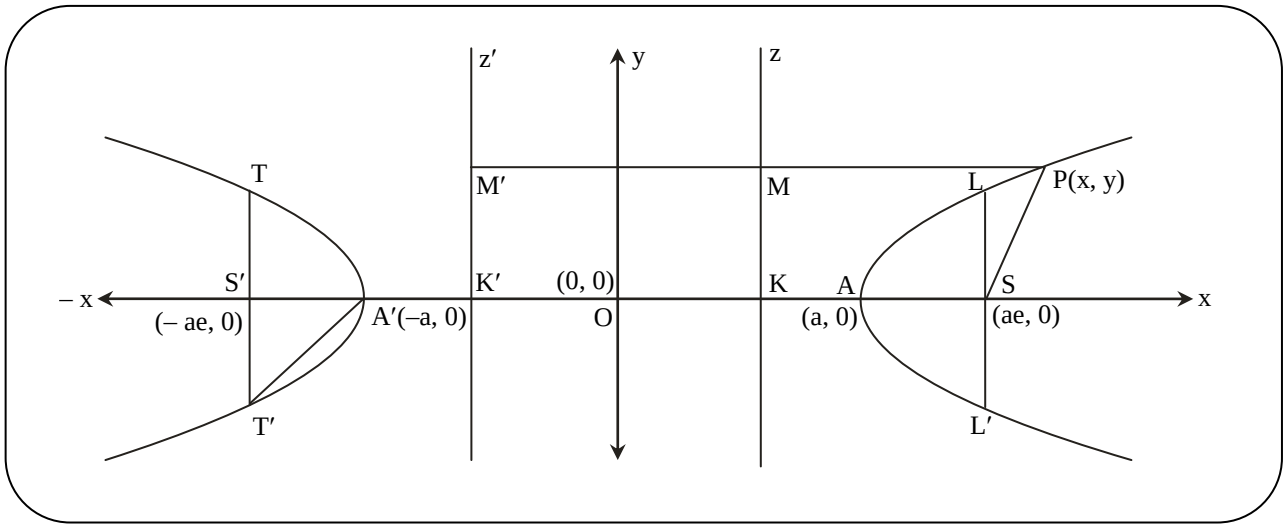
1. It is advised to the students that they should prepare a question bank for the revision as it is very difficult to solve all the questions at the time of revision.

2. Using above index you can prepare and maintain the questions for your revision.

KEY CONCEPTS

1. Standard Equation and Definitions

Standard Equation of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$



(i) Definition hyperbola :

A **Hyperbola** is the locus of a point in a plane which moves in the plane in such a way that the ratio of its distance from a fixed point (called focus) in the same plane to its distance from a fixed line (called directrix) is always constant which is always greater than unity.

(ii) Vertices :

The point A and A′ where the curve meets the line joining the foci S and S′ are called vertices of hyperbola.

(iii) Transverse and Conjugate axes :

The straight line joining the vertices A and A′ is called transverse axes of the hyperbola. Straight line perpendicular to the transverse axes and passes through its centre called conjugate axes.

(iv) Latus Rectum :

The chord of the hyperbola which passes through the focus and is perpendicular to its transverse axes is called latus rectum. Length of latus rectum = $\frac{2b^2}{a}$.

(v) Eccentricity :

For the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, $b^2 = a^2 (e^2 - 1)$

$$e = \sqrt{1+\left(\frac{2b}{2a}\right)^2} = \sqrt{1+\left(\frac{\text{Conjugate axes}}{\text{Transverse axes}}\right)^2}$$

(vi) Focal distance :

The distance of any point on the hyperbola from the focus is called the focal distance of the point.

Note : The difference of the focal distance of a point on the hyperbola is constant and is equal to the length of the transverse axes. $|S'P - SP| = 2a$ (const.)

2. Conjugate Hyperbola

The hyperbola whose transverse and conjugate axes are respectively the conjugate and transverse axes of a given hyperbola is called conjugate hyperbola.

Equation of conjugate hyperbola $-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Note :

(i) If e_1 and e_2 are the eccentricities of the hyperbola and its conjugate then $\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$

(ii) The focus of hyperbola and its conjugate are concyclic.

S.No.	Particulars	Hyperbola	Conjugate Hyperbola
		$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
1.	Co-ordinate of the centre	(0, 0)	(0, 0)
2.	Co-ordinate of the vertices	(a, 0) & (–a, 0)	(0, b) & (0, –b)
3.	Co-ordinate of foci	(± ae, 0)	(0, ± be)
4.	Length of the transverse axes	2a	2b
5.	Length of the conjugate axes	2b	2a
6.	Equation of directrix	x = ± a/e	y = ± b/e
7.	Eccentricity	$e = \sqrt{1+\frac{b^2}{a^2}}$	$e = \sqrt{1+\frac{a^2}{b^2}}$
8.	Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$
9.	Equation of transverse axes	y = 0	x = 0
10.	Equation of conjugate axes	x = 0	y = 0

3. Parametric equation of the Hyperbola

Let the equation of ellipse in standard form will be given by $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Then the equation of ellipse in the parametric form will be given by $x = a \sec \phi$, $y = b \tan \phi$ where ϕ is the eccentric angle whose value vary from $0 \le \phi < 2\pi$. Therefore coordinate of any point P on the ellipse will be given by $(a \sec\phi, b \tan \phi)$.

4. Position of a point $P(x_1, y_1)$ with respect to Hyperbola

The quantity $\frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} = 1$ is positive, zero or negative according as the point (x_1, y_1) lies inside on or outside the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

5. Line and Hyperbola

''The straight line $y = mx + c$ is a sacant, a tangent or passes outside the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ according as $c^2 > = < a^2m^2 - b^2$

6. Equation of Tangent

(i) The equation of tangents of slope m to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are $y = mx \pm \sqrt{a^2m^2 - b^2}$ and the co-ordinates of the point of contacts are

$$\left(\pm\frac{a^2m}{\sqrt{a^2m^2-b^2}}, \pm\frac{b^2}{\sqrt{a^2m^2-b^2}}\right)$$

(ii) Equation of tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point (x_1, y_1) is $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$

(iii) Equation of tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $(a \sec\theta, b \tan\theta)$ is $\frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1$

Note : In general two tangents can be drawn from an external point (x_1, y_1) to the hyperbola and they are $y - y_1 = m_1 (x - x_1)$ and $y - y_1 = m_2 (x - x_1)$, where m_1 and m_2 are roots of

$$(x_1^2 - a^2)\, m^2 - 2x_1y_1 + y_1^2 + b^2 = 0$$

SOLVED EXAMPLES

Ex.1 Find the equation of the hyperbola whose directrix is $2x + y = 1$, focus $(1, 2)$ and eccentricity $\sqrt{3}$.

Sol. Let P (x,y) be any point on the hyperbola. Draw PM perpendicular from P on the directrix.

Then by definition

$SP = e\, PM$

$\Rightarrow (SP)^2 = e^2(PM)^2$

$$\Rightarrow (x-1)^2 + (y-2)^2 = 3\left\{\frac{2x+y-1}{\sqrt{4+1}}\right\}^2$$

$\Rightarrow 5(x^2 + y^2 - 2x - 4y + 5\}$

$= 3(4x^2 + y^2 + 1 + 4xy - 2y - 4x)$

$\Rightarrow 7x^2 - 2y^2 + 12xy - 2x + 14y - 22 = 0$

which is the required hyperbola.

Ex.2 Find the lengths of transverse axis and conjugate axis, eccentricity and the co-ordinates of foci and vertices; lengths of the latus rectum, equations of the directrices of the hyperbola $16x^2 - 9y^2 = -144$.

Sol. The equation $16x^2 - 9y^2 = -144$ can be written as $\frac{x^2}{9} - \frac{y^2}{16} = -1$. This is of the form

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

$\therefore\ a^2 = 9, b^2 = 16 \Rightarrow a = 3,\ b = 4$

Length of transverse axis :

The length of transverse axis $= 2b = 8$

Length of conjugate axis :

The length of conjugate axis $= 2a = 6$

Eccentricity : $e = \sqrt{1+\frac{a^2}{b^2}} = \sqrt{1+\frac{9}{16}} = \frac{5}{4}$

Foci : the co- ordinates of the foci are $(0, \pm be)$, i.e., $(0, \pm 4)$

Length of Latus rectum :

The length of latus rectum $= \frac{2a^2}{b} = \frac{2(3)^2}{4} = \frac{9}{2}$

Equation of directrices :

The equation of directrices are $y = \pm \frac{b}{e}$

$$y = \pm \frac{4}{(5/4)} = \pm\frac{16}{5}$$

Ex.3 Find the position of the point $(5, -4)$ relative to the hyperbola $9x^2 - y^2 = 1$.

Sol. Since $9(5)^2 - (-4)^2 - 1 = 225 - 16 - 1 = 208 > 0$ so the point $(5, -4)$ lies outside the hyperbola $9x^2 - y^2 = 1$

Ex.4 The line $5x + 12y = 9$ touches the hyperbola $x^2 - 9y^2 = 9$ at the point

(A) $(-5, 4/3)$ (B) $(5, -4/3)$ (C) $(3, -1/2)$ (D) None of these

Sol.[B] We have : m = Slope of the tangent $= -\frac{5}{12}$

If a line of slope m is tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then the coordinates of the point of contact are

$$\left(\pm\frac{a^2m}{\sqrt{a^2m^2 - b^2}}, \pm\frac{b^2}{\sqrt{a^2m^2 - b^2}}\right)$$

Here, $a^2 = 9$, $b^2 = 1$ and $m = -5/12$

So, points of contact are $\left(\pm 5, \pm\frac{4}{3}\right)$

i.e. $\left(-5, \frac{4}{3}\right)$ and $\left(5, -\frac{4}{3}\right)$.

Out of these two points $\left(5, -\frac{4}{3}\right)$ lies on the line $5x + 12y = 9$. Hence, $\left(5, -\frac{4}{3}\right)$ is the required point.

Ex. 5 The equation of the common tangents to the parabola $y^2 = 8x$ and the hyperbola $3x^2 - y^2 = 3$ is -

(A) $2x \pm y + 1 = 0$ (B) $x \pm y + 1 = 0$

(C) $x \pm 2y + 1 = 0$ (D) $x \pm y + 2 = 0$

Sol.[A] Parabola $y^2 = 8x$

$\therefore\ 4a = 8 \Rightarrow a = 2$

Any tangent to the parabola is

$$y = mx + \frac{2}{m} \qquad ...(i)$$

If it is also tangent to the hyperbola

$\frac{x^2}{1} - \frac{y^2}{3} = 1$ i.e. $a^2 = 1, b^2 = 3$ then

$c^2 = a^2m^2 - b^2 \Rightarrow \left(\frac{2}{m}\right)^2 = 1.m^2 - 3$

or $m^4 - 3m^2 - 4 = 0 \Rightarrow (m^2-4)(m^2+1) = 0$

$\therefore$ $m = \pm 2$ putting for m in (i), we get the tangents as $2x \pm y + 1 = 0$

Ex.6 The locus of the point of intersection of the lines $\sqrt{3}x - y - 4\sqrt{3}k = 0$ and $\sqrt{3}kx + ky - 4\sqrt{3} = 0$ for different values of k is -

(A) Ellipse (B) Parabola

(C) Circle (D) Hyperbola

Sol.[D] $\sqrt{3}x - y = 4\sqrt{3}k$ ...(i)

and $\sqrt{3}kx + ky - 4\sqrt{3} = 0$

$\Rightarrow k(\sqrt{3}x + y) = 4\sqrt{3}$ ...(ii)

To find the locus of their point of intersection eliminate the variable K between the equations from (i) $K = \frac{\sqrt{3}x - y}{4\sqrt{3}}$ and putting in (ii), we get

$(\sqrt{3}x - y)(\sqrt{3}x + y) = (4\sqrt{3})^2$

$3x^2 - y^2 = 48$

or $\frac{x^2}{16} - \frac{y^2}{48} = 1$

Hence the locus is hyperbola

Ex.7 The eccentricity of the conic represented by $x^2 - y^2 - 4x + 4y + 16 = 0$ is -

(A) 1 (B) $\sqrt{2}$ (C) 2 (D) $\frac{1}{2}$

Sol.[B] We have $x^2 - y^2 - 4x + 4y + 16 = 0$

or $(x^2 - 4x) - (y^2 - 4y) = -16$

or $(x^2 - 4x + 4) - (y^2 - 4y + 4) = -16$

or $(x-2)^2 - (y-2)^2 = -16$

or $\frac{(x-2)^2}{4^2} - \frac{(y-2)^2}{4^2} = -1$

i.e. $e^2 = 1 + \frac{a^2}{b^2}$ ($\because$ conjugate hyperbola)

$e^2 = 1 + \frac{4^2}{4^2} \Rightarrow e = \sqrt{2}$

Ex.8 The equation $9x^2 - 16y^2 - 18x + 32y - 151 = 0$ represent a hyperbola -

(A) The length of the transverse axes is 4

(B) Length of latus rectum is 9

(C) Equation of directrix is $x = \frac{21}{5}$ and $x = -\frac{11}{5}$

(D) None of these

Sol.[C] We have $9x^2 - 16y^2 - 18x + 32y - 151 = 0$

$9(x^2 - 2x) - 16(y^2 - 2y) = 151$

$9(x^2 - 2x + 1) - 16(y^2 - 2y + 1) = 144$

$9(x-1)^2 - 16(y-1)^2 = 144$

$\frac{(x-1)^2}{16} - \frac{(y-1)^2}{9} = 1$

Comparing with $\frac{X^2}{a^2} - \frac{Y^2}{b^2} = 1$

where $X = x - 1$, $Y = y - 1$

and $a^2 = 16, b^2 = 9$ so

The length of the transverse axes $= 2a = 8$

The length of the latus rectum $= \frac{2b^2}{a} = \frac{9}{2}$

The equation of the directrix $X = \pm\frac{a}{e}$

$x - 1 = \pm\frac{16}{5} \Rightarrow x = \pm\frac{16}{5} + 1$

$x = \frac{21}{5}; x = -\frac{11}{5}$

Ex.9 For what value of λ does the line $y = 2x + \lambda$ touches the hyperbola $16x^2 - 9y^2 = 144$?

Sol. $\because$ Equation of hyperbola is $16x^2 - 9y^2 = 144$

or $\frac{x^2}{9} - \frac{y^2}{16} = 1$ comparing this with

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we get $a^2 = 9, b^2 = 16$ and

comparing this line $y = 2x + \lambda$ with $y = mx + c$; $m = 2$ & $c = \lambda$

If the line $y = 2x + \lambda$ touches the hyperbola $16x^2 - 9y^2 = 144$

then $c^2 = a^2m^2 - b^2 \Rightarrow \lambda = 9(2)^2 - 16$

$\therefore$ $\lambda = \pm 2\sqrt{5}$

Ex.10 Find the equation of the tangent to the hyperbola $x^2 - 4y^2 = 36$ which is perpendicular to the line $x - y + 4 = 0$.

Sol. Let m be the slope of the tangent since the tangent is perpendicular to the line $x - y + 4 = 0$.

$\therefore$ $m \times 1 = -1 \Rightarrow m = -1$

since $x^2 - 4y^2 = 36$

or $\frac{x^2}{36} - \frac{y^2}{9} = 1$

Comparing this with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$;

$\therefore$ $a^2 = 36$ & $b^2 = 9$ so the equation of tangents are

$y = (-1)x \pm \sqrt{36 \times (-1)^2 - 9}$

$\Rightarrow y = -x \pm \sqrt{27}$ or $x + y \pm 3\sqrt{3} = 0$

LEVEL- 1

Question based on **Equation & Properties of Hyperabola**

Q.1 The vertices of a hyperbola are at (0, 0) and (10, 0) and one of its foci is at (18, 0). The equation of the hyperbola is -

(A) $\frac{x^2}{25}-\frac{y^2}{144}=1$

(B) $\frac{(x-5)^2}{25}-\frac{y^2}{144}=1$

(C) $\frac{x^2}{25}-\frac{(y-5)^2}{144}=1$

(D) $\frac{(x-5)^2}{25}-\frac{(y-5)^2}{144}=1$

Q.2 If the latus rectum of an hyperbola be 8 and eccentricity be $\frac{3}{\sqrt{5}}$, then the equation of the hyperbola is-

(A) $4x^2 - 5y^2 = 100$ (B) $5x^2 - 4y^2 = 100$

(C) $4x^2 + 5y^2 = 100$ (D) $5x^2 + 4y^2 = 100$

Q.3 The foci of the hyperbola $9x^2 - 16y^2 + 18x + 32y - 151 = 0$ are-

(A) (2, 3), (5, 7) (B) (4, 1), (–6, 1)

(C) (0, 0), (5, 3) (D) None of these

Q.4 The foci of the hyperbola $4x^2 - 9y^2 - 36 = 0$ are-

(A) $[\pm\sqrt{11}, 0]$ (B) $[\pm\sqrt{12}, 0]$

(C) $[\pm\sqrt{13}, 0]$ (D) $[0, \pm\sqrt{12}]$

Q.5 Foci of the hyperbola $\frac{x^2}{16}-\frac{(y-2)^2}{9}=1$ are

(A) (5, 2); (–5, 2) (B) (5, 2); (5, –2)

(C) (5, 2); (–5, –2) (D) None of these

Q.6 The eccentricity of a hyperbola passing through the points (3, 0), $(3\sqrt{2}, 2)$ will be-

(A) $\sqrt{13}$ (B) $\frac{\sqrt{13}}{3}$

(C) $\frac{\sqrt{13}}{4}$ (D) $\frac{\sqrt{13}}{2}$

Q.7 Equation of the hyperbola with eccentricity 3/2 and foci at (± 2, 0) is-

(A) $\frac{x^2}{4}-\frac{y^2}{5}=\frac{4}{9}$ (B) $\frac{x^2}{9}-\frac{y^2}{9}=\frac{4}{9}$

(C) $\frac{x^2}{4}-\frac{y^2}{9}=1$ (D) None of these

Q.8 If the centre, vertex and focus of a hyperbola be (0, 0), (4, 0) and (6, 0) respectively, then the equation of the hyperbola is-

(A) $4x^2 - 5y^2 = 8$ (B) $4x^2 - 5y^2 = 80$

(C) $5x^2 - 4y^2 = 80$ (D) $5x^2 - 4y^2 = 8$

Q.9 The eccentricity of the hyperbola can never be equal to-

(A) $\sqrt{\frac{9}{5}}$ (B) $2\sqrt{\frac{1}{9}}$

(C) $3\sqrt{\frac{1}{8}}$ (D) $\sqrt{2}$

Q.10 The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the foci is-

(A) $\frac{4}{3}$ (B) $\frac{4}{\sqrt{3}}$

(C) $\frac{2}{\sqrt{3}}$ (D) None of these

Q.11 If the length of the transverse and conjugate axes of a hyperbola be 8 and 6 respectively, then the difference of focal distances of any point of the hyperbola will be-

(A) 8 (B) 6

(C) 14 (D) 2

Q.12 If m is a variable, the locus of the point of intersection of the lines $\frac{x}{3}-\frac{y}{2}=m$ and $\frac{x}{3}+\frac{y}{2}=\frac{1}{m}$ is a/ an-

(A) parabola (B) ellipse

(C) hyperbola (D) None of these

Q.13 The equation of the hyperbola whose foci are (6, 5) , (– 4, 5) and eccentricity 5/4 is-

(A) $\frac{(x-1)^2}{16} - \frac{(y-5)^2}{9} = 1$

(B) $\frac{x^2}{16} - \frac{y^2}{9} = 1$

(C) $\frac{(x-1)^2}{9} - \frac{(y-5)^2}{16} = 1$

(D) None of these

Q.14 The equation $\frac{x^2}{12-\lambda} + \frac{y^2}{8-\lambda} = 1$ represents

(A) a hyperbola if $\lambda < 8$

(B) an ellipse if $\lambda > 8$

(C) a hyperbola if $8 < \lambda < 12$

(D) None of these

Q.15 The equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a rectangular hyperbola if-

(A) $\Delta \neq 0$, $h^2 > ab$, $a + b = 0$

(B) $\Delta \neq 0$, $h^2 < ab$, $a + b = 0$

(C) $\Delta \neq 0$, $h^2 = ab$, $a + b = 0$

(D) None of these

Q.16 The equation $\frac{x^2}{1-k} - \frac{y^2}{1+k} = 1$, $k > 1$ represents-

(A) circle (B) ellipse

(C) hyperbola (D) None of these

Q.17 If e and e′ be the eccentricities of two conics S and S′ such that $e^2 + e'^2 = 3$, then both S and S′ are-

(A) ellipse (B) parabolas

(C) hyperbolas (D) None of these

Q.18 A point moves in a plane so that its distances PA and PB from two fixed points A and B in the plane satisfy the relation PA – PB = k ($k \neq 0$), then the locus of P is-

(A) a parabola

(B) an ellipse

(C) a hyperbola

(D) a branch of a hyperbola

Q.19 The equation of the conic with focus at (1, –1), directrix along x – y + 1 = 0 and with eccentricity $\sqrt{2}$ is-

(A) $x^2 - y^2 = 1$

(B) $xy = 1$

(C) $2xy - 4x + 4y + 1 = 0$

(D) $2xy + 4x - 4y - 1 = 0$

Q.20 The length of the latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$ is-

(A) $\frac{2a^2}{b}$ (B) $\frac{2b^2}{a}$

(C) $\frac{b^2}{a}$ (D) $\frac{a^2}{b}$

Q.21 The equation $16x^2 - 3y^2 - 32x + 12y - 44 = 0$ represents a hyperbola-

(A) the length of whose transverse axis is $4\sqrt{3}$

(B) the length of whose conjugate axis is 4

(C) whose centre is (–1, 2)

(D) whose eccentricity is $\sqrt{\frac{19}{3}}$

Q.22 The length of the transverse axis of a hyperbola is 7 and it passes through the point (5, –2). The equation of the hyperbola is-

(A) $\frac{4}{49}x^2 - \frac{196}{51}y^2 = 1$

(B) $\frac{49}{4}x^2 - \frac{51}{196}y^2 = 1$

(C) $\frac{4}{49}x^2 - \frac{51}{196}y^2 = 1$

(D) none of these

Q.23 The latus rectum of a hyperbola $\frac{x^2}{16} - \frac{y^2}{p} = 1$ is $4\frac{1}{2}$. Its eccentricity e =

(A) 4/5 (B) 5/4 (C) 3/4 (D) 4/3

Q.24 Consider the set of hyperbola $xy = k$, $k \in R$. Let e_1 be the eccentricity when k = 4 and e_2 be the eccentricity when k = 9. Then $e_1^2 + e_2^2 =$

(A) 2 (B) 3 (C) 4 (D) 1

Q.25 The eccentricity of the hyperbola $-\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is given by -

(A) $e=+\sqrt{\frac{a^2+b^2}{a^2}}$ (B) $e=+\sqrt{\frac{a^2-b^2}{a^2}}$

(C) $e=+\sqrt{\frac{b^2-a^2}{a^2}}$ (D) $e=+\sqrt{\frac{a^2+b^2}{b^2}}$

Q.26 If e and e' be the eccentricities of a hyperbola and its conjugate, then $\frac{1}{e^2}+\frac{1}{e'^2}=$

(A) 0 (B) 1

(C) 2 (D) None of these

Question based on **Line and Hyperbola**

Q.27 The equation of a tangent parallel to y = x drawn to $\frac{x^2}{3}-\frac{y^2}{2}=1$ is-

(A) x – y + 1 = 0 (B) x – y + 2 = 0

(C) x + y – 1 = 0 (D) x – y + 2 = 0

Q.28 The line $lx + my + n = 0$ will be a tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, if -

(A) $a^2l^2 + b^2m^2 = n^2$ (B) $a^2l^2 – b^2m^2 = n^2$

(C) $am^2 – b^2n^2 = a^2l^2$ (D) None of these

Q.29 The equation of tangents to the hyperbola $x^2 – 4y^2 = 36$ which are perpendicular to the line $x-y+4=0$

(A) $y = – x + 3\sqrt{3}$ (B) $y = x – 3\sqrt{3}$

(C) $y = – x \pm 2$ (D) None of these

Q.30 The line y = x + 2 touches the hyperbola $5x^2 – 9y^2 = 45$ at the point-

(A) (0, 2) (B) (3, 1)

(C) (–9/2, –5/2) (D) None of these

Q.31 Equation of tangent to the hyperbola $2x^2 – 3y^2 = 6$ which is parallel to the line y = 3x + 4 is-

(A) y = 3x + 5

(B) y = 3x – 5

(C) y = 3x + 5 and y = 3x – 5

(D) none of these

Q.32 If the straight line $x \cos\alpha + y \sin\alpha = p$ be a tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, then-

(A) $a^2 \cos^2\alpha + b^2 \sin^2\alpha = p^2$

(B) $a^2 \cos^2\alpha – b^2 \sin^2\alpha = p^2$

(C) $a^2 \sin^2\alpha + b^2 \cos^2\alpha = p^2$

(D) $a^2 \sin^2\alpha – b^2 \cos^2\alpha = p^2$

Q.33 The value of m for which y = mx + 6 is a tangent to the hyperbola $\frac{x^2}{100}-\frac{y^2}{49}=1$ is-

(A) $\sqrt{\frac{17}{20}}$ (B) $\sqrt{\frac{20}{17}}$

(C) $\sqrt{\frac{3}{20}}$ (D) $\sqrt{\frac{20}{3}}$

Q.34 Equation of one of common tangent to parabola $y^2 = 8x$ and hyperbola $3x^2 – y^2 = 3$ is-

(A) 2x – y – 1 = 0 (B) 2x – y + 1 = 0

(C) y + 2x + 1 = 0 (D) y – 2x + 1 = 0

LEVEL- 2

Q.1 The latus rectum subtends a right angle at other focus of a hyperbola then its eccentricity is-

(A) $\sqrt{3}+1$ (B) $\sqrt{2}+1$

(C) $-\sqrt{3}+\sqrt{2}$ (D) 2

Q.2 The equation of the hyperbola whose foci are the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ and the eccentricity is 2, is -

(A) $\frac{x^2}{4}+\frac{y^2}{12}=1$ (B) $\frac{x^2}{4}-\frac{y^2}{12}=1$

(C) $\frac{x^2}{12}+\frac{y^2}{4}=1$ (D) $\frac{x^2}{12}-\frac{y^2}{4}=1$

Q.3 A tangent to a hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ intercepts a length of unity from each of the coordinate axes, then the point (a, b) lies on the-

(A) $x^2 - y^2 = 2$ (B) $x^2 - y^2 = 1$

(C) $x^2 - y^2 = -1$ (D) none of these

Q.4 A common tangent to $9x^2 - 16y^2 = 144$ and $x^2 + y^2 = 9$ is -

(A) $y = \frac{3}{\sqrt{7}}x + \frac{15}{\sqrt{7}}$ (B) $y = 3\sqrt{\frac{2}{7}}x + \frac{15}{\sqrt{7}}$

(C) $y = 2\sqrt{\frac{3}{7}}x + 15\sqrt{7}$ (D) none of these

Q.5 The product of the lengths of the perpendiculars drawn from foci on any tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is -

(A) a^2 (B) b^2

(C) a^2b^2 (D) a^2 / b^2

Q.6 The area of quadrilateral formed by focii of hyperbola $\frac{x^2}{4}-\frac{y^2}{3}=1$ and its conjugate hyperbola is-

(A) 14 (B) 24

(C) 12 (D) None of these

Q.7 The equations to the common tangents to the two hyperbolas $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$ are-

(A) $y = \pm x \pm \sqrt{b^2 - a^2}$

(B) $y = \pm x \pm \sqrt{a^2 - b^2}$

(C) $y = \pm x \pm (a^2 - b^2)$

(D) $y = \pm x \pm \sqrt{a^2 + b^2}$

Q.8 A hyperbola has axes along coordinate axes. Its transverse axis is 2a and it passes through (h,k) then its eccentricity is-

(A) $\sqrt{\frac{h^2+k^2+a^2}{h^2-a^2}}$ (B) $\sqrt{\frac{h^2-a^2}{h^2+k^2+a^2}}$

(C) $\sqrt{\frac{h^2+k^2-a^2}{h^2-a^2}}$ (D) $\sqrt{\frac{h^2-a^2}{h^2+k^2-a^2}}$

Q.9 If the focii of the ellipse $\frac{x^2}{k^2a^2}+\frac{y^2}{a^2}=1$ and the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{a^2}=1$ coincides then value of k =

(A) $\pm\sqrt{3}$ (B) $\pm\sqrt{2}$

(C) $\sqrt{3}$ (D) $\sqrt{2}$

Q.10 The locus of the point of intersection of the lines bxt – ayt = ab and bx + ay = abt is -

(A) a parabola (B) an ellipse

(C) a hyperbola (D) None of these

Q.11 Let LL′ be the latus rectum through the focus S of a hyperbola and A′ be the farther vertex of the conic. If $\Delta A'LL'$ is equilateral then its eccentricity e =

(A) $\sqrt{3}$ (B) $\sqrt{3}+1$

(C) $(\sqrt{3}+1)/\sqrt{2}$ (D) $(\sqrt{3}+1)/\sqrt{3}$

Q.12 If the latus rectum subtends a right angle at the centre of the hyperbola then its eccentricity is

(A) e = ($\sqrt{13}$)/ 2 (B) e = ($\sqrt{5}$ –1) /2

(C) e = ($\sqrt{5}$ + 1)/2 (D) e = ($\sqrt{3}$ + 1)/2

Q.13 The equation x = $\frac{e^t + e^{-t}}{2}$; y = $\frac{e^t - e^{-t}}{2}$; t ∈ R represents

(A) an ellipse (B) a parabola

(C) a hyperbola (D) a circle

Q.14 If the tangent at the point (2 sec θ, 3 tan θ) of the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$ is parallel to 3x – y + 4 = 0, then the value of θ is-

(A) 45° (B) 60° (C) 30° (D) 75°

Q.15 The ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and the hyperbola $\frac{x^2}{25} - \frac{y^2}{16} = 1$ have in common-

(A) centre only

(B) centre, foci and directrices

(C) centre, foci and vertices

(D) centre and vertices only

Q.16 The tangents to the hyperbola $x^2 - y^2 = 3$ are parallel to the straight line 2x + y + 8 = 0 at the following points-

(A) (2, 1) (B) (2, –1)

(C) (–2, 1) (D) (–2, –1)

Q.17 P is a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, N is the foot of the perpendicular from P on the transverse axis. The tangent to the hyperbola at P meets the transverse axis at T. If O is the centre of the hyperbola, then OT.ON is equal to -

(A) e^2 (B) a^2 (C) b^2 (D) $\frac{b^2}{a^2}$

Q.18 If e_1, e_2 are the eccentricities of the ellipse $\frac{x^2}{18} + \frac{y^2}{4} = 1$ and the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ respectively, then the relation between e_1 and e_2 is -

(A) $3e_1^2 + e_2^2 = 2$ (B) $e_1^2 + 2e_2^2 = 3$

(C) $2e_1^2 + e_2^2 = 3$ (D) $e_1^2 + 3e_2^2 = 2$

Q.19 The value of m for which line y = mx + 2 $\sqrt{5}$ touches the hyperbola $16x^2 - 9y^2 = 144$ are the roots of the equation $x^2 - (a + b)x - 4 = 0$, then (a + b) is equal to-

(A) 2 (B) 4

(C) 0 (D) none of these

Q.20 The area of triangle formed by lines $x^2 - y^2 = 0$ and any tangent to the hyperbola $x^2 - y^2 = a^2$ is-

(A) $2a^2$ (B) $4a^2$

(C) a^2 (D) None of these

Q.21 If the distances between the foci and the distance between the directrices of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are in the ratio 3: 2 then a : b is-

(A) $\sqrt{2} : 1$ (B) $\sqrt{3} : \sqrt{2}$

(C) 1 : 2 (D) 2 : 1

Q.22 If the eccentricity of the hyperbola $x^2 - y^2 \sec^2 \alpha = 5$ is $\sqrt{3}$ times the eccentricity of the ellipse $x^2 \sec^2 \alpha + y^2 = 25$, then a value of α is-

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

Q.23 If the tangent at the point P(a sec α, b tan α) to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ meets its transverse axis at T, then ST (S focus) must be equal to

(A) a(e – cos α) (B) b(e + cosα)

(C) a(e + cos α) (D) $\sqrt{a^2e^2 + b^2 \cot^2 \alpha}$

Assertion-Reason: (Q. No. 24 to 27)

The following questions given below consist of an "Assertion" (1) and "Reason "(2) Type questions. Use the following key to choose the appropriate answer.

(A) Both (1) and (2) are true and (2) is the correct explanation of (1)

(B) Both (1) and (2) are true but (2) is not the correct explanation of (1)

(C) (1) is true but (2) is false

(D) (1) is false but (2) is true

Q.24 **Statement-(1):** If $P(x_1, y_1)$ is a point on $b^2x^2 + a^2y^2 = a^2b^2$ then area $\Delta SPS' = ae\sqrt{a^2 - x_1^2}$

Statement-(2) : A tangent to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ meets the transverse and conjugate axis in P and Q then $\frac{a^2}{CP^2} - \frac{b^2}{CQ^2} = 1$, where C is the centre of the conic. Which of the statements is correct?

(A) both 1 and 2 (B) only 1

(C) only 2 (D) neither 1 nor 2

Q.25 **Statement-(1) :** The conic $16x^2 - 3y^2 - 32x + 12y - 44 = 0$ represent a hyperbola.

Statement- (2) : The square of the coefficient of xy is greater than the product of the coefficient of x^2 & y^2 and $\Delta \neq 0$.

Q.26 **Statement-(1):** The latus-rectum of the hyperbola $x^2 - y^2 = a^2$ is equal to the length of its transverse axis.

Statement-(2): The semi latusrectum of the ellipse $b^2x^2 + a^2y^2 = a^2b^2$ is equal to $\frac{b^2}{a}$.

Q.27 **Statement- (1) :** The equation $x^2 + 2y^2 + \lambda xy + 2x + 3y + 1 = 0$ can never represent a hyperbola.

Statement- (2): The general equation of second degree represents a hyperbola if $h^2 > ab$.

Passage : (Q.No.28 & 29)

If parametric equation of hyperbola is $x = \frac{e^t + e^{-t}}{2}$ & $y = \frac{e^t - e^{-t}}{3}$ then.

Q.28 Eccentricity of hyperbola is

(A) $\frac{\sqrt{13}}{2}$ (B) $\frac{\sqrt{13}}{3}$ (C) $\frac{3}{2}$ (D) $\sqrt{13}$

Q.29 Eccentric angle of point $\left(2, \frac{2}{\sqrt{3}}\right)$ on hyperbola

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$ (D) None of these

Column Matching Questions: (Q.No.30 to 31)

Q.30 Match the column

Column I	**Column II**
(A) The number of tangents from (1, –1) to the hyperbola $9x^2 - 16y^2 = 144$	(P) 1
(B) The number of tangents from (–1, 1) to the ellipse $16x^2 + 9y^2 = 144$	(Q) 4
(C) If e_1 and e_2 are the eccentricities of $xy = c^2$ and $x^2 - y^2 = a^2$ then $e_1^2 + e_2^2 =$	(R) 3
	(S) 0
	(T) 2

Q.31 Match the following curves with their respective parameter points:

Column I	**Column II**
(A) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	(P) $\frac{a}{2}\left(t + \frac{1}{t}\right), \frac{b}{2}\left(t - \frac{1}{t}\right)$
(B) $xy = c^2$	(Q) $\left(ct, \frac{c}{t}\right)$
(C) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	(R) (a sec t, b tan t)
	(S) (c tan t, c cot t)
	(T) (a cos t, b sin t)

LEVEL- 3

(Question asked in previous AIEEE and IIT-JEE)

SECTION -A

Q.1 The latus rectum of the hyperbola $16x^2 - 9y^2 = 144$ is- **[AIEEE-2002]**

(A) 16/3 (B) 32/3

(C) 8/3 (D) 4/3

Q.2 The foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide. Then the value of b^2 is- **[AIEEE- 2003]**

(A) 9 (B) 1

(C) 5 (D) 7

Q.3 The locus of a point $P(\alpha, \beta)$ moving under the condition that the line $y = \alpha x + \beta$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is- **[AIEEE-2005]**

(A) an ellipse (B) a circle

(C) a parabola (D) a hyperbola

Q.4 The equation of the hyperbola whose foci are (–2, 0) and (2, 0) and eccentricity is 2 is given by - **[AIEEE-2011]**

(A) $x^2 - 3y^2 = 3$ (B) $3x^2 - y^2 = 3$

(C) $-x^2 + 3y^2 = 3$ (D) $-3x^2 + y^2 = 3$

SECTION -B

Q.1 If $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$ represents family of hyperbolas, where α varies then- **[IIT Scr.2003/AIEEE-07]**

(A) e remains constant

(B) abscissas of foci remain constant

(C) equation of directrices remain constant

(D) abscissa of vertices remain constant

Q.2 The point at which the line $2x + \sqrt{6}y = 2$ touches the curve $x^2 - 2y^2 = 4$, is- **[IIT Scr. 2004]**

(A) $(4, -\sqrt{6})$ (B) $(\sqrt{6}, 1)$

(C) $\left(\frac{1}{2}, \frac{1}{\sqrt{6}}\right)$ (D) $\left(\frac{\pi}{6}, \pi\right)$

Q.3 If a hyperbola passes through the focus of the $\frac{x^2}{25} + \frac{y^2}{16} = 1$and its transverse and conjugate axes coincide with the major and minor axis of ellipse, and product of eccentricities is 1, then **[IIT-2006]**

(A) Focus of hyperbola is (5, 0)

(B) Focus of hyperbola is $(5\sqrt{3}, 0)$

(C) The equation of hyperbola is $\frac{x^2}{9} - \frac{y^2}{25} = 1$

(D) The equation of hyperbola is $\frac{x^2}{9} - \frac{y^2}{16} = 1$

Q.4 A hyperbola, having the transverse axis of length $2\sin\theta$, is confocal with the ellipse $3x^2 + 4y^2 = 12$. Then its equation is- **[IIT-2007]**

(A) $x^2\text{cosec}^2\theta - y^2\sec^2\theta = 1$

(B) $x^2\sec^2\theta - y^2\text{cosec}^2\theta = 1$

(C) $x^2\sin^2\theta - y^2\cos^2\theta = 1$

(D) $x^2\cos^2\theta - y^2\sin^2\theta = 1$

Q.5 An ellipse intersects the hyperbola $2x^2 - 2y^2 = 1$ orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the ellipse are along the coordinate axes, then **[IIT-2009]**

(A) equation of ellipse is $x^2 + 2y^2 = 2$

(B) the foci of ellipse are $(\pm 1, 0)$

(C) equation of ellipse is $x^2 + 2y^2 = 4$

(D) the foci of ellipse are $(\pm\sqrt{2}, 0)$

Q.6 The line $2x + y = 1$ is tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If this line passes through the point of intersection of the nearest directrix and the x-axis, then the eccentricity of the hyperbola is - **[IIT-2010]**

(A) 2 (B) 3 (C) 4 (D) $\sqrt{3}$

Passage : (Q.7 to Q.8)

The circle $x^2 + y^2 - 8x = 0$ and hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ intersect at the points A and B **[IIT-2010]**

Q.7 Equation of a common tangent with positive slope to the circle as well as to the hyperbola is -

(A) $2x - \sqrt{5}\ y - 20 = 0$ (B) $2x - \sqrt{5}\ y + 4 = 0$

(C) $3x - 4y + 8 = 0$ (D) $4x - 3y + 4 = 0$

Q.8 Equation of the circle with AB as its diameter is

(A) $x^2 + y^2 - 12x + 24 = 0$

(B) $x^2 + y^2 + 12x + 24 = 0$

(C) $x^2 + y^2 + 24x - 12 = 0$

(D) $x^2 + y^2 - 24x - 12 = 0$

Q.9 Let the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be reciprocal to that of the ellipse $x^2 + 4y^2 = 4$. If the hyperbola passes through a focus of the ellipse, then - **[IIT-2011]**

(A) the equation of the hyperbola is $\frac{x^2}{3} - \frac{y^2}{2} = 1$

(B) a focus of the hyperbola is (2, 0)

(C) the eccentricity of the hyperbola is $\sqrt{\frac{5}{3}}$

(D) the equation of the hyperbola is $x^2 - 3y^2 = 3$

ANSWER KEY

LEVEL-1

Q.No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
Ans.	B	A	B	C	A	B	A	C	B	C	A	C	A
Q.No.	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**
Ans.	C	A	D	C	C	C	A	D	C	B	C	D	B
Q.No.	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**					
Ans.	A	B	A	C	C	B	A	B,C					

LEVEL-2

Q.No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
Ans.	B	B	B	B	B	A	B	C	A	C	D	C	C
Q.No.	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**
Ans.	C	D	B,C	B	C	C	C	A	B	A,C	C	A	A
Q.No.	**27**	**28**	**29**										
Ans.	D	B	C										

(30) A → T ; B → S ; C → Q **(31) A → T ; B → Q, S ; C → P, R**

LEVEL-3

SECTION-A

Qus.	**1**	**2**	**3**	**4**
Ans.	B	D	D	B

SECTION-B

Qus.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Ans.	B	A	A,D	A	A,B	A	B	A	B,D